ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

MISCELLANEOUS PAPERS,

1877—1887.

CAMBRIDGE:

1888.

THE papers here collected for convenience of reference form a part of the smaller publications prepared during the last ten years, either as a portion of the work of this Observatory, or as conveying the results of inquiries undertaken by its officers individually. The present collection includes papers separate copies of which were printed for distribution by their authors, and which are of suitable form to be bound together.

An asterisk is prefixed to such titles in the following list as correspond to papers contained in the present collection. The others are excluded, owing to the want of a sufficient supply of copies available for binding. Additional copies of most of the papers here collected, and of a few of the remainder, can for the present be supplied to those interested in the particular subjects to which they relate.

EDWARD C. PICKERING.

CAMBRIDGE, U. S.,
November 28, 1887.

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11.* On the Present State of the Question of Standards of Length. By W. A. ROGERS. (Proc. Am. Acad. of Arts and Sciences, XV. 273.)

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13. Description of a New Position Micrometer. By LEONARD WALDO. (Am. Journ. of Science and Arts, July, 1880; 3d Ser., XX. 49.)

14. On Tolles's Interior Illuminator for Opaque Objects. By W. A. ROGERS. (Journal of Royal Microscopical Society, III. 754.)

15. The Coefficient of Safety in Navigation. By W. A. ROGERS. (Proc. U. S. Naval Institute, VII. Nos. 3 and 4. Science, April, 1881; II. 171.)

16. On the Conditions of Success in the Construction and the Comparison of Standards of Length. By W. A. ROGERS. (Proc. Am. Soc. Micros., 1882, p. 231.)

17.* New Planetary Nebulæ. By EDWARD C. PICKERING. (Am. Journ. of Science, Oct. 1880; CXX. 303. The Observatory, March, 1881; IV. 81.)

18.* Thirty-fifth Annual Report of the Director of the Astronomical Observatory of Harvard College, presented to the Visiting Committee Dec. 6, 1880. (Cambridge, 1881. 8vo.)

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20.* Large Telescopes. By EDWARD C. PICKERING. (Proc. Am. Acad. of Arts and Sciences, XVI. 364. Nature, Aug. 1881; XXIV. 389.)

21.* Photometric Measurements of the Variable Stars *β Persei* and DM. 81° 25, made at the Harvard College Observatory. E. C. PICKERING, ARTHUR SEARLE, and O. C. WENDELL, Observers. (Proc. Am. Acad. of Arts and Sciences, XVI. 370.)

22.* Report of the Committee on Standards of Stellar Magnitude. (Proc. Am. Assoc. for Advancement of Science, 1881, XXX. 1.)

23.* On a Convenient Method of Expressing micrometrically the Relation between English and Metric Units of Length on the

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27. On a Method of Reducing Different Catalogues of Stars to a Homogeneous System. By W. A. ROGERS. (Proc. Am. Assoc. for Advancement of Science, 1882; XXXI. 99.)

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57. A Preliminary Examination of Metal Thermometers. By W. A. ROGERS. (Am. Meteor. Journ., Oct. 1885, p. 252.)

58. Description of a Series of Fifty Slides, illustrating the Action of a Diamond in Ruling upon Glass, presented to the Royal Microscopical Society of London. By W. A. ROGERS. (Journ. Roy. Micros. Soc., Oct. 1885.)

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60. On the Original Graduation of the Harvard College Meridian Circle, in Situ. By W. A. ROGERS and GEO. B. CLARK. (Proc. Am. Assoc. for the Advancement of Science, 1884; XXXIII. 111.)

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66.* Third Report of the Committee on Standards of Stellar Magnitude. EDWARD C. PICKERING, Chairman. (Proc. Am. Assoc. for the Advancement of Science, 1885, XXXIV. 1.)

67.* Possibility of Errors in Scientific Research due to Thought Transference. By EDWARD C. PICKERING. (Am. Soc. Psychical Research, I. 235.)

68.* Fortieth Annual Report of the Director of the Astronomical Observatory of Harvard College, presented to the Visiting Committee Dec. 3, 1885. (Cambridge, 1886. 8vo.)

69.* A Plan for the Extension of Astronomical Research. By EDWARD C. PICKERING. (Cambridge, 1886.)

70. Report on the Investigation of the Units of Length employed by the Pratt and Whitney Co. By W. A. ROGERS. 1886.

71. International Electrical Exhibition, 1884. Report of Sub-Committee on Railway Signals. W. A. ROGERS, Chairman. 1886.

72.* Accurate Mountain Heights. By EDWARD C. PICKERING. (Appalachia, IV. 215. Science, VII. 423.)

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76.* Observations of Variable Stars in 1886. By EDWARD C. PICKERING. (Proc. Am. Acad. of Arts and Sciences, XXII. 380.)

77.* Harvard College Observatory. Form of Records.

78.* Forty-second Annual Report of the Director of the Astronomical Observatory of Harvard College, presented to the Visiting Committee Dec. 2, 1887. (Cambridge, 1887. 8vo.)

[From Appalachia, Vol. I, No. 3.]

THE MICROMETER LEVEL.

By E. C. Pickering.

[From Appalachia, Vol. I, No. 3.]

THE MICROMETER LEVEL.

BY E. C. PICKERING.

Read November 8, 1876.

During last summer my work for the Club was mainly in the direction of topography. From Jefferson Hill, the Flume House and Jackson as headquarters, trips were taken to various points from which extended views were obtainable. The vertical and horizontal angles of all the principal summits visible were then measured by a micrometer level. This instrument consists of a surveyor's level with one end of the telescope on a hinge, and the other end supported by a micrometer screw. Instead of using a tripod, the instrument rested on a small cast iron triangle, and was placed directly on a projecting rock. Great steadiness was thus obtained. The instrument in its box weighed about twelve pounds, or fifteen pounds with the additional protection required for the more exposed summits. The probable error of the vertical angles was only about 6″, that of the horizontal angles, about 4′. The readings were made very rapidly, almost as fast as a companion could record. When alone on a mountain and making my own records, 72 vertical and 58 horizontal angles were measured in 46 minutes, or at the rate of 170 per hour. The positions of the stations, with the number of vertical and horizontal angles observed, are given below:

	Vertical angles.	*Horizontal angles.*
Israel River Bridge	30	27
Plaisted House (piazza)	65	56
" " (window)	241	67
R. R. Bridge, Lancaster	43	43
Hill near Jefferson	32	31
Bray's Hill	105	107
Jefferson Hill	67	62
Boy Mt.	75	74
Starr King	118	115
Owl's Head	146	135
Mt. Adams	223	215
Mt. Pleasant House	40	34
Mrs. Pendexter's, N. Conway	71	57
Intervale Station	20	15
Northern Kiarsarge (P. 1. 1)	194	178
Mt. Pleasant	123	109

	Vertical angles.	*Horizontal angles.*
Mt. Washington	70	66
Mt. Jefferson	260	208
Twin Mt. House	64	53
Flume House	18	20
Mt. Pemigewasset	76	70
Mt. Lafayette	185	164
Mt. Liberty	172	151
S. Doublehead C. S.	139	111
N. Moat	200	184
S. Moat	87	85
Chocorua	119	113
Conway Corner, R. R. Station	27	24
Thorn Mt.	155	138
Mt. Willey	197	190
Glen Station	53	48
Bemis Station	7	11
Upper Bartlett	28	26
Iron Mt.	83	80
Thorn Mt. House	41	37

The total number of vertical angles observed during my stay at Jefferson Hill was 1145, of horizontal angles, 932. On the trip to attend the field meeting of the Club, 802 vertical, and 720 horizontal angles were measured. At the Flume House, during a week's visit, 451 vertical, and 405 horizontal angles, were taken, and at Jackson, 1136 vertical, and 1047 horizontal. In all, 3534 vertical and 3104 horizontal, or 6638 angles. The portability and rapidity of the instrument is shown by the fact that in almost every case, leaving the hotel in the morning, the mountain was ascended, the observations taken, and the return effected, before dark. The observations were therefore made during the three or four hours which could be passed on the summit.

In one case only, on Mt. Adams, was a night passed in camp. Two other trips occupied two days. Leaving the Mt. Pleasant House in the morning, Mt. Pleasant was reached by the Crawford Path, and the night spent on Mt. Washington. An early start the following morning gave us a morning on Mt. Jefferson, and an hour or so on Mt. Adams. I reached Jefferson Hill the same evening on foot, at about nine o'clock. This amount of work would have been quite impracticable without the aid of

my friend Mr. G. C. Mann, in recording the observations and helping carry the micrometer level. On the other two-day trip, leaving Jackson in the morning, and taking the cars to the Intervale Station, we reached the top of N. Moat at an early hour. A walk over the ridge to S. Moat, across the Swift River and over the eastern ridge of Chocorua, brought us to the foot of that mountain by nightfall. The next day we ascended Chocorua, walked back to Conway Centre, and returned by cars and stage to Jackson the same evening. This trip gave 839 angles, 556 on the first day. Of the other trips, the ascent of Mt. Liberty is described in APPALACHIA No. 2, p. 122, and that of Mt. Willey, p. 120.

The other methods of determining mountain elevations are by levelling, by measuring vertical angles with a theodolite, and by a barometer or boiling point apparatus. The first of these can seldom be used from its slowness, as often several days would be required to determine the height of a single summit. The second method, which is that employed by the Coast Survey, differs from the one described here only in the form of instrument used. A more expensive and heavier instrument is required, and the work is much less rapid. As the uncertainty of atmospheric refraction is the principal source of error, the advantage of the superior accuracy of the larger theodolites is lost. The shorter lines of sight employed with the micrometer level, owing to the greater frequency of the stations occupied, gives it also a marked advantage. The barometer is open to the objection of large errors, and requires a visit to every point to be measured. A comparison of the work of the two instruments shows that more elevations can be determined with the micrometer level in a single day, than in a whole season with the barometer, and the errors will also be much less with the former instrument. A traveller may determine the altitudes of an entire group of mountains by ascending two of them and reading the angles with a micrometer level. The horizontal angles will furnish their position, and the vertical angles will give two independent measurements of their altitudes. It is of course necessary to determine the horizontal positions of two, and the altitude of one, observed point.

AN ADDRESS

ON

THE ENDOWMENT OF RESEARCH,

READ BEFORE THE

AMERICAN ASSOCIATION

FOR THE

ADVANCEMENT OF SCIENCE.

AT NASHVILLE, TENN,

AUGUST, 1877.

BY PROF. EDWARD C. PICKERING,
VICE PRESIDENT, SECTION A.

SALEM:
PRINTED AT THE SALEM PRESS.
1877.

ADDRESS

OF

PROFESSOR EDWARD C. PICKERING,

VICE PRESIDENT, SECTION A.

FELLOW MEMBERS OF THE ASSOCIATION —

LADIES AND GENTLEMEN : —

IT is a great disappointment to me that I am unable to address you this evening in person. But the heat, and length of the journey warn me that it is safer to remain at home. I have looked forward for some time to this opportunity of calling the attention of the Association to a subject in which many of you are, I know, already much interested, the Endowment of Research. We all admit that this is one of the great objects of this Association, as of almost all scientific bodies. But what steps are we taking to aid this object, beyond looking on with interest while our members and others investigate various scientific problems, listening approvingly to the results, and publishing the papers? Doubtless it will be said, and with justice, that we cannot do more at present for lack of means. But the question I wish to raise lies deeper than this. Many persons admit that there are enough public-spirited and liberal men in the community to furnish the money needed for any object which can be shown to be really worthy. Let us for the moment admit this argument, let us assume that abundant means can be obtained, provided that assurance can be given that the result will justify the act. Not that the institution

shall be made self-supporting or return an income, but that the money shall not be wasted and that the results shall be a continual increase of human knowledge, or improvement of the condition of the human race.

The first obstacle we encounter is the opinion widely maintained even by scientific men, that the original research of a country is a natural growth and that it is useless to try to force it. We might as well say that music and art are natural growths and that it is therefore useless to teach them! What should we have of ancient art were it not for the munificent encouragement of many ancient rulers? In later days how would literature and art have thrived had it not been for the support, often scanty it is true, of the public in purchasing books, paintings and sculpture? With the man of science it is quite different. There is generally little or no pecuniary reward for his success. The public do not eagerly crave each new memoir on the higher mathematics. A crowd does not gather around the bulletin board to read the discovery of a new asteroid or organic radical. The consequence is that since the man of science, like other men, must live, he is obliged to engage in some other occupation, generally teaching, which still allows him apparatus, a little time and generally less means, for research. Under these circumstances should we not be surprised, not that so little is accomplished, but rather that so much volunteer work is done? Can we doubt that far more would be accomplished if these same men were allowed to devote their entire energies to their investigations and were aided by the necessary appliances?

It is said that research is carried on by few and that but one man in a thousand is qualified for such work. But this is the strongest reason for making every effort to render the energy of such men most effective. The obvious solution is organization —carrying out a plan by which research should be rendered as systematic as are the processes of the mechanical arts. Suppose that a man should attempt to build and furnish his own house! He might devote his whole life to the work and still obtain a dwelling very inferior to what would be erected in a few months, by a small portion of that system which we call civilization. Suppose that he wished to travel, and should build his own boat or locomotive! To take a case more like that in question. Compare the old-fashioned watch where the maker was compelled to

make even the wheels himself, with the precision and uniformity of the chronometers of the present day. Notice also the result in scientific work where organization and division of labor has already been introduced. What would be the results of the Coast Survey if those who plan the work were obliged to erect the signals or take the soundings? At present an investigator is obliged to originate the subject, to plan the apparatus in detail and often even to construct it with his own hands. He must also be able to perform the experiment and discuss the results. It is no wonder that so many accomplishments are rarely united in one person. It would, on the other hand, be comparatively easy to find two or three persons who between them could divide these labors. Again, so much time is often spent in preparing the apparatus and making the preliminary observations that the college professor feels that he cannot afford to make the long-continued and oft-repeated experiments needed to bring his work to a proper conclusion. He, therefore, publishes a preliminary paper with the promise, seldom fulfilled, of completing it in the future. Let us take an actual example. Suppose the subject selected is the reflection of light. Much time and skill may be required to devise a suitable photometer and to bring it into proper working condition. When this is once done it is only necessary to find an assistant with good eyes who can repeat over and over the measurements of the light reflected by different substances at various angles of incidence. The work now becomes a simple routine, and if properly organized is not expensive. A large class of scientific work is of this kind. In fact most quantitative investigation, especially that serving to establish physical laws requires oft-repeated measurements of the same quantity. An assistant will often thus obtain even better results than his superior, because his time being less valuable will be less occupied with other matters, and he will be able to concentrate his entire energies on his work. Moreover, an assistant may be selected with special reference to his work, as in the present case for the sensitiveness of his eye.

Before proceeding further let us consider how far the field is already occupied, and what aid is now offered in this country to research. This may be stated in very few words. We have first the munificent bequest of one of the first Presidents of this Association, who thus added at his death another to the long list of

valuable contributions to science, to which his life was devoted. The income of the Bache fund amounts to two or three thousand dollars, and is expended in aiding scientific men in conducting original investigation. Secondly, the Rumford fund, although originally intended merely for giving medals for discoveries in light and heat, is now largely applied to aiding investigation in these sciences. The large income, considering the special nature of the subjects included, enables liberal aid to be extended to any worthy research in light or heat. The Smithsonian Institution, besides its many indirect aids to research, applies a portion of its income directly to this work. With these should be mentioned the Boyden premium, a sum of one thousand dollars offered for many years by the Franklin Institute for a specific investigation. I am informed, however, that no application for this reward has ever been made. The establishment by the Johns Hopkins University of Fellowships of which the incumbents are expected to devote their time mainly to research is an important step in the right direction. Many other colleges indirectly countenance or mildly encourage research, some actively, but most of them passively. Some persons, however, even go so far as to maintain that the time and energy of a college professor is paid for, that he may teach, and regard original work as outside occupation. Were this view general, small indeed would be the growth of science in this country.

I shall confine my remarks to the sciences included in Section A, which is defined by Mathematics, Physics and Chemistry. Mathematics as here used includes in its applications too wide a field for any one person. It would probably be best to omit some portions. Astronomy, for instance, could be better treated at the Observatories now existing. Not that the wants of this science are as yet supplied, but we have already too many unsupported Observatories, and far more could be accomplished with half the number having double the endowment. Geodesy might also be left to the Coast Survey, were it not for the excellent field still open in topography. This is especially the case in connection with the State Surveys which will, doubtless, be established throughout the country as soon as the people are educated up to an appreciation of their value. I cannot forbear in this connection calling attention to the need of a Summer School of Topography, which might furnish valuable results, at the same time

that it would give health and strength during the summer season. The problem of mountain surveying will well repay study, as there is at present no satisfactory solution. The best mountain contour maps are far from representing the actual surface of the ground. Under Mathematics should also be included Mensuration in its various branches and many portions of Engineering. The latter would be so closely allied to the work in Physics, in the subjects of Mechanics and Heat, that it would be difficult to draw the line between them. Every branch of Physics would be easily treated by this method, as in Mechanics, Sound, Light, Heat and Electricity, numerous problems are awaiting an experimental solution. The excellent results attained by students in many of our physical laboratories proves conclusively that assistants could be obtained capable of undertaking work of the greatest precision. Chemistry, besides its ordinary branches, should include the laws of molecular action, thermo-chemistry and the more difficult problems of animal and vegetable chemistry.

The working corps of an institution established for making researches in the subjects named above should be somewhat as follows: —

First, a President, who need not necessarily be an investigator, or even possess great scientific ability. He must have good executive ability, be a judge of men, and understand thoroughly the engineering principles of construction.

Secondly, a corps of investigators, men of acknowledged scientific ability, and selected for the originality of their ideas, even if they have not shown special skill in carrying them into practice. Three men to represent the subjects of Mathematics, Physics and Chemistry, would be capable of carrying on an immense amount of research. Each should have one or two deputies or Assistant Professors, capable of taking their places during temporary absence. Their duties would be mainly planning details, superintending the construction of apparatus, and answering the questions of their subordinates.

Third, a large corps of assistants, whose duty it should be to carry out the work laid out for them, but who would not necessarily be able to plan it. They form the hands, while the preceding class correspond to the head of our organization. Each investigator should be able to provide work for at least ten such assistants. They would generally work in pairs, one observing, the

other recording, and would change places at intervals. Their work would be mainly routine, and would require care and mechanical, rather than intellectual, skill. Many young men would be glad to secure such work temporarily, though it would be permanent for few, as the salary would be low.

Fourth, workmen, such as a mechanician, carpenter, tinman, etc., capable of constructing in wood or metal the apparatus devised. One person should also be employed whose duty it would be to see that the apparatus was always ready for use. This would be especially important for the chronograph, telegraph wires, electric light and other appliances liable to be used by several persons.

The subjects for investigation in each department would in general originate with the Professors in charge. They should also encourage their assistants to suggest subjects, and aid in planning them. Many other scientific men would, doubtless, avail themselves of an opportunity to have their theories tested when unable themselves to perform the necessary experimental work. The plan would in all cases be submitted to the President in writing, with an estimate of its cost, of the apparatus needed and of the probable time required to complete it. If found practicable and approved, the apparatus would be constructed or purchased, and tested under the direction of the Professor by his more skilful assistants. When they were able to obtain accordant results, they would show one or two of the younger assistants precisely how the measurement should be made and carefully supervise their first trials. A long series of results could now be obtained at small expense, with slight supervision and direction as to the most important variations to be tried. The results would finally be reduced and prepared for publication.

Let me now invite you to accompany me on a visit to this supposed Institution, that we may examine its structure more in detail. We shall find it where land is not too valuable, but near enough to some large city that workmen of all kinds may be obtained at short notice. It is set back from the road so as to be free from dust and the jar of heavy vehicles, and commands a distant view in at least one direction. This may be needed for experiments on atmospheric refraction or opacity, on the velocity of light or for many other purposes. A distant lighthouse forms an excellent object for such observations at night. Association with, or at least proximity to, some large college is much to be desired

to avoid duplication of the collections of books and apparatus. Many college professors have under their charge a dozen instruments, each of which could profitably occupy the entire time of one person. These instruments are now simply exhibited to their classes once a year.

The building itself is large but low, and resembles one or more blocks of two-story dwelling houses. No more common mistake is made than in wasting the money which should be used for equipment, on architectural effect. This although greatly desirable in itself, is often out of place in a building devoted to science, and in fact is not unfrequently the cause of serious inconvenience. The windows should be small, that they may be easily darkened by shutters, and the walls should not be too thick, or carry heavy mouldings or cornices, on account of the light. It is useless to hope for architectural beauty in this building, as the effect would be spoiled by attachments which might be made to the exterior. The rooms are numerous but most of them small, as only one or two persons would in general engage in the same research and one experiment would often disturb another. The President and Professors should all live under the same roof with their work, since it may often be necessary that they should be present at any hour of the day or night. Where an observation must be repeated at short intervals, inconvenient working hours must sometimes be maintained for a considerable time.

On entering the building we find that it is arranged like a hotel, with long entries running from one end to the other, and rooms leading off on each side. Two iron rails are laid on the lower entry and continued through a rear door in a straight line to some distance behind the building. By placing a car on them with one wheel graduated, as proposed by the Coast Survey, considerable distances may be measured with the greatest accuracy. These entries will have various other uses, as in photometry, in studying wave-motion, elasticity, resistance of pipes, etc. The cellars should be dry, properly finished and lighted. They would prove most useful for accurate measurements or other work requiring a uniform temperature. They would also contain furnaces, and a small engine for furnishing power throughout the building. Stone piers disconnected with the floors would pass through the building for the support of delicate instruments. Many of these might be attached to a single pier. Numerous pipes are laid under the floors

for carrying water, gas, oxygen, hydrogen, steam, compressed air, etc., to any desired point. Wires are also provided for transmitting electric currents. Batteries would be replaced by a magneto-electric machine driven by the engine which would thus enable the electric light to be supplied at a few minutes notice. Various auxillary small motors as turbines or gas engines, might also be desirable. Time would be transmitted electrically throughout the building, and a chronograph with several barrels would register observations in any portion of the building.

These examples serve to show the system of coöperation by which various appliances, which any investigator may need, might be rendered available for many. Any one who has engaged in such work will realize how much would be saved by having such means of measurement always ready for immediate use. Often nine-tenths of the time is spent in getting instruments ready which have not been used for months, or of which portions are used for other purposes.

I think no one will deny that the scheme here proposed would, if carried out successfully, greatly increase the original research of the country. It would act not only directly, but by stimulating those connected with other institutions. Doubtless, too, many amateurs would be ready to avail themselves of the facilities here collected, and contribute to its support, the money that would thus be saved. Were it desirable to let it conform to the demands of applied science much useful work might be done by offering opportunities for testing new inventions or products. For example, the power and economy of new motors, the strength of new brands of steel or other metals could here be determined with accuracy and economy. The advantages of a corps of unprejudiced observers whose position would place them above the suspicion of partiality, would prove of great value in many cases. As experts in a legal case they would have the advantage of having at hand every appliance for proving the correctness of their statements. But apart from these practical applications, in the realm of pure science no one can deny the value of the results likely to accrue. Not only could the more difficult problems be studied to better advantage, but those more formidable from their extent and now rarely undertaken by a single individual might easily be solved by coöperation. Now their only solution depends on an occasional Government appropriation where large portions

are often lost wilfully, or spent ignorantly by the many hands through which the money passes, before it is brought to bear on the scientific conditions of the problem.

Let us now return for a moment to the question of endowment. It is well known that there is no country in the world where so much money is given by private individuals to the encouragement of education and science. Such persons, not unnaturally wishing to be associated with their gifts, have in many cases established colleges bearing their names. Let no man think that he will now benefit the cause of higher education by so doing. The demand is more than supplied. We have too many colleges with far too little endowment. Each new college seriously injures its neighbors by drawing pupils from them, and if insufficiently endowed no one can be expected to contribute to the glory of another man's name. The consequence is a struggle for existence on the part of what should be active literary institutions, extending, as well as disseminating, human knowledge. The same remarks apply to other literary or scientific institutions, as libraries, observatories, or museums of natural history. No benefit accrues to science from an observatory without a telescope, or from a telescope without an observer. The true patron of science will select an object proportionate to his gift. If he will abundantly endow any one subject, no matter how limited, he will confer a real boon on his fellow men.

Finally, the advantages of establishing the institution which I have described are that it opens a new and unoccupied field. It does not encroach on existing institutions, or in any way injure them. On the contrary, if associated with a college it would prove a great benefit by the increased facilities which would thus be collected together, as furnishing instructive employment for a time to some of the graduates, and as increasing the scientific atmosphere of the place. It would benefit science by a constant extension of its boundaries, and if properly managed should become the headquarters of experimental science in the country. That the details given above are defective and open to criticism I do not doubt, but that the object is laudable I presume no one will deny. Whoever will supply this want will leave the name of one who extended human knowledge and benefited his fellow men.

XII.

ON THE LIMITS OF ACCURACY IN MEASUREMENTS WITH THE MICROSCOPE.

By Professor Edward W. Morley, of Western Reserve College.

Presented Oct. 9, 1878.

The following measurements of rulings on glass, by Mr. Rogers, were made with an objective of two tenths of an inch focus, and a cobweb micrometer. For a description of the ruled plates the reader is referred to page 178 of the present volume of the Proceedings. Light was thrown on the rulings by reflection from clouds: care was taken to have the light as uniform as possible. Such care is necessary in making accurate measurements with a lens of short focus. The screw for fine adjustment was permitted to be moved only through half a revolution during the measurements. The same parts of the micrometer screw were used throughout the measurements of a band. The image of the line ruled on the plate consists of a bright central line, with a darker line on each side; the wires of the micrometer were placed on this central brighter line, and so near its apparent left-hand limit that the bright line included between the dark wire and the dark border of the image of the ruled line was the minimum visible quantity, and was the same for both wires. Care was taken not to look at the index of the micrometer until the coincidence of the wires was finally established; and also to move the wires a considerable quantity before making a second measurement, except in perhaps five cases on the third plate. In two cases the coincidence thus finally established was re-examined after the reading had been taken, on account of divergence from a previous result, and in one of these the coincidence was found to be imperfect. With this exception, the figures given are absolutely the whole of the measurements on the rulings.

Two bands on the first plate, and one each on the second and third plates, were measured twice. The probable difference of two measurements of the same interval was found to be one three hundred and fifty-nine thousandth of an inch; from which the probable error of a single measurement may be presumed to be about two millionths of an inch. It happened that thirty-five of the differences between two measurements of the same space were less than the probable difference as computed by the usual formula, and thirty-five were greater.

The measurements on the third plate were more difficult than the other, partly because the lines were too fine for the easiest work, and partly on account of fatigue. They are, therefore, less satisfactory than the measurements on the other plates. The outer lines of some bands on the second plate were also troublesome, and the results for two or three not so good as for other spaces.

Mr. Rogers made measurements of the same plates, which he prepared for publication without knowing my results, but after the original micrometer readings of my measurements had passed beyond my control. Of his measurements I know nothing at the time of writing the following tabular results. By concert with him, my results are tabulated in the form adopted by him, for ease of comparison. My numbers for the spaces measured increase in the direction of the arrows on the ruled plates, if I have made no mistake, and also my numbers of the bands. The numbers of the plates are those written on them by Mr. Rogers. In the third plate I measured only spaces composed of five of the spaces of one twenty-four hundredth of an inch as ruled; the difficulty of the measurement of so faint lines, as well as the fear of incurring a return of a certain slight difficulty with one of my eyes, from which recovery was not then complete, led me thus to abridge this part of the work. It is to be regretted that this plate was not taken in hand earlier.

The figures in the columns of individual and accumulated errors represent millionths of an inch composed of twenty-four revolutions of the screw of Mr. Rogers's ruling engine. But in the case of the fourth plate they represent hundred-thousandths of a millimetre of a similar standard.

PLATE I.

Space.	Band I. Individual Errors.	Band I. Accumulated Errors.	Band II. Individual Errors.	Band II. Accumulated Errors.	Band III. Individual Errors.	Band III. Accumulated Errors.
1	— 2	— 2	— 3	— 3	+ 3	+ 3
2	— 6	— 8	—10	— 13	—10	— 7
3	—13	— 21	—13	— 26	—27	— 34
4	— 8	— 29	—21	— 47	—16	— 50
5	—16	— 45	—26	— 73	—14	— 64
6	—23	— 68	—28	—101	—31	— 95
7	—18	— 86	—16	—117	—21	—116
8	—27	—113	—24	—141	—29	—145
9	—18	—131	—26	—167	—21	—166
10	— 7	—138	—19	—186	—20	—186
11	—12	—150	—14	—200	—12	—194
12	—12	—162	—12	—212	— 9	—203
13	—12	—174	— 6	—218	— 1	—204
14	— 2	—176	+ 7	—211	— 5	—209
15	+ 4	—172	+12	—199	+12	—197
16	+15	—157	+22	—187	+23	—174
17	+20	—137	+27	—150	+20	—154
18	+19	—118	+15	—135	+26	—128
19	+21	— 97	+20	—115	+28	—100
20	+18	— 79	+26	— 89	+28	— 72
21	+20	— 59	+25	— 64	+23	— 49
22	+20	— 39	+21	— 43	+20	— 29
23	+17	— 22	+19	— 24	+20	— 9
24	+18	— 4	+21	— 3	+16	+ 7
25	+ 3	— 1	+ 3	+ 0	+ 0	+ 7

PLATE I.

Space.	Band IV. Individual Errors.	Band IV. Accumulated Errors.	Band IV. Individual Errors.	Band IV. Accumulated Errors.	Band V. Individual Errors.	Band V. Accumulated Errors.	Band V. Individual Errors.	Band V. Accumulated Errors.	Band VI. Individual Errors.	Band VI. Accumulated Errors.	Band VII. Individual Errors.	Band VII. Accumulated Errors.	Band VIII. Individual Errors.	Band VIII. Accumulated Errors.
1	+11	+ 11	+ 7	+ 7	+ 1	+ 1	+ 2	+ 2	+ 2	+ 2	— 5	— 5	+ 8	+ 8
2	—19	— 8	—14	— 7	—18	— 17	—16	— 14	—18	— 16	—18	— 23	—17	— 9
3	—15	— 23	—15	— 22	—21	— 38	—24	— 38	—22	— 38	—19	— 42	—23	— 32
4	—19	— 42	—23	— 45	—21	— 59	—25	— 63	—29	— 67	—28	— 70	—38	— 70
5	—27	— 69	—18	— 63	—33	— 92	—33	— 96	—33	—100	—36	—106	—39	—109
6	—27	— 96	—27	— 90	—38	—130	—33	—129	—31	—131	—39	—145	—41	—150
7	—22	—118	—16	—106	—28	—158	—37	—166	—25	—156	—32	—177	—32	—182
8	—33	—151	—30	—136	—20	—178	—22	—188	—19	—175	—26	—203	—26	—208
9	—20	—171	—25	—161	—17	—195	—16	—204	—12	—187	—19	—222	—21	—229
10	— 8	—179	—13	—174	—16	—211	—14	—218	—14	—201	—13	—235	—11	—240
11	— 3	—182	— 6	—180	+ 7	—204	+ 8	—210	+ 4	—197	+ 4	—231	+ 5	—235
12	— 2	—184	+ 1	—179	+13	—191	+17	—193	+12	—185	+18	—213	+13	—222
13	+15	—169	+17	—162	+19	—172	+20	—173	+27	—158	+31	—182	+31	—191
14	+21	—148	+22	—140	+30	—142	+29	—144	+32	—126	+26	—156	+27	—164
15	+26	—122	+29	—111	+26	—116	+25	—119	+29	— 97	+30	—126	+36	—128
16	+29	— 93	+28	— 83	+33	— 83	+31	— 88	+28	— 69	+30	— 96	+33	— 95
17	+32	— 61	+29	— 54	+28	— 55	+29	— 59	+25	— 44	+27	— 69	+29	— 66
18	+23	— 38	+20	— 34	+25	— 30	+23	— 36	+18	— 26	+29	— 40	+22	— 44
19	+27	— 11	+25	— 9	+20	— 10	+22	— 14	+13	— 13	+20	— 20	+24	— 20
20	+12	+ 1	+ 5	— 4	+16	+ 6	+11	— 3	+14	+ 1	+16	— 4	+14	— 6

Space	Plate II. Band I. Individual Errors.	Plate II. Band I. Accumulated Errors.	Plate II. Band II. Individual Errors.	Plate II. Band II. Accumulated Errors.	Plate II. Band III. Individual Errors.	Plate II. Band III. Accumulated Errors.	Plate III. Band I. Individual Errors.	Plate III. Band I. Accumulated Errors.	Plate III. Band I. Individual Errors.	Plate III. Band I. Accumulated Errors.	Plate IV. Band I. Individual Errors.	Plate IV. Band I. Accumulated Errors.
1	—17	—17	— 6	— 6	+10	+10	— 5	— 5	+ 3	+ 3	+15	+15
2	— 1	—18	—10	—16	—20	—10	+ 6	+ 1	+ 6	+ 9	+ 1	+16
3	+15	— 3	+ 2	—14	— 7	—17	+ 5	+ 6	+ 8	+17	— 2	+14
4	+ 1	— 2	+ 4	—10	+10	— 7	+ 1	+ 7	— 3	+14	—10	+ 4
5	— 9	—11	—11	—21	+ 1	— 6	— 2	+ 5	+ 3	+17	+ 7	+11
6	+ 6	— 5	— 6	—27	+ 2	— 4	+12	+17	+ 6	+23	— 3	+ 8
7	+ 6	+ 1	— 7	—34	—13	—17	+ 2	+19	+ 4	+27	— 7	+ 1
8	— 9	— 8	+ 1	—33	— 7	—24	+15	+34	+16	+43	— 6	— 5
9	— 3	—11	+ 4	—29	+ 7	—17	— 2	+32	+ 3	+46	+ 5	+ 0
10	+ 1	—10	+ 1	—28	— 2	—19	— 6	+26	— 6	+40	— 1	— 1
11	— 5	—15	— 4	—32	— 8	—27	— 6	+20	— 3	+37	— 5	— 6
12	— 1	—16	+ 0	—32	+10	—17	+ 2	+22	— 4	+33	+10	+ 4
13	— 3	—19	—21	—53	— 6	—23	+ 2	+24	— 2	+31	+16	+20
14	— 4	—23	+ 3	—50	— 5	—28	+ 9	+33	+ 0	+31	— 5	+15
15	+ 5	—18	— 2	—52	— 3	—31	—14	+19	— 8	+23	—12	+ 3
16	— 4	—14	+13	—39	— 3	—34	— 5	+14	+ 0	+23	+ 2	+ 5
17	+15	+ 1	+23	—16	+12	—22	— 9	+ 5	—10	+13	—10	— 5
18	— 2	— 1	— 2	—18	+ 7	—15	— 9	— 4	— 7	+ 6	+13	+ 8
19	— 2	— 3	+ 5	—13	+ 1	—14	— 5	— 9	— 6	+ 0	+ 5	+13
20	+ 7	+ 4	+ 4	— 9	+11	— 3	+12	+ 3	+ 3	+ 3	—12	+ 1
21	— 1	+ 3	+ 1	— 8	+ 3	+ 0						
22	— 2	+ 1	— 6	—14	— 5	— 5						
23	— 1	+ 0	+ 1	—13	+ 4	— 1						
24	+ 6	+ 6	+ 5	— 8	+ 0	— 1						
25	— 4	+ 2	+ 2	— 6	— 1	— 2						

XIII.

ON THE LIMITS OF ACCURACY IN MEASUREMENTS WITH THE TELESCOPE AND THE MICROSCOPE.

BY PROFESSOR WILLIAM A. ROGERS.

Presented Oct. 9, 1878.

IT is often desirable in astronomical observations to assign to a given result the degree of precision which the observations will justify. Usually the limit of precision is defined either by the probable error of a single observation, or of the mean of a given number of observations.

Let

x = any given numerical value.
n = the number of values of x.
v = the difference between each value of x and the arithmetical mean of all the values.
$[v]$ = the sum of the separate residuals, without regard to sign.
r = the probable error of a single value.
r_0 = the probable error of the arithmetical mean.

We shall then have, —

$$r = .6745\sqrt{\frac{[v\,v]}{m-1}}$$
$$r_0 = .6745\sqrt{\frac{[v\,v]}{m\,(m-1)}} \qquad (a)$$

Or, according to Peters, —

$$r = .8453\ \frac{[v]}{\sqrt{m\,(m-1)}}$$
$$r_0 = .8453\ \frac{[v]}{m\sqrt{(m-1)}} \qquad (b)$$

As an illustration, we assume the following values of x, *without defining their signification*: —

x	v	$v v$
61.70″	+ .05″	+ .02″
61.50	+ .25	+ .06
60.90	+ .85	+ .72
61.70	+ .05	+ .02
61.30	+ .45	+ .20
61.20	+ .55	+ .30
60.80	+ .95	+ .90
61.90	— .15	+ .02
61.60	+ .15	+ .03
61.50	+ .25	+ .06
62.80	— 1.05	+ 1.10
62.70	— .95	+ .90
60.80	+ .95	+ .90
61.30	+ .45	+ .20
63.00	— 1.25	+ 1.56
61.10	+ .65	+ .42
63.90	— 2.15	+ 4.62
Mean, 61.75		

From equations (a) we have, —

$$r = \pm .6745\sqrt{\frac{12.03}{16}} = \pm .585''$$

$$r_0 = \pm .6745\sqrt{\frac{12.03}{16 \times 17}} = \pm .142''$$

And from equations (b), —

$$r = \pm \frac{.8453 \times 11.15}{\sqrt{17 \times 16}} = \pm .571''$$

$$r_0 = \pm \frac{.8453 \times 11.15}{17\sqrt{16}} = \pm .139''$$

If we reject the last value of x, viz. 63″.90, we have: —

From (a), —

$$r = \pm .474''$$

$$r_0 = \pm .119$$

From (b), —

$$r = \pm .491$$

$$r_0 = \pm .123$$

Let us now inquire what interpretation can be safely given to these

values of r and r_0, and what conclusions can be drawn therefrom concerning the precision of x.

First, it will be seen that the two formulæ do not give precisely the same results, and the relation between the results is changed by rejecting one apparently discordant observation. The difference is, however, insignificant when compared with the actual error of observation. In general, the agreement will be more perfect the greater the number of values of x.

Second, it is obvious that if for x we write, $x \pm$ a constant, the values of v will not be thereby changed; hence the values of r and r_0 *will give no indication whatever with reference to the existence of any constant error involved in the values of x.*

Admitting, then, that there is no constant error in the given series, what degree of precision can be assigned to any single value of x, and to the mean value 61″.75?

It would hardly seem necessary to call attention to the erroneous assumption that, since the value of r is $\pm$ .57″, therefore no single value can be greater than 62″.32, nor less than 61″.18; or that since r_0 is $\pm$ .14″, therefore the value 61″.75 is true within this limit. The refutation of the first assumption is made sufficiently easy by an examination of the separate values of x, but it is not quite so easy to show the fallacy of the second.

Notwithstanding the absurdity of attempting to assign to the arithmetical mean the degree of precision indicated by the value of r_0, observers of limited experiences are continually found doing this, and the writer recalls two instances in which professional astronomers have committed themselves to the same fallacy.

In general, it is entirely unsafe to draw conclusions with respect to the degree of precision to be attached to the arithmetical mean from the magnitude of the probable error, until the signification of the values from which it is derived is defined.

If the values of x are found by successive readings of the four microscopes of a meridian circle for the same position of the telescope, the separate values are *simple functions* of the quantity required, and involve only the accidental errors of the observer, either in making the bisections of the divisions of the circle, or in reading the index of the micrometer screws. In this case the probable error of the mean is a tolerably accurate indication of the degree of precision which may be attached to it.

But the values of x given, represent the observed index errors of the meridian circle of Harvard College Observatory, as derived from

separate fundamental stars, observed January 5, 1872. They are given on page xxiii., Vol. X., of the Annals of the Observatory. Here x is a *complex function.* It involves not only the error of reading the microscopes, but several other *classes* of errors. They may be enumerated as follows: —

Errors depending on the observer.	(*a*) Error of reading microscopes. (*b*) Error of bisection of the star observed, or its equivalent.
Errors depending on the instrument.	(*c*) Errors of graduation of the circle, both accidental and systematic. (*d*) Error depending on the micrometer screws of microscopes. (*e*) Error due to the flexure of the instrument. (*f*) Error due to an imperfect figure of the pivots. (*g*) Error resulting from a change in the position of the instrument during the observations.
Errors independent of the observer and of the instrument.	(*h*) Error resulting from an erroneous place of the fundamental star observed. (*i*) Error depending on the state of the atmosphere, including the constant of refraction, imperfect thermometers, barometers, &c.

In this case, then, one must place quite a different interpretation upon the probable error of the mean value. In fact, the only safe interpretation that can be given to it, is the one which regards it as a means of comparing observations made by different observers under nearly the same conditions and in the same manner.

This subject may be considered in another way. It is a property of the arithmetical mean that it makes the sum of the squares of the residuals a minimum. The solution of a greater number of equations than the unknown quantities which they contain, by the process of least squares, rests upon the same basis; viz. that such values must be given to the unknown quantities as will, when substituted in the original equations, make the sum of the squares of the residuals a minimum. Theoretically, any unknown quantity may be made equal to a constant plus the sum of all the corrections which make up this quantity. We may always have

$$X = C + A\,a + B b + C c + D d, \text{ \&c.}$$

The only limit to the number of terms is the one which requires that the coefficients A, B, C, D, &c. shall be known. The solution of a series of equations of this form will give the most probable values of the constant C, and of the unknown quantities a, b, c, d, &c., provided

x is a simple function; but if x is a complex function the solution will no longer give the *true* values of the separate unknown quantities, though it may yield such values as will give the most probable sum of Aa, Bb, Cc, Dd, &c.. with respect to their effect upon x.

Let us take, as an illustration, the ordinary equation for the reduction of transit observations. The fundamental equation may be put under the following variety of forms: —

$$
\begin{aligned}
&(a)\ 0 = \Delta\mathrm{T} + [T - \mathrm{R.\,A.}] + Aa \\
&(b)\ = \Delta\mathrm{T} + [T - \mathrm{R.\,A.}] + Aa + Bb \\
&(c)\ = \Delta\mathrm{T} + [T - \mathrm{R.\,A.}] + Aa + Bb + Cc \\
&(d)\ = \Delta\mathrm{T} + [T_0 - \mathrm{R.\,A.}] + \tau h + Aa + Bb + Cc \\
&(e)\ = \Delta\mathrm{T} + [T_0 - \mathrm{R.\,A.}] + \tau h + Aa + Bb + Cc + Dd \\
&(f)\ = \Delta\mathrm{T} + [T_0 - \mathrm{R.A.}_0] + Ee + \tau h + Aa + Bb + Cc + Dd \\
&(g)\ = \Delta\mathrm{T} + [T_0 - \mathrm{R.A.}_0] + Ee + \tau h + Aa + Bb + Cc + Dd + C
\end{aligned}
$$

If the level b and the collimation c are obtained independently of the observations by direct measures, then, neglecting the small terms which follow, for any time, T, and with the given right ascension, R. A., the only unknown quantities in equation (a) are the clock error ΔT and the azimuth term Aa. A solution of a series of equations of this form will give the most probable *individual* values of a and ΔT.

If the level term Bb is unknown, the general equation takes the form (b). Notwithstanding the fact that the equation is somewhat more complex in its structure, the solution by least squares will give the most probable individual values of a and b, if the stars are selected with reference to a proper distribution of positive and negative values for A and B.

If the collimation term Cc is unknown, the equation takes the form (c). Here a solution by least squares will *not* give the most probable individual values of a, b, and c, unless the observations are arranged with proper reference both to the magnitude and the sign of A, B, and C. Even when these precautions are observed, the value of c from the solution will rarely agree exactly with the value obtained from reversal or from collimators.

If, for any star, the observed time T is written $T_0 + \tau h$, the term τh being the hourly rate of the clock multiplied by the interval τ between T and T_0, the equation takes the form (d). We now introduce an unknown quantity depending on another instrument, viz. the clock.

We may still further introduce the term $D\,d$, representing the diurnal aberration, giving the form (e), and by substituting R. A.$_0$ + $E\,e$ for R. A. where $E\,e$ represents a term depending on 2 ☽, we get the form (f).

Finally, if we represent by the constant C the personal equation between bright and faint wires, bright and faint stars, &c., we have the form (g).

Of course it is wholly absurd to introduce the terms $D\,d$, $E\,e$, and C as unknown quantities, and these forms are given only to show that one must exercise sound judgment in the formation of the equations in order that the solution by least squares shall give correct results. It is useless to expect that the solution will separate errors which appertain to different instruments. For example, in form (g) it would seem hardly necessary to say that the solution will entirely fail in assigning to the telescope the correct values of a, b, and c; to the clock, the true values of ΔT and $\tau\cdot h$, to yield the physical constant which enters into the diurnal aberration, and the coefficient which results from the variable motion of the moon; and to refer to the observer the constant which involves the various forms of personal equation. Yet, according to the common acceptation of the theory, this form of the equation is allowable, since all the unknown quantities have known coefficients.

Again, as soon as the equation involves unknown quantities which pertain to different instruments, it becomes so complex in its character that we can no longer assume that even the *sum* of the terms which affect ΔT is the most probable value that can be found, for in so doing we assume that ΔT is a constant, whereas the solution requires it to be a variable.

Let us now inquire how far these views are confirmed by the facts of observation.

In my own case, the probable error of a *single* reading of four microscopes of the meridian circle is $\pm.094''$. If, therefore, as many as 10 observations are obtained, the probable error of the mean will be not far from $\pm.03''$. The probable error of a single difference between myself and my assistant, Mr. Joseph F. MacCormick, is for a single reading of four microscopes $\pm.125''$.

The probable error of a *single* complete observation in declination is, in my own case, about $\pm.36''$, and of the mean of 10 observations is $\pm.11''$. The probable error of a *single* complete observation in right ascension is, for an equatorial star, $\pm.026^s$ and for the mean of 10 observations $\pm.008^s$.

If, therefore, the probable error can be taken as a measure of the accuracy of the observations, there ought to be no difficulty in obtaining, from a moderate number of observations, the right ascension within .02^s and the declination within 0''.2. Yet it is doubtful, after continuous observations in all parts of the world for more than a century, if there is a single star in the heavens whose absolute co-ordinates are known within these limits. In 1866 the illustrious Argelander proposed a list of stars for simultaneous observation by different observers, for the purpose of investigating the systematic differences which he found to exist in all modern catalogues. This scheme was carried out only to a limited extent. But in 1878 the fortunate requirements of a special problem secured data which will go far towards the establishment of the existence of these errors, even with the present methods of refinement in observation, if indeed they do not for the present reveal their cause.

During that year Mr. David Gill, recently appointed Director of the Cape of Good Hope Observatory, solicited the co-operation of astronomers in determining the co-ordinates of 28 stars, which he used in his heliometer observations of the planet Mars for obtaining the solar parallax. The observatories named below made the observations required, which were forwarded to Mr. Gill upon the completion of the reductions. The results are published in Vol. XXXIX., page 99, of the Monthly Notices of the Royal Astronomical Society.

In the following table are given the differences between the least and the greatest results for each star, both in right ascension and in declination.

Star.	$\Delta\alpha$	$\Delta\delta$	Star.	$\Delta\alpha$	$\Delta\delta$	Star.	$\Delta\alpha$	$\Delta\delta$
	s.	''		s.	''		s.	''
1	0.189	1.45	10	0.460	2.84	19	0.242	3.14
2	.169	2.04	11	.287	1.77	20	.333	2.70
3	.377	2.63	12	.077	1.74	21	.220	2.16
4	.098	2.57	13	.350	1.80	22	.283	2.31
5	.224	2.04	14	.263	2.34	23	.260	1.77
6	.166	1.86	15	.270	1.80	24	.203	2.78
7	.193	3.47	16	.183	1.86	25	.219	1.62
8	.190	1.93	17	.225	3.47	26	.207	2.06
9	.300	3.15	18	.287	1.37	27	.298	2.57
						28	.264	2.47

Even after the observations were reduced to a homogeneous system, Mr. Gill finds the following outstanding errors: —

Authority.	Δα	Δδ	Authority.	Δα	Δδ
	s.	″		s.	″
Königsberg,	+.006	—0.71	Leiden,	—.053	—0.19
Melbourne,	+.026	—0.49	Paris,	+.055	+0.01
Pulkowa,	+.005	+0.36	Washington,	—.120	+0.78
Leipzig,	+.049	+0.40	Harvard College,	—.072	+0.09
Greenwich,	+.009	—0.56	Cordoba,	—.032	—0.20
Berlin,	+.044	+0.67	Oxford,	+.076	+0.21

These systematic discordances, especially in right ascension, are so alarmingly large that, unless they can be reconciled, the heliometer observations are comparatively worthless. Mr. Gill, therefore, proposed a second list of 12 stars, one half comparatively bright and the other half faint. The observations of these stars are now completed, but the only series yet at hand, are those of Königsberg and Harvard College. Here the discordance is very large, and varies with the magnitude of the star observed. Professor Pickering, the Director of Harvard College Observatory, early in this investigation, proposed the artificial reduction of the magnitude of the bright stars by holding circular diaphragms of varying diameters in front of the object-glass of the telescope. By alternating between bright and faint images of the same star, on different groups of the transit threads, the personal equation between bright and faint stars can be found. This plan was followed in the investigation at Harvard College Observatory, at Leiden, and probably at some other observatories. At Harvard College Observatory, also, a sensible difference was found between results obtained with bright and faint fields of the telescope, this difference varying with the magnitude of the star.

A similar investigation is now being made in another class of observations, viz. the measurement of the position angle and distance of double stars with the filar-micrometer. The range of systematic discordances between the measures of different observers is of course here far less than will always be found in the determination of position in space, for such observations are entirely relative in their character; but the outstanding errors are still so large as to demand a special investigation. Even with observers of skill and long experience, such as Struve, Hall, Dembowski, Burnham, and Stone, there are residual errors in the measurements of the same components far exceeding the limits indicated by the magnitude of the probable error of any single observer.

Finally, it is even an open question whether any real advance has

been made in the absolute precision of observations with the telescope for the last forty years, if we except the tentative investigations of the last five or six years. Argelander's Åbo Catalogue of 1830, and the Pulkowa Catalogue of 1845, are as yet pre-eminent for that kind of accuracy which answers to the crucial test of agreement with future observations. After a lapse of nearly fifty years, Argelander's positions of the thirty-six stars known as the "Maskelyne fundamental stars" are at least as near the truth as the mean of the observations of these stars made during the last ten years.

The great need of instrumental astronomy is a rigid investigation of all the classes of error to which observations are now subject, not simply for any one observer, but for all the principal observers of the world, and *upon a common plan.* If some competent and recognized authority, like the *Astronomischen Gesellschaft*, would arrange a scheme of observations having this object in view, and take measures to secure the co-operation of all the principal observatories in this work, it would seem that the foundation for a real advance might be made in the precision with which observations can be made.

In the investigation which follows I have endeavored to ascertain the limits of accuracy in measurements with the microscope by a process similar to that by which observers with the telescope are now seeking to reach the ultimate limit of precision. The remarks already made with regard to the degree of reliability to be attached to conclusions drawn from the magnitude of the probable errors of observation apply with equal force to measures made under the microscope. Neither the probable error of a single observation nor the probable error of the mean of a given number of observations furnishes a safe criterion by which the real measure of accuracy may be estimated. For example, with the comparator for short lengths described in the April number of the *American Quarterly Microscopical Journal*, it is the experience of the writer that, in an unlimited number of repetitions of measures of the same space, the pointer will in every case fall upon the same tenth of a division of the index of the screw. Hence, if the readings are taken to tenths only, the resulting probable error will always be zero, without regard to the value of one division. In this particular instrument the value of one tenth of one division is one eighty-thousandth of an inch, but the probable error would still remain zero if the readings were carried to tenths of one division only for any change whatever in the pitch of the screw, and consequently for any reduction in the value of one division, provided the pointer always falls within this tenth.

The probable error of a *single* measure with the comparator for short lengths is about two millionths of an inch. If the probable error can be taken as a measure of precision, it ought not to be difficult to measure one millionth of an inch with entire certainty by repeating the measures a sufficient number of times.

Let us see if this theoretical accuracy is attainable.. Before proceeding to the discussion, it may be worth while to say that a sharp distinction must be drawn between absolute accuracy and a superficial appearance of accuracy. If I determine the value of a centimeter within one ten-thousandth of its whole length, I can use the equivalent expression, one millionth of a meter; but it does not follow that I can measure a meter within this limit. I say that a given space, corresponding to one thousandth of an inch, requires a correction of one millionth of an inch; but it makes a wide difference whether I ascertain this fact by direct measurement, or whether I get it by dividing the correction for an entire inch by one thousand. Extending the number of figures in the quotient does not give a corresponding increase of accuracy. The index of the screw of my dividing engine can be set to correspond to a motion of one billionth of an inch with entire certainty as far as the mechanical indication of this degree of accuracy is concerned; yet previous to May, 1877, the actual errors of a given ruled plate amounted, under certain conditions, to as much as one seven-thousandth of an inch. Even now, after four epochs of improvement, I can hardly say of a given space that it is certainly true within one eighty-thousandth of an inch until a careful investigation has been made with the comparator. Again, it does not follow that, because the spaces of a closely ruled band of lines, like Nobert's bands, appear to be equal under an objective of high power, they are therefore to be taken as the measure of the real accuracy of the graduations. It is far more difficult to subdivide an inch into one hundred equal parts, than to make a further subdivision of one of these parts. As I shall presently show, almost all of the errors of a given graduation are periodic in their character, but the increments proceed by such minute variations in the case of closely ruled bands that they can only be detected when their sum amounts to an appreciable quantity. Thus, if the accumulated error of a screw having a pitch of one in twenty amounts to one two-thousandth of an inch for half a revolution of the index, the average periodic error for each two-thousandth of an inch will be one hundred-thousandth of an inch. It will thus be seen that, for even the first of Nobert's bands, which are about ten thousand to the inch, the systematic error for any single space is inap-

preciable. But even in this case, only ten increments are required in order to produce an error of measurable magnitude.

A simple and direct way to determine the degree of precision with which measures under the microscope may be made, is to compare measurements of the same space made by different observers and under different conditions. I may get results which show an agreement *inter se* quite within the limits of the accuracy required, but which are yet wide of the truth. But if another equally skilful observer obtains substantially the same results from a series of measurements made under entirely different conditions, the inference of their general correctness may be drawn with tolerable safety.

In carrying forward this investigation I was fortunate in securing the co-operation of Professor Edward W. Morley, of Hudson, Ohio, whose paper will be found on page 164 of this volume of the Proceedings.

The rulings selected for joint measurement, are described as follows: —

Plate I. consists of eight bands. The first three bands are composed of twenty-six lines each. The distance between the lines is $\frac{1}{480}$ of an inch. The remaining five bands are composed of twenty-one lines each, the distance between the lines being $\frac{1}{800}$ of an inch. *All the rulings of this plate involve the periodic errors which belong to the ruling screw.*

Plate II. consists of three bands of very heavy lines, each band being composed of twenty-six lines. The interval between the lines is the same as in the corresponding three bands of Plate I. The lines are filled with graphite and are mounted in balsam. *In this plate the errors which are a function of one revolution of the screw were corrected during the process of ruling.*

Plate III. consists of 101 lines, separated by an interval of $\frac{1}{2400}$ of an inch, *and freed as nearly as possible from errors of all kinds.*

Plate IV. consists of 21 lines, separated by an interval of $\frac{1}{20}^{mm.}$, *corrected for systematic errors.*

The results given in the following tables under the head "Corr." represent the corrections which must be applied to each space of a given band in order to make it equal to a mean of all the spaces. They are *expressed in millionths of an inch*, except in Plate IV., in which the unit is *one hundred-thousandth of a millimeter*. The results given under the head Σ represent the accumulated errors reckoned from the first line of each band. In Plate I. the values given were formed by successive additions of the individual errors. In Plates II.,

III., and IV., the values in column Σ were obtained by measuring the accumulated errors directly with the comparator for short lengths, and the individual errors were found by successive subtractions.

The first band of Plate I. was measured with great care with a filar-micrometer made by Powell and Leland, with a glass eye-piece micrometer, with a comparator screw by Merz of Munich, and with the Clark screw mentioned above. The results from the Clark screw are somewhat discordant, as they were obtained before the instrument was fairly completed. They are, however, taken into account on the principle adopted of including *every* measure taken. The values of Plate III. were found by taking the mean of the accumulated errors of each successive group of five spaces, measured directly with the Clark comparator for short lengths. The separate results given under the first and fourth bands of Plate I. are given for the purpose of deducing the probable error of observation. They are not simple repetitions of measures made at one time. Each column refers to a different date. As the different sets of measures were only brought together from the note-books after all the work was done, I had no previous knowledge of the degree of agreement to be expected from separate measures of the same space. In fact, the comparison was made for the first time, soon after receiving the results communicated by Professor Morley.

PLATE I.—BAND I.

Space.	Corrections to each Space from Measures with Eye-piece Micrometer and ⅛-inch Objective.						Corrections to each Space from Measures with Merz Screw and ⅛-inch Objective.					Correction to each Space from Measures with Filar Micrometer and ⅛-inch Objective.		Correction to each Space from Measures with Clark Screw.	
	I.	II.	III.	IV.	Mean Corr.	Σ	I.	II.	III.	Mean Corr.	Σ	Corr.	Σ	Corr.	Σ
1	+3	−4	−6	+6	+0	+0	−8	−20	−6	−11	−11	+5	+5	−12	−12
2	−2	−4	−8	−14	−7	−7	−9	−6	−9	−8	−19	−11	−6	−13	−25
3	−14	−7	−11	−17	−12	−19	−17	−22	−11	−17	−36	−20	−26	−8	−33
4	−17	−18	−9	−17	−15	−34	−11	−21	−11	−14	−50	−18	−39	−23	−56
5	−19	−17	−14	−20	−18	−52	−25	−18	−16	−20	−72	−14	−53	−14	−70
6	−25	−25	−26	−23	−25	−77	−24	−16	−20	−20	−92	−28	−81	−28	−98
7	−25	−29	−21	−20	−24	−101	−28	−18	−16	−21	−113	−26	−107	−24	−122
8	−27	−27	−29	−26	−27	−128	−28	−18	−27	−24	−137	−24	−131	−11	−133
9	−22	−26	−25	−17	−25	−153	−21	−28	−30	−26	−163	−22	−153	−37	−170
10	−23	−14	−29	−17	−21	−174	−19	−14	−25	−19	−182	−18	−171	−8	−178
11	−21	−10	−21	−20	−18	−192	−5	−19	−12	−12	−194	−15	−186	−12	−190
12	−9	−8	−10	−11	−10	−202	+9	+0	−11	−1	−195	−7	−193	−11	−201
13	−12	−7	−8	−6	−8	−210	+23	−4	−8	+5	−190	+1	−192	+3	−198
14	+6	−1	+4	+6	+4	−206	+7	−4	+6	+8	−187	+6	−186	+8	−190
15	+9	+13	+8	+6	+9	−197	+0	+0	+9	+3	−184	+7	−179	+5	−185
16	+16	+18	+19	+12	+16	−181	+10	+16	+9	+12	−172	+12	−167	+17	−168
17	+21	+19	+22	+12	+19	−162	+16	+11	+11	+13	−159	+15	−152	+34	−134
18	+21	+21	+22	+18	+21	−141	+13	+26	+15	+18	−141	+16	−136	+7	−127
19	+25	+22	+22	+27	+24	−117	+20	+30	+29	+26	−115	+20	−116	+28	−99
20	+23	+23	+25	+24	+24	−93	+22	+30	+21	+24	−91	+25	−91	+20	−79
21	+23	+20	+24	+30	+24	−69	+23	+30	+28	+27	−64	+31	−60	+32	−47
22	+23	+23	+25	+27	+24	−45	+25	+25	+31	+27	−37	+21	−39	+12	−35
23	+16	+17	+19	+18	+18	−27	+12	+20	+18	+17	−20	+13	−26	+9	−26
24	+18	+17	+22	+18	+18	−9	+21	+15	+19	+18	−2	+21	−5	+26	−0
25	+7	+3	+8	+6	+5	−4	+0	+0	+0	+0	−2	+5	+0	+0	−0

PLATE I.

SPACE.	BAND II.		BAND III.		BAND IV.					FILAR MICROMETER.		BAND V.		BAND VI.		BAND VII.		BAND VIII.	
	Corr.	Σ	Corr.	Σ	I.	II.	III.	Mean Corr.	Σ	Corr.	Σ	Corr.	Σ	Corr.	Σ	Corr.	Σ	Corr.	Σ
1	+ 0	+ 0	+ 1	+ 1	+21	+14	+14	+16	+ 16	+19	+ 19	+ 7	+ 7	+ 4	+ 4	+ 4	+ 4	+ 3	+ 3
2	-11	- 11	- 6	- 5	- 2	- 7	-10	- 6	+ 10	- 7	+ 12	-12	- 5	-11	- 7	-13	- 9	-10	- 7
3	-18	- 29	-21	- 26	-17	-10	-22	-16	- 6	-14	- 2	-23	- 28	-17	- 24	-17	- 26	-18	- 25
4	-20	- 49	-19	- 45	-19	-25	-13	-19	- 25	-22	- 24	-33	- 61	-23	- 47	-27	- 53	-27	- 52
5	-24	- 73	-26	- 71	-28	-28	-33	-30	- 55	-34	- 58	-35	- 96	-32	- 79	-36	- 89	-37	- 89
6	-30	-103	-28	- 99	-32	-31	-33	-32	- 87	-34	- 92	-35	-131	-33	-112	-35	-124	-39	-128
7	-26	-129	-26	-125	-26	-27	-31	-28	-115	-25	-117	-31	-162	-29	-141	-32	-156	-30	-158
8	-24	-153	-29	-154	-29	-31	-33	-31	-146	-33	-150	-25	-187	-28	-169	-31	-187	-24	-182
9	-27	-180	-26	-180	-22	-23	-25	-23	-169	-24	-174	-18	-205	-19	-188	-23	-210	-23	-205
10	-22	-202	-25	-205	-11	-12	-10	-11	-180	-15	-189	-15	-220	-15	-203	-18	-228	- 9	-214
11	-16	-218	-17	-222	+ 0	- 7	+ 2	- 2	-182	+ 1	-188	+ 1	-219	+ 8	-195	- 2	-225	+ 2	-212
12	-11	-229	- 7	-229	+ 3	+ 6	+ 8	+ 6	-176	+ 4	-184	+16	-203	+10	-185	+15	-210	+12	-200
13	- 8	-282	- 1	-230	+15	+19	+20	+18	-158	+21	-163	+22	-181	+20	-165	+26	-184	+28	-172
14	+ 6	-226	- 1	-231	+20	+21	+20	+20	-138	+27	-136	+24	-157	+29	-136	+30	-154	+26	-146
15	+13	-213	+17	-214	+20	+28	+23	+24	-114	+26	-110	+27	-130	+28	-108	+29	-125	+34	-112
16	+23	-190	+20	-194	+27	+32	+32	+30	- 84	+33	- 77	+30	-100	+28	- 80	+30	- 95	+34	- 78
17	+27	-163	+25	-169	+29	+28	+29	+29	- 55	+29	- 48	+31	- 69	+27	- 53	+29	- 66	+27	- 51
18	+23	-140	+27	-142	+21	+29	+26	+25	- 30	+22	- 26	+26	- 43	+28	- 25	+27	- 39	+25	- 26
19	+19	-121	+25	-117	+21	+20	+23	+21	- 9	+21	- 5	+25	- 18	+19	- 6	+22	- 17	+21	- 5
20	+24	- 97	+27	- 90	+ 6	+ 6	+14	+ 9	+ 0	+ 5	+ 0	+17	- 1	+ 7	+ 1	+14	- 3	+ 8	+ 3
21	+22	- 75	+22	- 68															
22	+24	- 51	+24	- 44															
23	+23	- 28	+22	- 22															
24	+21	- 7	+19	- 3															
25	+ 6	- 1	+ 4	+ 1															

Space.	Plate II.						Plate III.		Plate IV.	
	Band I.		Band II.		Band III.		Band I.		Band I. $= \frac{1}{10}^{mm.}$	
	Corr.	Σ	Corr.	Σ	Corr.	Σ	Corr.	Σ	Corr.	Σ
1	+ 2	+ 2	− 6	− 6	− 3	− 3	+3	+3	+10	+10
2	− 4	− 2	− 1	− 7	+ 8	+ 5	+0	+3	−11	− 1
3	− 4	− 6	− 1	− 8	+ 3	+ 8	+0	+3	+ 0	− 1
4	+ 3	− 3	+ 4	− 4	−15	− 7	−4	−1	− 7	− 8
5	+ 3	+ 0	− 6	−10	+21	+14	+3	+2	+ 2	− 6
6	− 6	− 6	+ 1	− 9	+ 8	+22	−5	−3	+ 1	− 5
7	− 6	−12	+ 0	− 9	−22	+ 0	−1	−4	+ 3	− 2
8	−10	−22	− 8	−17	+ 9	+ 9	−2	−6	+ 2	− 0
9	+ 0	−22	− 2	−19	− 7	+ 2	−1	−7	+ 2	+ 2
10	+ 1	−21	− 8	−27	− 6	− 4	+0	−7	+ 5	+ 7
11	− 2	−23	− 5	−32	+ 9	+ 5	+0	−7	− 2	+ 5
12	− 8	−31	+ 3	−29	− 1	+ 4	+1	−6	+ 2	+ 7
13	+ 8	−23	+15	−14	− 6	− 2	+1	−5	+14	+21
14	+ 2	−21	− 5	−19	− 1	− 3	+1	−4	−17	+ 4
15	+ 0	−21	− 6	−25	+ 1	− 2	+5	+1	− 2	+ 2
16	+ 6	−15	+ 6	−19	+ 6	+ 4	+2	+3	− 1	+ 1
17	+ 9	− 6	− 2	−21	− 8	− 4	−3	−3	+ 0	+ 1
18	− 9	−15	+ 4	−17	+ 6	+ 2	+6	+3	+ 8	+ 8
19	+ 5	−10	+12	− 5	+ 1	+ 3	−1	+2	+ 0	+ 8
20	+10	+ 0	+ 4	− 1	+ 2	+ 5	+1	+3	−10	− 2
21	+ 0	+ 0	− 4	− 5	+12	+17				
22	+ 0	+ 0	− 1	− 6	− 5	+12				
23	+ 0	+ 0	− 4	−10	− 6	+ 6				
24	+ 2	+ 2	− 2	−12	− 8	− 2				
25	− 2	+ 0	+12	+ 0	+ 0	− 2				

From a comparison of the separate values obtained by myself and by Professor Morley, the following conclusions are drawn: —

(*a*) By comparing the separate values of Bands I. and IV. of Plate I., obtained with the eye-piece micrometer, with the corresponding mean values, the average probable error of the measure of a single space is found to be 19 ten-millionths of an inch, the greatest deviation from the mean in 156 measures being 8 millionths of an inch.

(*b*) Comparing with the mean value, the separate results obtained with the eye-piece micrometer, the Merz screw, and the filar micrometer, from the first and fourth bands of Plate I. we find the following average deviations: —

For the eye-piece micrometer, 17 ten-millionths of an inch.
For the Merz screw, 25 ten-millionths of an inch.
For the filar micrometer, 18 ten-millionths of an inch.

(*c*) Comparing the measures of the separate spaces made by Professor Morley with my own, made in the first band of Plate I. with

the eye-piece micrometer, the Merz screw, and the filar-micrometer, and in the remaining bands of this plate with the eye-piece micrometer only, we find the following deviations expressed in millionths of an inch: —

Number of millionths,	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of cases of agreement,	16	30	39	24	15	17	13	8	2	3	4	2	2

The mean deviation is 34 ten-millionths of an inch.

(*d*) Comparing the accumulated errors of the middle point obtained by Professor Morley and by myself, we have: —

	Band.	Rogers.	Morley.	R. — M.
Plate I.	1	—197	—174	—23
	2	—232	—218	—14
	3	—230	—204	—26
	4	—181	—177	— 4
	5	—219	—211	— 8
	6	—199	—199	+ 0
	7	—224	—233	+ 9
	8	—213	—237	+24
Plate II.	1	— 23	— 19	— 4
	2	— 14	— 53	+39
	3	— 2	— 23	+21
Plate III.	1	— 7	+ 31	—38

The numerical value of the average deviation is therefore 17 millionths of an inch. It is to be noted, however, that only the bands of Plate I. are strictly comparable, since these only were obtained in the same way, viz. by successive additions. The values for Plates II. and III., it will be remembered, were obtained in my own case by direct measurement, in which the degree of accuracy may be taken as nearly equal to that of the measure of any individual space, while those of Professor Morley were obtained by direct addition, in which case an error at any point is carried through the whole of the remaining series.

(*e*) In explanation of the disagreement in the maximum values of the accumulated errors, even in the bands of Plate I., which were ruled at the same time, it is to be said that the graduation was done before I had learned the necessity of dispensing with oil or grease as

a lubricant. Without doubt, a part of the discordance is due to errors of measurement, but recent experience has convinced me that, when oil is used as a lubricant, every precision-screw has a variable periodic error, depending on the position of the nut in the line of its motion, especially when there is a decided change of temperature. It is sufficient to say here, that, since adopting a substitute for oil, the errors of the screw have remained practically constant.

(*f*) It will be seen from simple inspection that nearly all of the errors under discussion are periodic in their character. This may be shown conclusively in the following way: —

If we have a series of errors depending on one revolution of the screw, which are *strictly periodic in their character*, and from which all accidental errors are excluded, the given series can be represented *exactly* by a series of equations of the form, —

$$n = + a \sin x + b \cos x + a' \sin 2x + b' \cos 2x, \text{ \&c.};$$

in which

n represents any given value of the series;
x is an aliquot part of one revolution of the screw;
a, b, a', b', are unknown coefficients.

If, therefore, an expression of this form is found which will represent all the given errors of a series, it may be safely affirmed that the errors themselves are entirely periodic in their character.

Making n equal, successively, to the mean of the values of the individual errors of spaces 1, 2, 3, &c. of the first three bands of Plate I., we have a series of equations whose solution by least squares will give the normal equation,

$$n = -26.6 \sin x + 5.9 \cos x - 0.1 \sin 2x + 0.8 \cos 2x.$$

In like manner we get from the mean of the separate values of Bands IV., V., VI., VII., VIII. of the same plate,

$$n = -29.1 \sin x + 9.9 \cos x - 2.0 \sin 2x + 1.3 \cos 2x.$$

Substituting in these equations the known values of x, we get values of n which are directly comparable with the observed values given in the tables. The observed and the computed values are given in the following table, as well as the deviations of both the individual and the accumulated errors from the computed values.

Plate I. — Bands I., II., III.						Plate I.— Bands IV., V. VI., VII., VIII.					
Individual Errors.			Accumulated Errors.			Individual Errors.			Accumulated Errors.		
Observed value of n.	Computed value of n.	Δ	Observed.	Computed.	Δ	Observed value of n.	Computed value of n.	Δ	Observed.	Computed.	Δ
— 1	+ 0	—1	— 1	+ 0	—1	+ 7	+ 1	+6	+ 7	+ 1	+6
— 9	— 8	—1	— 10	— 8	—2	—11	— 9	—2	— 4	— 8	+4
—18	—14	—4	— 28	— 22	—6	—16	—18	+2	— 20	— 26	+6
—19	—19	+0	— 47	— 41	—6	—26	—26	+0	— 47	— 52	+5
—23	—24	+1	— 70	— 65	—5	—84	—31	—8	— 81	— 83	+2
—27	—28	+1	— 97	— 93	—4	—85	—83	—2	—116	—116	+0
—25	—28	+8	—122	—121	—1	—80	—82	+2	—147	—148	+1
—25	—27	+2	—147	—148	+1	—27	—28	+1	—174	—176	+2
—27	—25	—2	—174	—178	—1	—21	—21	+0	—195	—196	+1
—21	—21	+0	—195	—194	—1	—18	—11	—2	—208	—207	—1
—16	—15	—1	—211	—209	—2	+ 2	+ 0	+2	—207	—207	+0
— 8	— 8	+0	—219	—217	—2	+12	+ 9	+3	—195	—198	+8
— 1	— 2	+1	—220	—219	—1	+23	+18	+5	—172	—180	+8
+ 4	+ 6	—2	—216	—218	—8	+26	+27	—1	—146	—153	+7
+12	+12	+0	—204	—201	—3	+29	+31	—2	—118	—122	+4
+19	+16	+3	—185	—185	+0	+31	+83	—2	— 87	— 89	+2
+24	+22	+2	—161	—163	+2	+29	+82	—3	— 58	— 57	—1
+22	+24	—2	—139	—139	+0	+26	+27	—1	— 33	— 30	—3
+28	+26	—3	—116	—118	—8	+22	+20	+2	— 11	— 10	—1
+25	+26	—1	— 91	— 87	—4	+11	+11	+0	+ 0	+ 1	—1
+24	+26	—2	— 67	— 61	—6						
+23	+22	+1	— 44	— 39	—5						
+20	+18	+2	— 24	— 21	—3						
+20	+14	+6	— 4	— 7	+3						
+ 4	+ 7	—3	+0	+ 0	+0						

Finally, we have a severe test in the agreement of the values of n computed for parts of the revolution which were not observed. In order to adapt the equations, for example, to successive ten degrees of revolution, the coefficients of the first equation must be multiplied by $\frac{10}{14.4} = .694$, and those of the second equation must be multiplied by $\frac{10}{18} = .556$. We shall then have: —

$$(1)\ n = -18.5 \sin x + 4.1 \cos x - 0.1 \sin 2x + 0.6 \cos 2x;$$

$$(2)\ n = -16.2 \sin x + 5.4 \cos x - 1.1 \sin 2x + 0.7 \cos 2x.$$

Substituting $x = 0°, 10°, 20°$, &c. in these equations, and comparing the results, we have the following discordances expressed in millionths of an inch: —

Number of millionths,	0	1	2	3	4
Number of cases,	5	7	17	3	4

The average deviation is 19 ten-millionths of an inch, and there are only 7 cases in which the disagreement exceeds 2 millionths of an inch.

(*g*) The relative advantages of the eye-piece micrometer, the filar micrometer, and the screw comparator, for narrow intervals, are nearly equal, as will be seen from the following comparison of the individual values derived by each method of observation, with the normal values found from the equation

$$n = -23.8 \sin x + 5.4 \cos x - 0.1 \sin 2x + 1.0 \cos x,$$

which represents the mean curve for the first band of Plate I.

Number of millionths,		0	1	2	3	4	5	6	7	8	9	10
Number of cases,	Eye-piece micrometer,	7	8	3	5	0	1	0	1	0	0	0
	Filar-micrometer,	4	2	3	5	2	1	5	1	2	0	0
	Merz screw,	4	2	4	2	5	2	2	2	0	1	1

(*h*) It appears from this investigation that it is possible to reduce the errors of a precision-screw for short intervals to about one hundred-thousandth of an inch by applying the corrections derived from the equation which represents the periodic errors. Since the rejection of oil as a lubricant, the errors have been considerably reduced.

(*i*) In a meridian circle having a diameter of 30 inches, one second of arc is equal to .0000727 of an inch. It appears, therefore, from this investigation, that, even if the attached microscopes have the same power as those used in this investigation, the ultimate limit of accuracy in the matter of bisection and reading only, must be at least 0.″05. But the microscopes of the meridian circle of Harvard College Observatory magnify only 51 diameters, while the magnifying powers used in this series of measures were 194, 290, 560, and 870. Moreover, this limit has reference only to repeated readings of the microscopes for the same position of the instrument. It has, therefore, only a *relative* value. When, in addition to the errors of simple pointing and reading, we take into account the accidental and the systematic errors of division in the graduated circles and the outstanding errors always found in measures of large arcs of a circle, the present limit of precision cannot fall much below 0.″2.

Since the completion of this investigation a further opportunity of comparing the results of measures of the same intervals by different observers has occurred. Through the kindness of Professor George F. Barker, of the University of Pennsylvania, I obtained the loan of a

ruled plate from the precision-screw of Mr. L. M. Rutherfurd. This plate is marked "$\frac{68}{360}$ rev." It consists of 11 lines, covering a space as nearly equal to one millimeter as the even notches of the index of the screw will give this value. A transverse line subdivides the vertical lines. The lines are apparently filled with graphite, and they are protected by a coating of transparent varnish.

I first measured the ten spaces of this plate in May, 1878. The plate was then sent to Professor Morley. After its return, and before comparing the results already obtained, it was again measured. In April and May, 1879, still another series of measures was made. The plate was then placed in the hands of Mr. J. R. Edmands, who is a skilful and careful observer with the microscope.

The measures made by Professor Morley and myself were of the separate spaces, and the accumulated errors were found by successive additions. Mr. Edmands made some measures in this way, but he also measured the distance of the successive lines from each of the end lines. The differences between the errors of the adjacent lines were then compared with the direct measures of the spaces, while the sums of the errors of the spaces were compared with the errors of the individual lines. The values given by him were obtained by giving to each determination its proper relative weight.

In accordance with the prearranged plan of observation, each observer remained in ignorance of the results obtained by the other observers until the work of measurement was completed. The following are the values of the corrections required to reduce each space of this particular millimeter to the mean of all the spaces. They are expressed in *millionths of a centimeter.*

INDIVIDUAL ERRORS.

Spaces.	Rogers. May, 1878.	Morley.	Rogers. October and November, 1878.	Rogers. April and May, 1879.	Edmands.	Mean Values.
1	—36	— 8	—37	—16	—14	—22
2	—14	—12	—11	—19	—16	—14
3	— 6	—10	—18	—27	—16	—15
4	— 4	— 1	— 2	+11	+ 2	+ 1
5	— 7	— 1	— 3	— 1	— 3	— 3
6	—28	— 4	—23	— 4	— 3	—12
7	+ 0	— 1	+ 2	— 1	— 1	+ 0
8	+ 8	— 3	+ 4	+ 2	— 3	+ 2
9	+35	+ 3	+33	+13	+10	+19
10	+52	+35	+55	+42	+44	+46

ACCUMULATED ERRORS.

Spaces.	Rogers. May, 1878.	Morley.	Rogers. October and November, 1878.	Rogers. April and May, 1879.	Edmands.	Mean Values.
1	—36	— 8	—37	—16	—14	—22
2	—50	—20	—48	—35	—30	—37
3	—56	—30	—66	—62	—46	—51
4	—60	—31	—68	—51	—44	—51
5	—67	—32	—71	—52	—47	—54
6	—95	—36	—94	—56	—50	—66
7	—95	—37	—92	—57	—51	—66
8	—87	—40	—88	—55	—54	—65
9	—52	—37	—55	—42	—44	—46
10	— 0	— 2	— 0	— 0	— 0	— 0

It is somewhat doubtful whether the mean values given, represent the actual errors of the spaces. There are some indications that a shrinkage of the film of varnish has occurred since it was first applied, and this action may have produced some effect upon the graphite with which the lines are filled. Although graphite is an impalpable powder, I have seen many instances in which it has been lifted in mass from the filled lines and thrown a distance as great as one thousandth of an inch without breaking the continuity of the particles. This action seems to take place only when the lines and the filling are protected by a thin cover-glass, closely cemented to the slip on which the lines are ruled. Sometimes an explosion seems to take place, scattering the graphite in all directions, leaving it in curves having nearly a uniform shape. I have never been fortunate in seeing this action, but in the case of one ruled plate an actual observation limits the time within which the explosion must have occurred to about ten days. In this case the lines remained perfect for about four months after

they were ruled and filled, but between the 1st and the 10th of April of the present year nearly one half of the powder was completely removed from the lines.

It is possible also, that some of the discordances are due to the fact that the measures were not all made along the same horizontal line. My earlier measures were made along a line just above the transverse line, while the later ones were made along a line a little below the transverse line. My observations of April and May of the present year, and those made by Mr. Edmands, ought to be comparable, since they were made at nearly the same time and along the same line. The greatest disagreement is in the third space; and that the discordance is not an accidental one is shown by the fact that *all* my observations agree in giving —27, while *all* of his agree in giving —16. But, admitting that the discordances are all due to errors of observation, it will be seen that the average deviation from the mean is only 6 millionths of a centimeter.

ANNUAL REPORT

OF THE

DIRECTOR

OF

HARVARD COLLEGE OBSERVATORY.

PRESENTED TO THE VISITING COMMITTEE
NOVEMBER 14, 1878,

BY

PROFESSOR EDWARD C. PICKERING.

CAMBRIDGE:
PRESS OF JOHN WILSON AND SON.
1879.

REPORT

OF THE

DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF HARVARD COLLEGE

FOR 1877–78.

TO THE PRESIDENT OF THE UNIVERSITY: —

SIR, — During the past year, both of the large telescopes belonging to the Observatory have been kept actively in use. It would be most unfortunate that either of these superb instruments should again be idle, or be employed without the hope of reducing the observations made with it, as has been the case in past years. Even now, the expenditure involved in using both instruments considerably exceeds the income of the Observatory, which must be enlarged unless the use of one telescope at least is greatly restricted. But it is hoped that the results of the year's work, which are stated below, will induce the friends of astronomy to prevent the recurrence of this necessity.

I shall first describe the work of the large equatorial telescope, and the various operations connected with it; then the observations made with the meridian circle; and afterwards the extension in the distribution of standard time signals, the progress made in reducing and publishing the past observations, the condition of the library, the present state of the attempt to increase the income of the Observatory, and the work proposed for next year.

The equatorial observations have been made under my own immediate direction, by Messrs. Searle and Upton, and by myself. The East Equatorial has been used on two hundred and thirty-nine nights during the year ending Oct. 31, 1878, mainly in a continuation of the photometric work projected last year. About 18,000 photometric observations have been obtained (22,000 in the fifteen months during which the telescope has been at work).

The light of all the known satellites, except the two inner satellites of Uranus, which are too faint for observation with our instrument, has been carefully measured. The relative light of the components of all known double stars of the fifth magnitude or brighter, and visible in this latitude, has also been determined.

Four comparisons constitute a set, and the observations of each star comprise at least ten sets. Moreover, each star has been observed on three or more nights, and by two — or oftener by three — observers, to avoid errors due to the condition of the air or to individual peculiarities.

Other precautions have been taken to guard against various systematic errors which were from time to time apprehended. A careful study of the work shows that the probable error of the final results amounts only to about one-twentieth of a magnitude. Some sources of error, which may seriously influence ordinary estimates of stellar magnitude, have been detected. It appears that the proximity of a bright star may affect the estimate of the light of a faint star to the extent of two or three magnitudes! In the present series of measurements, this source of error is avoided.

By combining a spectroscope with a photometer like that used in the work just described, various parts of the spectra of β *Cygni* and γ *Andromedæ* have been examined, and the progressive increase in the relative light of their blue components towards the more refrangible end of the spectrum has been clearly shown.

The measurement of the very faint test objects selected and in part observed last year is now completed. The constant of the photometer with which these observations were made, has also been redetermined with great care.

One of the most important series of the equatorial observations of the year relates to the eclipses of Jupiter's satellites. Much value was formerly attached to these phenomena, as a means not only of determining the orbits of the satellites themselves, but of measuring the distance of the sun or the velocity of light, and of obtaining terrestrial longitudes. But for these purposes the observation of the mere appearance or disappearance of a satellite cannot well be made sufficiently exact. Large differences occur between the results of the work done by different observers or with different instruments. The state

of the atmosphere at the time of any particular observation may also greatly affect it. Errors of this kind are much lessened by photometric observations of the satellites, as they gradually enter or emerge from the shadow of Jupiter, using the planet itself or another satellite as a standard. Each comparison thus obtained gives an independent determination of the time of the eclipse, free from the errors due to the condition of the air or the power of the telescope employed, and less likely to be affected by personal equation than the observation of a disappearance or reappearance. Since June 23, 1878, twenty-three eclipses have been observed, nine of them with both the East and West Equatorials. These thirty-two series comprise over eight hundred observations. Omitting those required to determine the full brightness of the satellites just before or just after their eclipses, there remain nearly three hundred from each of which the time of an eclipse may be deduced. This gives on the average nine observations in each series; and, supposing the value of each observation to be only equal to one of those usually made, gives us in one year an amount of material which it would ordinarily require nine years to collect. This estimate does not take into account the numerous cases in which many photometric observations of an eclipse may be obtained when the actual disappearance or reappearance cannot be observed, owing to clouds or accident of any kind. Moreover, by the ordinary method, an observation during twilight can have little value, while good photometric observations may be made as well then as at any other time. It is even possible to make them before sunset.

For the development, by the various kinds of work above described, of a branch of science which has heretofore received little attention, it has been necessary to devise a new class of instruments. These are designated in the records of the Observatory by the letters of the alphabet, the first which was constructed being called Photometer A, and the last Photometer O. The work of the year has mainly been done with Photometers H and I. Photometers L and M, which have as yet not been brought into systematic use, require special notice. Photometer M consists of a Rochon micrometer, in which the images of the two stars may be rendered equal in brightness by a Nicol. The positions and distances of the components of double stars may thus be measured with great precision, while the systematic

errors occurring in measurements of the usual kind may be almost wholly eliminated. Photometer L is intended to measure the brightness, positions, or distances of any two stars the angular separation of which lies between 6′ and 40′. It resembles a double-image micrometer or heliometer, except that the full aperture of the telescope is used. The loss of light and deterioration of the images in these instruments is thus avoided.

By the liberal gifts to which reference is made on page 13, the construction of the photometers just mentioned, especially of those adapted for micrometric work, was rendered possible. Great credit is also due to Mr. George B. Clark, of the firm of Alvan Clark & Sons, without whose ingenuity and skill their construction would have been attended with great difficulty.

Besides the photometric observations made with the East Equatorial, it has occasionally been used in determining the places of asteroids, and for other purposes.

All the available telescopes of the Observatory were in use during the transit of Mercury last May, for observations of the diameter of that planet, as well as of its ingress and egress. The weather, unfortunately, seriously interfered with these observations. Photographs of the transit were also taken in co-operation with the U. S. Naval Observatory, upon dry plates sent from Washington, and returned for development and measurement. A few additional photographs were obtained, for preservation at the Observatory.

No expedition was sent from this Observatory to observe the Total Eclipse of the Sun on July 29, 1878. In view of the pressing need of money for our immediate work, I did not feel justified in an expenditure the results of which might be negative. Aid was freely offered to other observers, by the loan of any instruments not in use at the Observatory. Three or four parties availed themselves of this opportunity, and instruments valued at two or three thousand dollars were thus used during the Eclipse. These instruments have been returned in good condition, and their use has been handsomely acknowledged in the printed reports. Leave of absence was given at this time to Mr. Leonard Waldo, who observed the Eclipse in Texas, and to Mr. Upton, who joined Professor Stone's party in Colorado.

A new method of measuring the transparency of the air has been employed during the past two months, by comparing the light of stars near the zenith and horizon. This promises to furnish a simple means of determining the coefficient of atmospheric absorption at different times and places.

Observations with the meridian circle have been made on one hundred and fifty-eight nights during the year. The observer in charge of the instrument has been, as in previous years, Professor W. A. Rogers, who has been assisted in reading the microscopes by Mr. J. F. McCormack. The observations of the stars between 50° and 55° north declination have continued without interruption. This great work, which has been carried on for eight years, will be completed by the end of December, 1878. Of eight thousand stars to be observed, only one hundred and thirty-four remain, all situated between 23^h and 5^h of right ascension.

Observations of a list of two hundred and fifty-eight stars, drawn up by the U. S. Coast Survey, at whose request the work was undertaken, were begun in January, 1878. Six observations of each star have thus far been made, with four or five exceptions. The method of observation in use here allows the observations to succeed each other at very short intervals, so that, although in some cases several of the stars are situated within a single minute of right ascension, the whole work will be completed by the end of 1878.

The whole number of observations made during the year is as follows: —

Zone Stars	1,510
Fundamental Stars	1,338
Coast Survey Stars	1,285
Miscellaneous observations	167
Total	4,300

The chronograph sheets have all been read as far as October, 1878; and the results, together with the circle readings, have been copied in permanent form. The reduction of last year's observations of the planet Mars, and of the comparison stars in the lists of Mr. Gill and Professor Eastman, has been completed.

The reductions of the Polar Catalogue of 1872–73 are completed, but considerable work still remains to be done in preparing the results for the press.

The observations of 1874–75, made upon the stars of the General Catalogue of Vol. X. of the Annals of the Observatory, have been reduced, and the work is now in process of printing. The discussion of these observations is still to be made.

The reduction of the observations made upon the stars of the Coast Survey has been somewhat more than half completed by Mr. Upton, who has this work in charge. The fundamental stars upon which these observations depend are one hundred and sixteen in number. The co-ordinates of these stars for 1878.0 have been derived from the following authorities: —

Greenwich	from 1870 to 1875,	inclusive.
Washington	„ 1870 to 1874,	„
Cambridge	„ 1871 to 1875,	„

The positions taken from these sources have been reduced to the system of the Berlin Ephemeris of five hundred and thirty-nine stars, by applying corrections both for constant and for systematic deviations from this system.

A series of photometric observations have also been made, in which Professor Rogers has estimated the brightness of stars seen with various apertures. This furnishes a means of reducing to absolute measure the scale of magnitudes he has employed in the zone observations.

The general management of the time-service remains in charge of Mr. Leonard Waldo, who has been assisted, since July 1, by Mr. Frank Waldo. The signals have been distributed over a wider area than before. By the intervention of railroad and telegraph companies, they now reach Bangor, Lennoxville in Canada, Albany, and New York. The most important extensions of the service consist in the arrangements by which the signals are now sent through the northern part of Massachusetts to North Adams, and in the establishment of a time-ball at Boston. This last enterprise was made practicable by the liberal co-operation of the Equitable Life Assurance Co., and of the United States Signal Service. The ball is of copper, four feet in diameter, and weighs about two hundred and fifty pounds. It has a fall of about fifteen feet, and is held in place, when raised to the top of its mast, by a powerful electro-magnet. It is released at noon by the clock at Cambridge, which acts automatically through the telegraph line to the building of the Equitable Life Assurance Co., on Devonshire Street, where the ball

is mounted. If, from any cause, the ball fails to drop at noon, it is released, in the manner just explained, at precisely five minutes after noon. Since July 12, a record of its performance has been kept, which is compared below with the records for a period of five years of the performance of the time-ball at Deal, England, dropped by the Royal Observatory at Greenwich. The figures show the percentage of results of each kind mentioned: —

	Time-ball at Boston.	Deal.
Dropped by telegraph at $12^h\ 0^m$	89	86
,, ,, ,, ,, $12^h\ 5^m$	7	
,, ,, hand	1	9
Failed to drop	3	5

The success attained in the working of the time-ball at Boston is largely due to the zeal and skill displayed by Mr. Orrin Parker, Sergeant U. S. Signal Service, who has been stationed at Boston, and in charge of the ball since its establishment. He has been efficiently assisted by Messrs. J. H. Righter and F. W. Conrad, also of the Signal Service.

Arrangements are in progress with the Western Union Telegraph Co., for the distribution of time-signals to various cities desirous of receiving them from the Observatory. Inquiries have been made of the superintendents of New England railroads with regard to several points connected with proposed extensions of the time-service, the answers to which may be stated in brief as follows: —

Ninety per cent of the railroads consider it a convenience to travellers to have all trains run by the same time throughout New England. Ninety-three per cent consider that a uniform standard of time distributed over their lines would lessen the risk of accident; ninety-three per cent advocate a general distribution of the signals over all the New England lines at the same instant of time. A recourse to legislation to produce uniformity in railroad time is not generally favored, fifty-three per cent of the roads considering that the proposed object can be best accomplished by mutual agreement. For a time to use as the standard, sixty per cent favor Boston (the signals now sent from the Observatory), seventeen per cent New York, and the rest various intermediate standards. The standard now in use is in forty-seven per cent of the cases Boston time; in

twenty-three per cent New York time; and in others various local times.

It may here be observed that no inconvenience can follow from the use of one standard of time over a large tract of country, even if it differs several minutes from the local time; and again, that if the use of local time is preferred, the same signals will still serve different places, which have only to consider the moment at which a signal is received as differing by a fixed number of minutes from noon or any other moment of Boston time (for example) selected for the transmission of the signal.

The new clock-room, described last year, has given complete satisfaction. Some changes have been made in the telegraphic circuits, so that either of the two lines from Cambridge may be available for dropping the time-ball as well as for distributing the signals. Messrs. E. Howard & Co. have placed one of their clocks, No. 191, upon the granite pier in the clock-room mentioned last year as designed for this purpose. This clock will be available as a substitute in case of need for the standard mean-time clock, Bond 394. Its performance is good, and it is kept within a small fraction of a second of the standard clock. The error of the standard clock, Bond 394, at 10 A.M. of each day (determined by clock comparisons, and of course not known with certainty when a long period of cloudy weather has occurred), has exceeded seven-tenths of a second on two days only during the year. Since July 1 it has exceeded one-tenth of a second on nine days only.

The reduction and publication of the past observations has been pushed as vigorously as our means permitted during the year. The two gaps in our series of Annals have been filled by the publication of Volume IV. Part 2, and Volume IX., so that the set is now complete from Volume I. to X. The amount of material still to be published will fill nine or ten volumes; that is, it will occupy almost as much space as all that the Observatory has published for thirty years past. If the observations were published in detail, as is the case at the Observatories of Greenwich and Washington, and as has been done heretofore at this Observatory, over a dozen volumes would be required to contain the observations already made. This includes only the material of whose publication there can be no question. Much remains which will not be issued, the lapse of time and other causes having seriously impaired its value.

This accumulation of valuable observations, the result of years of persistent labor, and of the expenditure of many thousands of dollars, is at present liable to destruction by fire or other accident. There is no absolute security from this danger until the volumes are published and distributed. The greatest need of the Observatory at the present time is the means of preparing this material for the press. By a sufficient increase in our clerical force, the expense need not be very great. It is not good economy, however, that much of my own time and that of my assistants should be spent on work which may be done by any copyist, as is now the case. Delay in publication not only diminishes the value of the work, but greatly increases the difficulty of preparing the manuscript for the printer. A delay of several years often renders it almost impossible to supply the details necessary to give the volume its greatest value. Moreover, there is danger that good work done here may be needlessly repeated elsewhere, if its publication be long delayed.

It seems hard that Professor Winlock, although for nearly ten years Director of this Observatory, did not live to see any of his own observations published, or even to complete the work of his predecessors. To avoid the increased accumulation of unreduced observations, a system of book-keeping has been introduced in the equatorial work, by which every day the observations of the previous evening are reduced as far as possible, and the results entered on sheets ready for the printer. This has enabled us to begin already the publication of the photometric work of the equatorial, although the entire system of observations is not yet completed.

During the past year an approach for wagons to the west wing of the Observatory has been laid out. A gardener has been employed for a large part of the summer in various improvements of the grounds, which have not been charged to the Observatory or College. No important change has been made in the building. Some upholstering, refurnishing, and additional book-shelves are much needed in the library and computing rooms. More pressing demands have prevented such an expenditure as is desirable on the library, and no shelf room now remains for the later accessions. The instruments remain in good condition, and have not suffered the deterioration to which they are liable when not used. The convenience of using the

large equatorial has been greatly increased by the addition of large finding circles, and during the present month by the substitution of a new tail-piece.

The peculiar advantages already enjoyed by this Observatory make it apparent that even a comparatively small increase of its resources may reasonably be expected to yield more immediate and valuable returns than can in any ordinary cases be looked for from money laid out in the encouragement of science. Besides the libraries, scientific collections, and laboratories of the University, we have the similar institutions of Boston close at hand. Instrument makers of the very first rank in their profession are established in the neighborhood, and the instrumental equipment of the Observatory is in many respects unsurpassed elsewhere. Funds for the publication of observations which have been made ready for the press, and for the payment of current expenses (such as repairs, fuel, lights, and stationery), are already at our disposal. The true test of the power of an observatory, however, must be the income which is available for its strictly scientific work, after the mere cost of its maintenance has been paid. The amount of work done will obviously be directly proportional to this surplus, so long at least as it remains insufficient to give full employment to the principal instruments of the institution.

In the present case, any increase of the income of the Observatory may be directly applied to the promotion of scientific researches, and will accordingly yield unusually large returns in the form of published observations.

A year ago, the Visiting Committee, recognizing the great need of an increased income for the Observatory, undertook to supply this want. To these gentlemen my best thanks are due for their vigorous measures, which promise to instil new life into the Observatory. I must especially mention Messrs. Agassiz, Amory and Bowditch, as it was their energy and enthusiasm which overcame all the obstacles which presented themselves. A subscription was started to raise five thousand dollars a year for five years, in sums not less than fifty dollars annually. It was hoped that if the immediate needs of the Observatory could thus be met, results would be obtained that would render it easy hereafter to secure an increase in our permanent endowment.

A circular was distributed asking for subscriptions, payable in case the entire amount was raised. During the year ending Oct. 31, 1878, to which this Report relates, subscriptions amounting to thirty-two hundred dollars annually have been made. Since then, the subscription has proceeded very satisfactorily; and it is confidently expected that in a short time the entire amount will be received. As a complete list of the subscribers cannot, therefore, be given, it appears best to defer until next year's Report an acknowledgment of the individual subscriptions. I desire at this time to express to those who have already promised their assistance the great obligation the Observatory is under to them for the generous manner in which they have responded to its needs. The assurance of friendly sentiments toward the Observatory given by those who were unable to aid in this work has also been most gratifying.

Three subscriptions, by Mrs. Forbes, Mrs. Brooks, and Mr. Appleton, were made without the condition that the entire sum should be secured. This generous action has permitted the construction of the new photometers referred to above.

If the subscription succeeds, both telescopes will be kept in active use. The Equatorials will be employed in photometric observations of Jupiter's satellites, of variable stars, and of the standards with which they have heretofore been compared. A chart will be prepared of all the stars in the immediate vicinity of the pole, and standards for future reference of star magnitudes will be measured. The two micrometers noted above, if they accomplish all they promise, will open wide fields of work. It is hoped that observations with them may be reduced to a routine, and the work of the large telescope carried late into the night by observers employed for this purpose only. Next autumn, special attention will probably be paid to the planet Mars, which will then be more favorably situated for observation than will again be the case for many years. An extended plan of work has been laid out for the Meridian Circle. It is proposed to determine the absolute positions of a series of standard stars, independently of previous observations. The errors now carried from one catalogue to another will thus be eliminated. This will involve observations at short intervals during a greater portion of the day and night. The reduction of the work with both instruments will be kept up with the

observations, as far as possible. It is hoped that at no distant day the past observations will be published, and new volumes of Annals issued within a few months of the completion of the observations.

It constantly happens that accurate measures of length are required in various portions of our astronomical work. The Prime Vertical Room is well adapted for this purpose, and will probably be fitted up for it. At present, it is used only for storing the duplicates of the Library, and other works not often consulted.

To recapitulate: Both telescopes have been in constant use, the distribution of standard time has been extended, and much progress has been made in the reduction of the past observations; but the present income of the Observatory will not permit this rate of work to be maintained. An increase of five thousand dollars a year for five years has been solicited, and if obtained will, during that time, more than double the scientific results of the Observatory, whose endowment, grounds, buildings, instruments, and library, represent a sum of more than $300,000. But little remains to be promised in order to secure the whole. If this can be done, rapid progress can be made in the publication of the accumulated mass of past observations, which are in danger of destruction by fire so long as they remain unprinted; and an activity can be maintained throughout the institution commensurate with its reputation at home and abroad.

EDWARD C. PICKERING.

[*From the American Quarterly Microscopical Journal, April, 1879.*]

ON TWO FORMS OF COMPARATORS FOR MEASURES OF LENGTH.

BY PROF. W. A ROGERS.

THE subdivision of a given unit into exactly equal parts, is a problem of extreme difficulty, and the difficulty rapidly increases with the length of the unit to be divided. The measurement of the subdivisions, presents almost equal difficulties, though of a somewhat different character.

Two methods offer themselves for the solution of the problem:

1. We may assume the smallest subdivision to be an aliquot part of the entire unit, and then obtain that unit by successive increments of this constant quantity. If the number of subdivisions is large, it will be found practically impossible to measure and repeat this constant with sufficient accuracy to obtain the whole unit with the required precision. Suppose, for example, we require the meter—even granting that the true value of the millimeter could be found, it would still be impossible to get an exact value of the meter by 1000 repetitions. An error of only one ten-thousandth of a millimeter in the assumed value will, in the whole meter, amount to one tenth of a millimeter .Through unavoidable accidental errors, the final deviation from an exact meter would doubtless exceed this amount. A fundamental objection to this method is found in the fact that so much time would be required to complete the measurements, that the changes introduced through a variation of the temperature could not be neglected.

2. We may assume the entire unit, and then obtain the subdivisions according to the following scheme:

(*a*) Subdivision into 2 equal parts.
(*b*) Subdivision into 4 equal parts.
(*c*) Subdivision into 5 equal parts.
(*d*) Subdivision into 10 equal parts.

The fundamental principle which must govern the construction of a comparator, is the requirement that these large subdivisions shall be easily made with great precision and within so short a time that the effect of a slight change of temperature can be neglected. As a check we have:

$$(b)=2\ (a).$$
$$(c)=2\tfrac{1}{2}\ (a).$$
$$(d)=2\ (c).$$
$$(d)=5\ (a).$$
$$(d)=2\tfrac{1}{2}\ (b).$$

When the relations between the subdivisions into 10 equal parts have been found, each one of these tenths may be again subdivided as before, without danger of accumulating either accidental or systematic errors, or even errors which would be introduced through a change of temperature, if not more than ten or fifteen minutes is required for the entire operation. The writer has found that for air contact with a long bar of metal, whether of iron, brass, or steel, a change of several degrees in the temperature, as indicated by a thermometer in air, requires over thirty minutes to produce a perceptible change in the length.

When we reach those subdivisions which fall within one field of the microscope it would seem better to modify, in some respects, the requirements which should govern the construction of a comparator.

The ordinary method of measuring short lengths is either by means of an eye-piece micrometer, in which the lines in the focus of the ocular are ruled on glass, or with the ordinary filar micrometer. There are quite a large number of micrometers of the latter class in use in this country. The one which I have used with great satisfaction was loaned to me for some experiments by Mr. F. Habirshaw, of New York. It was made by R. & J. Beck, and is excellent in every way. I believe Powell & Leland also make excellent micrometers of this class. I am not aware that any American maker has given attention to this important adjunct of the microscope.

These two methods of measuring short lengths are both open to two serious objections, both of which are inherent in the method of construction and use. First, even with a ¼ objective one is limited to a field not much exceeding ¼ mm., and secondly, in neither case can the contact of the lines of

the eye-piece with the lines under the objective be made in the center of the field.

These two forms of micrometers for measurements of short lengths are, I think, the only kinds in use in this country. Abroad, however, a micrometer screw which carries the object to be measured in a plane perpendicular to the line of collimation of the microscope is in frequent use. The screw has a divided head, from which the magnitude of the space measured is read directly. The writer has seen only the form made by Merz & Son, of Munich.

The introduction into this country of this form of a comparator for short lengths is due to Mr. Leonard Waldo, assistant in Harvard College Observatory. During his visit to the continent, in the summer of 1877, he purchased of Merz & Son two of these comparators. One of them he attached to a grand stand by Crouch, constructed upon a special order, with reference to great stability. It is described in a paper on "Measures of Short Lengths," printed in the *Proceedings of the American Academy of Science and Art*, Vol. XIII., page 352. The other one was bought for the physical laboratory of Professor John Trowbridge, and was kindly loaned to me by him for some experiments. It was fitted to the Tolles stand, shown in Fig. 8. It is marked *A* in the drawing.

It was found, after a somewhat extended trial, that this comparator answered the purpose of its construction admirably, simply as a comparator, in which the same part of the screw was used to compare different spaces, but it was found inadequate to the measurement of absolute lengths in terms of the screw. It will be seen from the cut that the screw passes through a short nut (*a*). As the plate which carries the object to be measured is held against the oval end of the screw by parallel springs, which have a maximum tension of four pounds, the screw has an inevitable tendency to wabble, as the leverage becomes greater by running the screw out its entire length.

In order to remedy these defects, Mr. Geo. B. Clark, of the firm of Alvan Clark & Sons, constructed for me a comparator of the design shown in the accompanying illustration. The comparator proper consists of a bed-plate, within which is fitted a slide, carried by the precision screw *b*. The object to be measured is held in position upon the moving plate by the clips shown in the cut. Instead of two parallel springs

there is a single cord attached to the center of the moving slide, which runs on the guide pulley (*d*), and is attached to a spring, not shown in the cut, which is fastened to a pin on the back side of the bed, a little to the right of and below *b*. The action of the spring, therefore, is wholly in the line of the screw, and as the direction of the cord falls a little below the motion of the slide, it has a slight tendency to keep the slide in contact with its seat without introducing friction. The screw (*c*) moves the whole bed-plate, including the precision screw (*b*). The whole comparator has a circular movement in the socket (*f*), attached to the original sub-stage (*e*) of the microscope. The Beck filar micrometer is shown at *h*, and an eye-piece with a micrometer, having some advantages over the usual form, is shown at *i*. Slow motion to the tube is given through the lever *g*.

The operation of using the comparator is as follows:

After the slide containing the graduations to be compared has been placed in proper position under the objective, with the right hand, the screw-head (*b*) is set at the zero of position; with the left hand, line 1 is brought in contact with a single line of the eye-piece micrometer; with screw *b*, line 2 is brought in contact with the fixed line of the eye-piece micrometer, and the number of revolutions and parts of a revolution are read off. Screw *b* is then brought back to zero, and the setting is made on line 2 by means of the screw *c*. In moving over the space from line 2 to line 3, with the screw *b*, it will be seen that *the same part of the screw is used as in going from line 1 to line 2*. Hence, the comparison of these two spaces is *independent of the errors of the comparing screw*.

The number of spaces which can be compared in this way is only limited by the length of the screw *c*, and the length of the opening through the bed-plate.

Again, suppose we measure the spaces 1, 2, 3, 4,..100, by a continuous forward motion of the screw. Such measures will involve all the errors of the screw itself. But if after the measures are made, we set the screw *b* back at zero, turn the ruled plate around 180°, and set on line 100 with screw *c*, the continuous forward motion of the screw *b* from line 100 to line 1 will be over the same part of the screw as from line 1 to line 100. In the first case, the screw measures the accumulated errors of the ruled plate from line 1 to any point up to line

100, but such measures involve the errors of the comparing screw. In the second case, the accumulated errors are measured in the same way from line 100 to line 1. But if we subtract the measures from line 1 to line 100 from the corresponding measures from line 100 to line 1, *the difference will give twice the accumulated errors at any point, independent of the errors of the comparing screw.* The only exception to this rule is found when the curve of errors takes a wave form. In a general way, this will be the case when the maximum error falls near line 25, and the minimum near line 75.

As an illustration of the character of the work which may be done with a comparator of this form, I give the measures of five standard micrometers, ruled at different times. As these micrometers are somewhat different in form from any with which the writer is acquainted, a brief description will be necessary.

I. A half inch is divided into 50 equal parts, the 1st, 25th and 50th spaces being again subdivided into 10 equal parts. The length of the lines is about one-eighth of an inch, the 5th and 10th lines being a little longer.

II. After arranging the position of the ruling carriage, so that the lines of the second series of graduations should begin near the point where those of the first end, coincidence is made mechanically with the first line of the series already ruled. For a short distance the ruling point goes over the same ground twice. A centimeter is then subdivived into 10 equal parts. The 1st, 5th, and 10th spaces are again subdivided into 10 equal parts, and one of the middle subdivisions is still further subdivided, giving .01 mm. Near by is a band of 21 lines, each space being equal to .001 mm.

The first five columns of the following table give the number of divisions of the comparing screw corresponding to each space of .01 inch. The values for No. 7 are the mean of two readings. The remaining values represent single measures. The mean values given in the sixth column, without doubt, nearly represent the actual errors of ruling, the accidental errors of measuring being nearly eliminated in taking the mean of the corresponding measures of the separate plates.

The last column represents the accumulated errors at every point between line 1 and line 50, expressed in millionths of an inch. With the sign given, the values represent *corrections* to the corresponding spaces:

MEASURES OF 50 SPACES. EACH SPACE=.01 INCH.

Spaces.	No. 7.	No. 8.	No. 10.	No. 11.	No. 12.	Mean.	Cor. in terms of screw-head.	Cor. in millionths of an inch.	Accumulated errors in millionths of an inch.
1	79.95	80.02	80.02	80.00	80.03	80.00	—.03	—4	—4
2	79.92	79.90	79.90	79.95	80.00	79.93	+.05	+6	+2
3	79.88	80.00	79.98	80.00	79.83	79.94	+.03	+4	+6
4	79.91	80.00	79.97	80.00	79.85	79.95	+.03	+4	+10
5	80.02	80.00	80.03	80.05	80.00	80.02	—.05	—6	+4
6	79.99	80.00	80.00	80.00	79.94	79.99	—.01	—1	+3
7	79.90	79.90	79.98	79.92	80.00	79.94	+.03	+4	+7
8	79.98	80.00	80.05	79.98	79.98	80.00	—.02	—2	+5
9	79.99	80.05	80.00	79.97	80.00	80.00	—.03	—4	+1
10	80.03	79.92	79.92	80.00	79.93	79.96	+.02	+2	+3
11	79.95	79.89	79.97	79.90	79.99	79.94	+.03	+3	+6
12	79.99	80.02	80.00	79.94	80.00	79.99	—.01	—1	+5
13	79.99	79.90	80.00	79.98	79.94	79.96	+.01	+1	+6
14	80.01	80.00	80.06	80.01	80.00	80.02	—.04	—4	+2
15	80.00	79.92	80.00	79.97	79.90	79.96	+.02	+2	+4
16	80.00	79.98	80.00	79.99	79.92	79.98	—.01	—1	+3
17	79.98	80.00	79.96	80.00	80.01	79.99	—.01	—1	+2
18	79.99	79.97	79.90	79.96	79.92	79.95	+.03	+4	+6
19	79.90	79.97	80.00	79.90	80.00	79.95	+.02	+3	+9
20	79.98	79.99	79.90	79.93	79.90	79.94	+.04	+5	+14
21	80.06	79.92	80.00	80.00	79.98	79.99	—.02	—3	+11
22	79.97	79.92	79.85	79.89	79.97	79.92	+.06	+7	+18
23	79.91	79.90	79.92	79.90	79.96	79.92	+.05	+6	+24
24	79.90	80.00	80.00	80.00	79.97	79.97	+.01	+1	+25
25	80.04	80.03	80.00	79.93	80.00	80.00	—.03	—4	+21
26	79.92	80.02	79.99	80.01	80.03	79.99	—.01	—1	+20
27	80.01	79.98	79.98	80.00	80.00	79.99	—.02	—2	+18
28	79.90	80.00	79.92	80.05	80.00	79.97	+.01	+1	+19
29	80.00	80.04	80.00	79.95	80.03	80.00	—.03	—4	+15
30	80.02	80.06	80.10	79.90	80.05	80.03	—.05	—6	+9
31	79.98	80.00	79.96	79.95	79.96	79.97	+.00	+0	+9
32	79.99	79.93	79.90	80.00	80.04	79.97	+.01	+1	+10
33	80.02	79.97	80.02	80.03	80.00	80.01	—.03	—4	+6
34	80.01	80.00	79.99	80.00	80.06	80.01	—.03	—3	+3
35	79.98	80.05	80.00	79.97	79.88	79.98	—.01	—1	+2
36	79.99	80.00	80.00	79.89	80.00	79.98	+.00	+0	+2
37	79.98	80.02	79.90	79.96	80.00	79.97	+.01	+1	+3
38	79.95	80.00	79.89	79.94	80.02	79.96	+.02	+3	+6
39	80.00	80.10	80.03	79.93	80.03	80.04	—.06	—7	—1
40	79.93	79.90	79.90	79.94	79.93	79.92	+.05	+6	+5
41	80.00	80.03	80.02	80.00	80.00	80.01	—.03	—4	+1
42	79.90	79.98	79.98	79.93	79.91	79.94	+.03	+4	+5
43	79.98	79.96	80.03	80.00	80.00	79.99	—.02	—2	+3
44	80.05	79.99	79.93	79.97	80.00	79.99	—.01	—1	+2
45	80.01	80.00	80.00	79.98	79.98	79.99	—.02	—2	+0
46	79.93	80.03	80.00	79.98	80.00	79.99	—.01	—1	—1
47	79.99	80.07	79.98	79.98	79.98	80.00	—.03	—4	—5
48	79.93	79.98	79.94	79.93	79.95	79.95	+.02	+2	—3
49	79.98	79.93	79.95	79.95	80.00	79.96	+.01	+1	—2
50	79.95	79.96	79.95	79.99	79.97	79.96	+.01	+1	—1
Means.	79.97	79.99	79.98	79.97	79.98	79.98			

One division=.00012504 inch.

MEASURES OF MILLIMETER DIVISIONS.

Spaces	Divisions of Comparator. Number 7.	Number 8.	Number 10.	Number 11.	Number 12.	Mean.	Corr. in terms of Screw-head.	Corr. in hundred thousandths of a cm.	Accumulated error, in hundred thousandths of a cm.
1	314.87	314.98	314.90	315.00	315.03	314.96	—.03	—1	—1
2	314.85	314.87	314.88	314.98	314.93	314.90	+.03	+1	+0
3	314.90	314.85	314.95	315.00	315.02	314.94	—.01	—0	+0
4	314.95	315.06	315.05	315.08	314.98	315.02	—.09	—3	—3
5	314.92	314.83	314.85	314.90	314.97	314.89	+.05	+2	—1
6	314.88	314.85	314.86	314.93	314.90	314.88	+.05	+2	+1
7	314.93	314.92	314.93	314.90	314.98	314.93	+.00	+0	+1
8	314.97	314.98	314.95	315.00	314.91	314.96	—.03	—1	+0
9	314.86	314.85	315.00	314.98	314.92	314.92	+.01	+0	+0
10	314.95	314.85	314.97	314.90	314.95	314.92	+.01	+1	+1
	314.91	314.90	314.93	314.97	314.96	314.93			

One division=.0003175 cm.

It will be seen that the individual errors of graduation are practicably insensible. It is not supposed that the figures which represent millionths of an inch are reliable to the last unit. They are given in the sixth decimal place in order to make sure of the figure in the fifth place.

It is now to be noted that the errors thus obtained are entirely relative errors. They give no indication whatever of the absolute value of any of the spaces measured. If the entire length of the half inch is, *e.g.*, .001 inch too long, to each of the corrections given in the table, must be applied still further the correction, .000020 inch.

It is, therefore, necessary to make a careful investigation of the entire length of the half inch and of the centimeter.

This is done with a comparator adapted to the comparison of spaces, ranging from coincidence to an entire yard or an entire meter. Comparators of this class are usually constructed with two sliding plates, each carrying its own microscope. A fundamental objection to this form is found in the fact that the microscopes cannot be brought much nearer together than three inches, by any direct means. For lack of space and of illustrations, only a general description of the form which the writer has had constructed, can be given here. It consists

of an iron bed, 60 inches long and 14 inches wide. V-shaped grooves, 6 inches apart, run the entire length. In the center of the bed, a fine-toothed rack reaches from end to end. Two sliding plates are carried along the ways by means of a pinion set in the center of the plates, and working so loosely in the rack that the slides are free to follow the law of gravity. A microscope is attached to each plate, giving the form usually adopted. Instead of two microscopes, however, it is found better to use but one. The microscope plate is followed on either side by plates terminating in tempered steel stops, which are at will either made free or clamped firmly to the bed of the comparator. If one wishes to compare two meters, the method of proceeding is as follows:

(*a*) One stop is set at or near one end of the bed.

(*b*) The meter with which comparison is to be made is placed in position under the microscope, so that contact is made between the end line and the zero line of the eye-piece micrometer.

(*c*) The microscope plate is then moved by means of the rack and pinion till the other end line forms contact with the zero line of the micrometer.

(*d*) The second stop is then brought up against the other end of the plate and adjusted so that when contact takes place between the stops, contact also takes place between the end line and the zero line of the micrometer.

(*e*) Having made the adjustment of the stops perfect, the meter to be compared is then placed in position. When contact is made with the first stop, by mechanical adjustment, the end line is brought in contact with the zero line of the micrometer. The microscope plate is then brought into contact with the second stop. If the other end line is now in coincidence with the zero line of the micrometer, the two meters have the same length. By noting the number of divisions which the end line falls short of, or passes beyond the zero line of the micrometer, the difference in the entire length can be found; the only element yet unknown being the value of one division of the micrometer.

After the comparison has been made, it is better, or a matter of precaution, to again compare the standard with the distance between the stops. Since the stops can be set in actual contact with the microscope plate at either end, it is obvious that this method admits of a comparison of short spaces as well as of long ones. The only criticism which I imagine will be

urged against this form of construction, is that founded on a doubt whether the contact between the stops always indicates the same measured space. The arm of the pinion has a head of about 2½ inches in diameter. In my own case the sense of touch has been so far cultivated that I am able to make 100 successive contacts without a single deviation exceeding .000035 inch, and very few deviations reach .000010 inch. A comparator of this form possesses one decided advantage over all others, viz., that after the stops are once set, *any adjustment of the microscope may be made without interfering with the comparison.* The only condition required is that the relation between the stops and the bed shall remain unchanged during the short time required for the comparison. This does not usually take over 10 minutes.

In order to compare separate subdivisions of the same standard we proceed as follows:

The stops are set, *e. g.*, equal to 1 decimeter. After the reading of the first decimeter has been taken, as indicated above, the bar is then moved along till the first line of the second decimeter forms a contact with the zero of the eye-piece micrometer, when at the same time contact is formed with the first stop. Moving the plate to the second stop the reading for the second decimeter is taken. A comparison of the several values obtained with the mean value, will show how much each is in error, provided the entire length is correct.

For the present, all standards of length made by the writer will be referred to a line yard and meter bar, constructed by the U. S. Coast Survey for Stevens' Institute, Hoboken, and very kindly loaned to me by Professor Mayer. It is marked Y. & M., No. 2.

Professor J. E. Hilgard, Assistant in charge of the U. S. Coast Survey Office, has kindly communicated to me the statement that the yard is standard at 62° F. It was constructed from the yard known as "Bronze 11." The meter was constructed from the iron meter in the possession of the Coast Survey, which was one of four bars from which the platinum bar of the Archives was constructed. It is believed to be the only one of the originals now in existence. There is still some doubt about the exact temperature at which copies of this bar are standard. It is somewhere between 67° and 69° F. A careful series of comparisons made at the Coast Survey Office during the Summer of 1878 gave 67.7°.

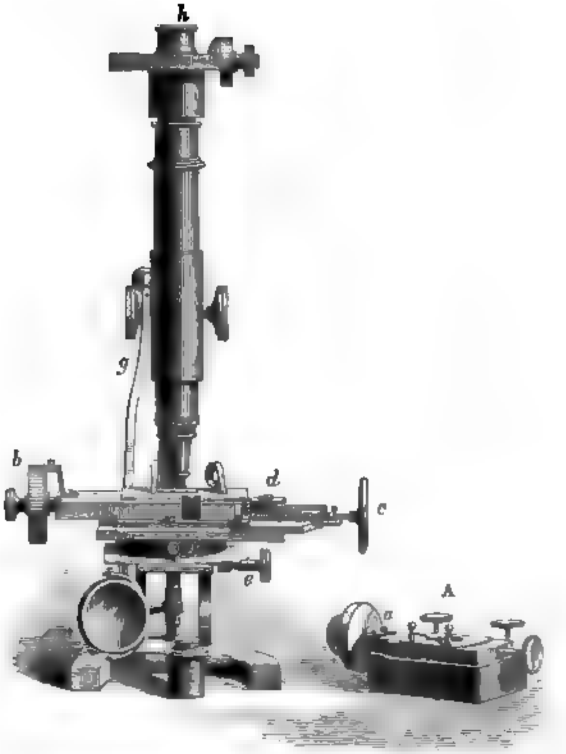

FIG. 8.

The bar which I have for the present adopted as a standard is of steel, and has the yard and its subdivisions on one side of the edge, and the meter and its subdivisions on the other. The bar is 40½ inches long, 1¾ inches deep, and ½ inch wide.

COMPARISON OF THE STEEL YARD WITH THE STANDARD.

Both the steel bar and the brass standard were placed in a wooden trough, with their graduated surfaces nearly in the same horizontal plane. The stops were set to correspond nearly to the end lines of the standard, after the trough had been filled with water having nearly the temperature of 62° F. The temperature of the room was then brought up to such a point that after several hours the thermometer immersed in water indicated 62°. The following comparisons were then made:

Series.	Thermometer.	Steel Yard too Long.
I.	61.0	.00134 inch.
II.	61.1	.00146
III.	61.1	.00132
IV.	64.1	.00065
V.	66.8	.00047
VI.	68.3	.00025
VII.	68.5	.00031

In order to get the error at 62° we may assume:

$$\begin{aligned} 134 &= a \text{———} \\ 146 &= a + .1b \\ 132 &= a + .1b \\ 65 &= a + 3.1b \\ 47 &= a + 5.8b \\ 25 &= a + 7.3b \\ 31 &= a + 7.5b \end{aligned}$$

Solving by least squares, we get:

$$\begin{aligned} a &= +131 \\ b &= -17 \end{aligned}$$

For 62° we have for the error (E):

$$\begin{aligned} E &= a + 1.0b \\ &= 131 - 17 = +114 \end{aligned}$$

At 62°, therefore, the steel yard is:

.00114 inch too long.

COMPARISON OF STEEL METER WITH THE STANDARD.

Series I. Steel meter was found to be .000050 cm. too long at 68.6°.

Series II. Steel meter was found to be .000008 cm. too short at 68.7°.

On account of the doubt still remaining with regard to the temperature at which the brass meter is standard, it will be assumed for the present that the two standards have the same length.

Comparison of decimeter divisions of steel bar:

Spaces.	Corrections.
1	+000170 cm.
2	+000178
3	−000050
4	+000000
5	+000050
6	−000114
7	+000093
8	−000253
9	+000040
10	−000112

Since the fourth decimeter has no correction, this space was chosen for an examination of the centimeters composing it. The following errors were found :

Cm. Spaces.	Corrections.
1	—.00029 cm.
2	—.00008
3	—.00033
4	+.00038
5	+.00082
6	—.00090
7	+.00033
8	+.00008
9	—.00096
10	+.00104

By applying these corrections, any one of these centimeters becomes a standard *with referencc to the standard chosen.*

The steel bar is now replaced in the water-bath, and comparison is made directly with any centimeter whose value is desired. Comparing with space 2 and space 8, the error of No. 12 of the above series was found to be as follows :

Compared with space 2 at 68°, No. 12 is .000019 cm. too long.

Compared with space 8 at 61°, No. 12 is .000005 cm. too long.

When the last degree of precision is required, the stops should be set independently for each centimeter of the standard bar, with its proper correction applied, and the centimeter whose value is desired should then be compared with the distance between the stops in each case.

Finally, we have a severe test of the accuracy of the graduation in the value of the centimeter in terms of the inch. The end line of the centimeter falls between line 39 and line 40 of the inch. The following measures give the distances from line 39 to the end line of the centimeter:

	Div. of Comparator.	In parts of Centimeter.
No. 7	= 29 56 =	.003696
8	= 29.74 =	.003719
10	= 29.72 =	.003716
11	= 29.56 =	.003696
12	= 29.64 =	.003706
	Mean,	.003707

The value of the centimeter in terms of the inch is

1 cm.= 393707 inch.

The value generally given is

.393708

Since the value obtained involves the mechanical error of making a coincidence between the first lines, this agreement is rather more close than ought to be expected.

Harvard College Observatory.

XV.

ON THE PRESENT STATE OF THE QUESTION OF STANDARDS OF LENGTH.

BY W. A. ROGERS.

Presented April 14, 1880.

IT is not my intention in this paper to enter into a minute account of the construction and comparison of the various standards of length which have been made the basis of measurements, either in trigonometrical surveys of the earth's surface, or in more strictly physical investigations. Many of these possess a certain historical interest, even when they have but little inherent value. For information of this kind, the reader is referred to the references at the end of this paper.

I shall confine myself to a consideration of those standards of length which are in actual use, and which have the authority and sanction of either national or international law.

Three natural units have been proposed as the basis of a standard of length, as follows: —

I. The length of a pendulum beating seconds in a vacuum at the level of the sea in the latitude of London.

II. One ten-millionth part of a quadrant of the earth's surface.

III. The length of a wave of light of given refrangibility.

It is generally supposed that the yard of Great Britain was founded upon the first of the natural units named, but it will be seen from the act of Parliament legalizing the standards prepared by the Royal Commission, signed June 17, 1824, that the reference of the standard of length to this unit refers to its restoration in case of loss or destruction, and not to its original construction. Notwithstanding many experiments were made at this time by Kater and others for the purpose of ascertaining the length of the standard expressed in terms of the length of a seconds pendulum, the yard actually legalized was con-

structed from Graham's scale by Bird, in 1760. It is marked, "Standard 1760."

The reason assigned for selecting Bird's "Standard 1760" was its close agreement with Shuckburgh's scale (0 – 36$^{in.}$), made by Troughton in 1798, with which the pendulum and the meter had been compared, and of which a fac-simile was known to exist at Geneva. It is interesting to note, in passing, that, previous to Shuckburgh, all transfers were made by means of beam compasses, and all comparisons were made in the same way. The first use of optical means of comparison must be credited to Troughton in 1798, when he transferred Bird's scale for Shuckburgh. The following is the act legalizing the standards:—

"SECTION I. Be it enacted . . . that from and after the first day of May one thousand eight hundred and twenty-five, the Straight Line or Distance between the Centres of the Two Points in the Gold Studs in the Straight Brass Rod, now in the Custody of the Clerk of the House of Commons, whereon the Words and Figures 'Standard Yard 1760' are engraved, shall be and the same is hereby declared to be the original and genuine Standard of that Measure of Length or lineal Extension called a Yard; and that the same Straight Line or Distance between the Centres of the said Two Points in the said Gold Studs in the said Brass Rod, the Brass being at the temperature of Sixty-two Degrees by Fahrenheit's Thermometer, shall be and is hereby denominated the 'Imperial Standard Yard.' . . . .

"SECTION III. And whereas it is expedient that the said Standard Yard, if lost, destroyed, defaced, or otherwise injured, should be restored to the same Length by reference to some invariable natural Standard; And whereas it has been ascertained by the Commissioners appointed by His Majesty to inquire into the subject of Weights and Measures, that the said Yard hereby declared to be the Imperial Standard Yard, when compared with a Pendulum vibrating Seconds of Mean Time in the Latitude of London in a Vacuum at the Level of the Sea is in the proportion of Thirty-Six Inches to Thirty-Nine Inches and one thousand three hundred and ninety-three ten-thousandth Parts of an Inch: Be it therefore enacted and declared, That if at any Time hereafter the said Imperial Standard Yard shall be lost or shall be in any Manner destroyed, defaced, or otherwise injured, it shall and may be restored by making a new Standard Yard, bearing the same Proportion to such Pendulum as aforesaid as the said Imperial Standard Yard bears to such Pendulum."

On the 16th of October, 1834, both houses of Parliament were con-

sumed by fire. The bar of 1760 was recovered, but in a condition which rendered it useless as a standard, one of the gold plugs having been melted out. It now became necessary to decide whether it should be restored in accordance with the act of June 17, 1824. Since the passage of the act, it had been shown that all the elements which were defined in the act for restoration were subject to some doubt. Dr. Young had shown that the reduction to the level of the sea was doubtful. Both Bessel and Baily had shown that the reduction for the weight of the air was erroneous. Baily had thrown doubt upon the estimated specific gravity of the pendulum employed, and upon the accuracy of the agate planes, while Kater himself showed that sensible errors had been introduced in comparing the pendulum with Shuckburgh's scale.

In view of these facts, all attempts to restore the lost standard in accordance with the act of June, 1824, were abandoned. Instead, it was decided to attempt the restoration of the lost standard from the various standards which had been previously compared with it. There were available for this purpose, —

Shuckburgh's scale ($0-36^{\text{in.}}$);

Shuckburgh's scale, with Kater's authority;

The yard of the Royal Society, constructed by Kater;

The Royal Astronomical Society's brass tubular scale;

Two iron bars, marked A_1 and A_2, belonging to the Ordnance Department, and preserved in the office of the Trigonometrical Survey.

The restoration of the standard was placed in the hands of Sir Francis Baily. At his death, in August, 1844, he had done little more than complete the provisional inquiries required before attempting the final construction. He had, however, after many experiments, decided upon the material of which the new standard should be composed. It has since his time been known as Baily's metal. Its composition is, copper 16, tin 2½, zinc 1.

Upon the death of Mr. Baily, the work of restoration was committed to the Rev. R. Sheepshanks. Sir George Airy, in his account of the construction of the new national standards of length, has given an exceedingly interesting communication from Mr. Sheepshanks (Philosophical Transactions, 1857, p. 661), detailing the means he employed for the restoration of the lost standard. He first constructed a brass bar, as a working standard. This "brass bar 2" was compared with all the standards which Mr. Sheepshanks considered properly available for this purpose, with the following results: —

	Inches.	
Brass bar 2	= 36.000280	by Shuckburgh [10 – 46 in.].
	= 36.000084	by Shuckburgh [0 – 36 in., Kater].
	= 36.000229	by Kater's scale.
	= 36.000303	by A_1 compared in 1834.
	= 36.000275	by A_2 compared in 1834.
Mean,	36.000234	

He assumed: —

Brass bar 2 = 36.00025 inches of the lost imperial standard at 62° Fahrenheit.

It is stated both in Appletons' and in Johnson's Encyclopædias that the yard of the Astronomical Society is the principal authority upon which the new standard rests. It will be seen from the above that this statement is erroneous.

The Imperial Standard Yard, known as "Bronze 19," or, according to the new nomenclature, as No. 1, was constructed according to this equation. It is made of Baily's metal, and has the following dimensions, viz.: —

Length = 38 inches.
Width = 1 inch.
Depth = 1 inch.

Gold plugs are inserted in wells sunk one half of the depth of the bar. The graduations are on these gold plugs.

Through the kindness of Mr. Chaney, the Warden of the Standards, I have recently had the pleasure of assisting him in comparing my own working standard yard with this bar. I have never seen lines better adapted to exact measurements. They have remarkably smooth edges, and are about one three-thousandth of an inch in width. Even for the most rigid scientific investigations, they are in every respect of unrivalled excellence.

Bronze No. 1 is the national standard, and is kept in what is known as the "Strong Room" of Old Palace Yard. Besides this bar, four Parliamentary copies were made, of which one copy is kept at the Royal Mint, one is in charge of the Royal Society, one is immured in the new Westminster Palace, and the other is kept at the Royal Observatory, Greenwich. Forty copies were prepared on Baily's metal. Of these only two are exactly standard at 62°, viz. Bronze 19, and Bronze 28, which is kept at the Royal Observatory as an accessible representation of the national standard. All the other copies have an equation with respect to No. 1; but instead of giving this equation,

the degree at which each bar is standard is given. These bars have all been distributed among different governments.

The standards prepared by Mr. Sheepshanks were legalized by act of Parliament passed July 30, 1855.

Two platinum bars — one a line-measure and the other an end-measure — form the basis of the metric standard of length at present in use in Great Britain. These bars have the following dimensions: —

	Line-Meter.	End-Meter.
Length,	41.0 inches.	39.37 inches.
Breadth,	1.0	1.00
Thickness,	0.211	0.287

The line-meter has the words "Royal Society, 45," engraved on the under side. The defining lines run nearly across the face of the bar, and there is no cross line to indicate the exact points from which measures are to be made. Arrows, arbitrarily placed, now indicate the points from which all the later measures have been made. The lines on this bar have become so obliterated that it is found impossible to see them with the method of illumination formerly in use. At the request of Mr. Chaney, I employed Mr. Crouch, of London, to construct two new objectives for the microscopes of the comparator, to which were adapted two of Tolles's interior illuminators for viewing opaque objects. The magnifying power was increased between five and six times, and the illumination was so much improved that there was no difficulty whatever in seeing the lines of this bar.

The end-meter has the words "Mètre à Bouts" engraved on one side, and the words "Fortin à Paris, Royal Society 44," on the other side. At the present time, this standard is not in a condition to admit of accurate measurements. The edges of the end surfaces are indented, and there is a raised burr on one end. These bars, together with the original bars compared by Hassler in 1832, are the only recognized standards which have ever been compared with the Meter of the Archives. They form, therefore, the real basis of our present knowledge of the relative length of the meter and of the yard. I shall presently recur to this important matter.

The line-meter was transferred by Troughton and Simms to a bar of Baily's metal in 1869. Kater's reduction to the Meter of the Archives was applied in the transfer. The Imperial Yard was also transferred to the same bar. The lines are drawn upon gold plugs, which are not, however, inserted in wells. At the distance of 39.37 inches from the first plug the "meter plug" is inserted. This meter is

the one from which Chisholm obtained his value of the equation between the meter and the yard. It is now the working standard meter of the Exchequer.

A second natural unit, viz. one ten-millionth part of a quadrant of the earth's surface, was adopted at the close of the last century as the basis of the metric system. In March, 1791, a committee of the Institute of France, consisting of fifteen members, including Borda, Lagrange, Laplace, Mongé, and Condorcet, recommended this unit as a standard of linear measure. It is interesting to note that the Commission was largely international in its composition. Besides the members of the Institute, there were delegates from the Republic of the Netherlands, from Sardinia, Denmark, Spain, Tuscany, the Roman Republic, the Cisalpine Republic, the Ligurian Republic, the Swiss Confederation, and from Piedmont. Their report was sanctioned by the Assembly, and an arc of the meridian passing through Paris, and extending from Dunkirk to Barcelona (stations differing about 10° in latitude), was measured with great care by Méchain and Delambre. By combining the results of this survey with arcs previously measured in Peru and Sweden, the length of a meridional quadrant passing through Paris was ascertained. For the measurement of base lines, two separate toise end-measures were employed, each assumed to be equivalent to the toise of Peru. From these standards, four iron bars were prepared, having their ends carefully ground and polished until they were exactly comparable with each other, and until each had the required length. One of these original bars, bearing upon it the stamp of the Commission, is now in the possession of the United States Coast Survey. It is understood to be the only copy now in existence. One bar was chosen as the standard of France, and from it was constructed the platinum "Mètre des Archives." Two similar meters, made at the same time and in the same manner, are now in existence, viz. the "Mètre du Conservatoire," and the "Mètre de l'Observatoire." All of these meters are end-measures, and have about the same dimensions as the English platinum meters already described. The equation between the "Mètre des Archives" and the "Mètre du Conservatoire" is small. That between the "Mètre des Archives" and the "Mètre de l'Observatoire" is not well determined. Only one line-measure is known to have been made at the time of the construction of the end-measures. This was transferred from the "Mètre de l'Observatoire," probably about two years after the adoption of the "Mètre des Archives" as a standard.

In 1870, a commission was formed at Paris, under the title "Com-

mission International du Mètre," for the purpose of settling all doubts in regard to the value and permanence of the unit bases. In 1874, this commission decided to maintain at Paris an "International Bureau of Weights and Measures, to be supported by *pro rata* contributions from all the signing powers, and charged with the care of the prototype standards, and with the duty of constructing and verifying copies of those standards, not only for the powers interested, but for other governments, for corporations, and even for private individuals who should apply for them, and who should be willing to pay the expense attending their construction and comparison."

It is not necessary to record here the considerations which led the Commission to abandon all attempts to establish a standard which should conform to the natural unit, in accordance with which the original meters were constructed. It is sufficient to say that the Commission decided that the "Mètre des Archives" shall be recognized and perpetuated forever as the true base of the measure of extension, without regard to the doubtful questions which have been raised concerning its correspondence with its theoretical value. The first resolution of the "Convention du Mètre," signed May 20, 1875, reads as follows: "For the execution of the international meter, the Mètre des Archives, in the state in which it is found, is taken as the point of departure."

The Convention adopted twenty-one resolutions respecting the meter, of which the most important are the following: —

"IV. Though deciding that the new international meter shall be a line-meter, of which every country shall receive identical copies, constructed at the same time as the line prototype, the Commission will nevertheless undertake to construct a certain number of end-measure standards for the countries which shall express a desire for them, and the equations of these end-meters with respect to the new line prototype shall be determined with equal care by the International Commission.

"V. The international meter shall be of the length of the meter at 0° C.

"VI. In the fabrication of the meters, an alloy shall be employed, composed of 90 parts of platinum and 10 parts of iridium, with an allowance of two per cent variation, more or less."

"IX. The bars of platinum-iridium, upon which the lines are to be traced, shall be 102 centimeters in length, and their transverse section shall be represented by a model described in a note by M. Tresca."

"XIII. The method of M. Fizeau shall be employed in determin-

ing the expansion of the platinum-iridium, of which the meters are formed.

"XIV. The prototype shall be submitted to the best possible process for the determination of the absolute coefficient of expansion of the whole meters. These measures shall be made separately, at least at five different temperatures comprised between 0° and 40° C."

"XVII. The comparisons shall be made by immersing the new standards in a liquid and in air, but with the reservation that the standard of the Archives shall not be immersed in any liquid until the close of the operations.

"XVIII. The tracing of the line-meters, and their first comparison with the 'Mètre des Archives,' shall be effected by the process of M. Fizeau."

In order to gain a clear understanding of what has been accomplished in accordance with these resolutions, it is necessary to distinguish between the operations of the French section of the Commission and those of the International Bureau. The construction of the prototypes from the "Mètre des Archives" was naturally committed to the French section, the primary standard being the property of the French government, the responsible authority for its preservation in a state of perfect integrity.

The prototypes thus prepared, after delivery to the International Commission, and after their formal acceptance by the Commission, are to be subjected to the various tests provided for by the articles of the convention. Finally, either one of these prototypes, or, perhaps, the mean of two or three of them, will be officially declared to be the true and final base of reference in all measurements of linear extension.

The International Bureau is now prepared for its share of the work. The buildings are completed, the instruments of comparison are in position, and nearly all of the provisional investigations required have been completed. The present delay must be charged to the unfortunate dead-lock between the Commission and the French section, the Commission refusing to accept the prototypes prepared by M. Tresca, mainly on the ground that the material of which the standards are made is not, in its present condition, a pure alloy of platinum and iridium, but that it contains, as is asserted, nearly two per cent of iron.

The International Bureau at the present time has no standards which have been compared directly with the "Mètre des Archives." It has three provisional meters, all of which have been indirectly compared with the primary standard. It has a meter made by Repsold, one made by Hermann and Pfister, of Berne, Switzerland, and one

made by Brunner Frères, of Paris, which is presumably a copy of the line-meter derived from the meter of the Observatory. The meter by Brunner Frères, the tracings on which are very good, is taken as the present provisional standard.

The buildings of the International Bureau are situated on the summit of a high hill at Breteuil, on the direct road from Paris to Versailles. The location is an admirable one in every point of view. During my recent visit to Paris, a very kind letter of introduction from Professor St. Claire Deville to Dr. Pernet, the Director of the Bureau, led to a very interesting visit to this establishment. There are three principal observing-rooms, perhaps eighty meters square. The walls are composed of corrugated metal, the waves running longitudinally. The rooms are lighted by a circular skylight only.

In one room is found the apparatus for comparing standards of weight, under the charge of Dr. Marek. There is here a marvellously perfect apparatus for weighing in a vacuum, made by Bunge, of Hamburg. The observer is enabled to perform every part of the operation of weighing, stationed at a distance of six or eight meters. There is also a fine balance by Rupprecht, of Vienna.

In a second room, the instruments for determining the coefficients of expansion are mounted. This department is under the charge of Dr. Benoit. Both the method by immersion in a liquid, and the method of M. Fizeau, are employed. The latter method is described in the *Comptes Rendus* for 1866. It is also described in the Proceedings of the Royal Society of Nov. 30, 1866. I quote from the Proceedings: "In M. Fizeau's observations, he has availed himself of the possibility of forming Newton's rings with the monochromatic sodium light, when one of the interfering rays is 52.205 waves in advance of the other, a fact which, conjointly with M. Foucault, he announced in 1849. Using the length of a wave of sodium light 0.0005888 mm. [$0.00002318^{\text{in.}}$] as a standard of measure, the position of a ring being observable to within $\frac{1}{10}$ of the distance between two consecutive rings, the *variation* of the distances between the two surfaces producing Newton's rings can be measured within 0.00002944 mm. or $0.0001159^{\text{in.}}$" The results by this method, and those obtained by immersion in a liquid, are found to show a good agreement. The chief objection to its use is found in the fact that the method is only applicable to pieces of metal not much exceeding one centimeter in length. It must be assumed that the whole bar has the same coefficient as the small portion of it upon which the observation rests. In the discussion of the coefficient of the Mètre des Archives, it will be interesting to compare the value finally derived with the value

found by M. Fizeau from a small section of the original bar which has been preserved in the Archives.

A third room of the building is devoted to the comparison of standards. This department is under the charge of Dr. Pernet. Here is mounted a fine comparator by Brunner Frères, built at a cost of 15,000 francs. A universal comparator by Starke, of Vienna, and costing 28,000 francs, will soon be in position. This apparatus will allow comparisons between standards two meters in length. There are also attachments for comparing subdivisions.

The Bureau has also a very perfect apparatus for determining the zero point and the boiling point of the thermometers employed in the comparisons. The *minimum* point of freezing is first found. To this minimum reading are applied the *variations* of the zero point, which are investigated for each thermometer. The thermometers are made by Baudin, of Paris. He makes two kinds. In one, the tubes are divided into a scale of equal parts; in the other, all the errors, except those of the zero and boiling points, are included in the graduations. The thermometers are read to single hundredths of a degree. It is the experience of Dr. Pernet that all the standard thermometers of Baudin will agree *inter se* within three hundredths of a degree.

I now pass to a consideration of the operations of the French Section, which are conducted in the building of the "École des Arts et Metiers," usually called the Conservatory.

Through the kindness of M. Tresca, who is the secretary of the Section, and, since the death of General Morin, the acting Director of the Conservatory, I was able, during a recent visit to Paris, to make a careful study of the entire operation of converting the "Mètre des Archives" into an equivalent line-meter. Notwithstanding the pressure of the official duties of M. Tresca, which were at the time of my visit especially great, owing to the illness and death of General Morin, the Director of the Conservatory, he gave me several hours each day, explaining in detail each step of the operation, from the melting of the platinum-iridium to the final comparison of the completed bar with the "Mètre des Archives." As the result of this somewhat critical study, I express the unqualified opinion that M. Tresca is entirely master of the problem. His methods and his results are at least beyond present criticism. In this I do not, however, include the method adopted of comparing end-measures with line-measures. This method, which is sometimes credited to Fizeau, and sometimes to Wild, seems to me to be radically defective.

To the end of a plate having the same thickness as the "Mètre

des Archives," a strip of very thin platinum, terminating in a sharp point, is attached. The reflection of this point from the end of the bar gives the means of observing the point of contact without actually touching the surface. This method seems to be a necessity in this case, since a statute law forbids contact of any kind whatever. If it was not for this necessary limitation, a much better method could be employed. Still, it is the opinion of M. Tresca that the absolute error of the line prototype can be reduced below 1 $\mu = .001$ mm.

On account of the difficulties attending the transfer from an end-measure to a line-measure, M. Tresca has adopted the plan of transferring one line-meter with the utmost precision. This copy, which he calls his "working meter," has occupied his attention for several months. He is confident that every source of error has been eliminated. This line-meter will be the basis of the final standard, since the Commission will accept without question the definitive transfer offered by the French Section after the difficulties with respect to the question of material are removed. On this point there is no dispute. The real issue is this. The International Bureau ask of the French Section one or more bars of pure platinum, and several of platinum-iridium, with the definitive comparison with the "Mètre des Archives." The Commission contends that the bars already offered contain two per cent of iron, transferred through the process of drawing through dies. It contends also, that, by the process of drawing through dies, the bars remain in a state of strain. M. Tresca, on the other hand, contends that it is impossible that these criticisms can hold.

He admits that iron is transferred during the passage of the bars through the rollers, but this iron is extracted after each of the two hundred passages through the dies. M. Tresca also admits that the bars would be in a state of strain if left in the state in which they come through the dies, but he anneals them finally. As a proof of the correctness of this view, he gets exactly the same coefficient of expansion after each melting and casting of a given bar. Some bars have been melted and recast as many as ten times.

I heard but little said on the question of the alloy. The standard weights of the Bureau are made of platinum-iridium, and there seems to be no question about them.

The comparing rooms of the Conservatory are built as follows. First (*a*), there are the thick stone walls of the building; then (*b*), space filled with hay; (*c*), wooden walls; (*d*), space filled with cotton; (*e*), wooden walls, covered on the inside with paper. A copper room,

entirely enclosing the comparators, is built in the central part of the enclosed space. The microscopes of the comparators are 90 centimeters in length. Only the micrometer screws project through the upper wall of the copper room, so that the heat of the body of the observer can produce but a very slight effect. The value of one division of the micrometer is 0.301 μ.

There are two rooms of the same form and dimensions. One contains the longitudinal comparator, on which the transfers are made, and on which a series of comparisons is also made directly after the transfers, the two bars remaining in the same relative positions. The other room contains the transverse comparator, with which the final and definitive comparisons are made. These comparators were built by M. Froment.

The transfers and comparisons are only made after the bars have been in position, and subject to the same temperature, for a period of forty-eight hours. All the transfers and comparisons are made in air. The tracings are made between one and two o'clock in the morning. Even then, the jar of passing vehicles is very perceptible. The disturbance is, however, strictly local. No matter how great the tremors, no permanent movement of the bars can be seen.

In the *exposé* of the condition of the labors of the French Section to Sept. 22, 1879, will be found a complete statement of the present state of this great work.

A third natural unit of length has been proposed in the length of a wave of light of given refrangibility. It is extremely doubtful, however, whether this unit will ever come into extensive use. In the present state of the measurements of wave-lengths, the total number in a meter is known with a far less degree of exactness than can be assigned to the comparison of different meters. Probably the wave-length of the line D is as well known as that of any other line of the spectrum, and yet the measures by different investigators show large discordances. The meter as determined by different observers shows the following errors when compared with the value given by Angström: —

Authority.	Error.
Fraunhofer	— 1.1 mm.
Ditscheiner	+ 0.8
Angström	+ 0.0
Van der Willigen	— 0.3
Mascart	— 1.0
Bernard	— 1.0

In the wave-length equation $\lambda = \epsilon \sin \theta$; ϵ represents the *mean* distance between the lines of the grating. A certain number of isolated errors may occur in the ruling, which may entirely escape detection under the spectroscope. For example, one of the provisional gratings, ruled with the machine built for the writer at the Waltham Watch Factory, has errors of spacing which are easily measurable, and yet the grating will show 7 and possibly 10 lines between b_1 and b_2, notwithstanding the fact that it is ruled upon commercial plate glass without finish.

Angström, in his investigations, found the distance between the extreme lines of the different gratings employed in terms of the Meter of the Archives; and the value of ϵ was found by dividing the entire distance by the number of spaces. I cannot find that he made any investigation, either of the accidental or the systematic errors of the gratings. I am aware that it is commonly asserted that it is impossible for systematic errors of appreciable magnitude to exist in a grating which shows the solar lines sharply defined; but there are many evidences that not only isolated accidental errors, but periodic errors of a small but measurable magnitude, are not incompatible with apparently perfect definition. Since the periodic error of a screw may undergo considerable variations in value through a change of temperature, especially if this change is abrupt and violent, it does not seem possible to overcome them entirely, except by a rigid investigation immediately preceding the ruling of a given grating, and by the application of the corrections derived, during the process of ruling. It is without doubt true that the *mean* interval between the ruled lines can be expressed with a far greater degree of accuracy than any given space can be measured under the microscope; but I believe it to be possible to measure the errors of lines widely separated when there is no evidence of their existence in the appearance of the solar lines. Even the best gratings which have thus far been produced show traces of systematic error when they are examined with monochromatic light.

Briefly, then, whenever the yard with its subdivisions is adopted as the measure of length, the unit to which all measures must be referred is the Bronze bar deposited in the "Strong Room" of Old Palace Yard, London, known as the "Imperial Yard No. 1." It has been shown that all attempts to express the length of the Imperial Yard in terms of a natural unit have been abandoned.

Wherever the metric system has been adopted, either by legal enactment or by actual use in the absence of definite legislation, the plati-

num end-measure meter, deposited in the Archives of Paris, is the only ultimate standard of reference. Since sixteen governments have entered into the Convention, which, through the International Commission, has decided that this particular bar at 0° Centigrade shall be the unit to which all measures of extension shall be referred, there is hardly a possibility that a different unit will ever be adopted. Great Britain is the only prominent government that has declined to enter the Convention. The two platinum meters which have hitherto been the standard of reference in that country are, however, no longer adapted to the purposes of exact measurement. Besides, even here the ultimate reference is the Meter of the Archives, through the equation determined by Kater and Arago in 1818.

The only exception to the entire abandonment of all attempts to refer the meter to a natural unit is the indirect determination by Clarke and others of the length of this unit expressed in the ten-millionth part of a quadrant of the earth's surface. With the highest possible respect for the work which has been done by Colonel Clarke, it does not seem likely that his value of this unit will ever become generally adopted. In the first place, his arc of the meridian does not follow the definition upon which the Meter of the Archives was founded. In the reference to any given unit, the standard of length determined must be ascertained with a greater degree of exactness than that attainable in the comparison of different copies of this standard. It is the experience of the writer that the error involved in the comparison of different meters need not exceed one millionth of a meter. It is not probable that an arc of a meridian of the earth's surface extending over 90° of latitude can be measured with sufficient exactness to warrant the assignment of this degree of accuracy to the aliquot part of this distance, which we call a meter.

In the United States, the particular yard which, previous to 1856, was taken as a standard, is the distance between the 27th and the 63d inch of a scale by Troughton. It has never had other than an indirect legal authority. It was never legalized by act of Congress. It was, however, adopted by the Treasury Department. The first standards distributed to the States by the authority and direction of Congress were copies of this particular bar.

In 1856, "Bronze bar No. 11" was presented by the British Board of Trade to the United States. It is standard at 61°.79 Fahrenheit. Since this date, all measures of length which are expressed in terms of the yard have been referred to this particular unit. This change, by whatever authority it was made, was one clearly demanded in the inter-

est of science, and by the legalization of Bronze No. 1 as the Imperial Standard Yard, with which it had been most carefully compared; but I am not aware that it has ever been sanctioned by act of Congress.

For the metric system, the iron meter mentioned above has always been taken as the standard of reference. It has, however, no legal sanction. Neither in the case of the yard nor of the meter are comparisons usually made directly with the original standards.

The Saxton comparator consists of a brass bed-plate having V-shaped ways running the entire length. A slide carrying a microscope slides freely over these ways. A series of brass posts form a part of this bed, through which pass steel screws having conical ends, which have been tempered and polished. There are stops for the yard and for its subdivision into feet, and of one foot into inches. There are also stops for the meter and for its subdivision into decimeters, and of one decimeter into centimeters. By a very ingenious arrangement, the arm attached to the moving microscope plate can be brought into contact with any stop without loss of motion. The end stops for the yard and for the meter were many years ago set to correspond with "Bronze No. 11" at 58° nearly for the yard, and with the iron meter at 68° nearly. It is understood that the position of these stops with respect to the brass bed have never been changed. The standards which have been distributed since 1856 have been transferred from these distances at the temperatures at which they are standard. The yard in actual use at the Bureau of Weights and Measures, therefore, may be defined to be the distance between two steel stops attached to the bed of the Saxton comparator which corresponds to the length of "Bronze No. 11" at 58° nearly, and the meter may be defined to be the distance between two steel stops of the Saxton comparator which corresponds to the length of the iron meter corrected for the difference between its length at 32° and at 68° nearly. Recent comparisons indicate that these temperatures should be diminished, by a trifling amount, for the present distances between the stops, both for the yard and for the meter.

In May, 1878, by the kindness of Prof. J. E. Hilgard, Assistant in Charge of the United States Coast Survey, I was able to secure copies of both the yard and the meter, together with their subdivisions. On May 14, Dr. Clarke, who has charge of the standards, transferred the yard to a glass bar which I had previously prepared, and on the morning of May 17 the meter with its subdivisions was transferred to the same bar.

Upon my return to Cambridge, the relative relations between the

subdivisions were investigated, with the following results. A positive sign indicates that the measured space is too short; a negative sign, that it is too long.

Subdivisions of the Yard.

Feet. No.	Feet. Corrections.	Inches. No.	Inches. Corrections.	Inches. No.	Inches. Corrections.
1	+ .00069 in.	1.	+ .00075 in.	7	— .00037 in.
2	— .00022	2.	— .00004	8	+ .00062
3	— .00047	3.	— .00031	9	+ .00009
		4.	+ .00024	10	— .00097
		5.	+ .00064	11	— .00020
		6.	— .00011	12	— .00036

Subdivisions of the Meter.

No.	Decimeters. Corrections.	No.	Centimeters. Corrections.
1	— .00080 cm.	1	— .00132 cm.
2	— .00018	2	+ .00021
3	— .00211	3	+ .00006
4	+ .00088	4	+ .00070
5	+ .00105	5	— .00070
6	+ .00142	6	+ .00003
7	+ .00041	7	+ .00100
8	— .00168	8	— .00019
9	— .00194	9	+ .00039
10	+ .00288	10	— .00016

These corrections involve the errors of transfer. No attempt was made to determine and apply the corrections due to the curvature of the ways upon which the slide carrying the microscope moves.

The subdivisions of the centimeter rest upon the authority of a centimeter subdivided into 100 equal parts by Brunner Frères, of Paris, for the office of the Coast Survey. By the permission of Professor Hilgard, I have made an extended series of comparisons between this unit and a centimeter derived from the screw of my own dividing engine. The comparisons were made by means of the comparator for short lengths, described in the American Microscopical Quarterly for April, 1879. The Brunner scale is divided on silver inlaid in brass. In my own scale, the graduations are upon glass.

Date.	Brunner. No. div. of comparator in 1 cm.	Thermometer. °	Rogers. No. div. of comparator in 1 cm.	B. — R. No. div.
1878, May 14	3149.02	70.4	3148.80	+ 0.22
15	3149.95	74.8	3150.11	— 0.16
16	3149.33	63.0	3149.53	— 0.20
16	3149.51	63.8	3149.53	— 0.02
16	3149.63	64.0	3149.21	+ 0.42
17	3149.66	72.1	3149.20	+ 0.46
17	3149.66	72.1	3149.40	+ 0.26
18	3149.18	62.0	3149.44	— 0.26
18	3149.58	68.3	3149.37	+ 0.21
18	3149.57	68.8	3149.37	+ 0.20
18	3149.42	70.0	3149.12	+ 0.30
19	3149.52	69.8	3149.17	+ 0.35
20	3149.30	71.3	3149.01	+ 0.29
21	3149.55	71.1	3149.09	+ 0.46
Means,	3149.49	68.7	3149.31	+ 0.18

In terms of the Brunner scale, therefore, my own unit is .000317 cm. × 0.18 = .000057 cm. too short. Subsequent investigations have shown that both units contain errors of considerable magnitude when compared with the hundredth part of the Coast Survey meter at 68°.

The following are the relative errors of each millimeter expressed in terms of the entire length of the centimeter: —

	Brunner. Correction.	Rogers. Correction.
Mm. No. 1	+ .000082 cm.	— .000016 cm.
2	— .000091	+ .000025
3	— .000079	+ .000012
4	— .000070	+ .000016
5	+ .000038	+ .000029
6	+ .000002	— .000016
7	+ .000056	— .000044
8	+ .000035	— .000022
9	— .000080	— .000006
10	+ .000105	+ .000013

Notwithstanding the fact that great advances have been made in the science of exact measurements since the construction of the normal standards now in use, there are several problems connected with this subject which require further investigation. It is a well-known fact,

that, while the different results obtained in comparing two standards by one observer and with a given instrument usually indicate marvellous precision in the comparisons, a different observer with a different instrument will probably get results equally accordant *inter se*, but which nevertheless do not agree with those obtained by the first observer. Until all the sources of error involved in comparisons are investigated and eliminated, it will be useless to expect an agreement between different observers. Among the points which demand investigation, the following require special attention.

(*a.*) *The magnifying power of the microscopes employed, which is the best adapted to secure the greatest absolute accuracy in measurements.*

In all the earlier comparisons, microscopes of very low power were employed, varying from forty to sixty diameters. The International Commission, relying largely upon the recent investigations of Förster, have decided upon the low power of from forty to fifty diameters. M. Tresca, of the French section, on the contrary, is a firm believer in high powers; he prefers a power of about 400. The writer has had considerable experience on this subject, and always with results favorable to high powers. With a proper illumination, and with lines having smooth edges, a magnifying power of 900 can be used with great ease, even in the comparison of two meters upon a longitudinal comparator.

New microscopes have been recently attached to the microscopes of the meridian-circle of Harvard College Observatory. In order to be able to read the divisions of the circle, it was necessary to have one eye-piece with the same power as that furnished by the maker. A second eye-piece, giving nearly double the magnifying power, was attached to a swinging arm in such a manner that either eye-piece can be used at will. A sufficiently extended series of observations has now been made to justify the conclusion that the high-power eye-piece gives the most accurate results. Again, in the investigation of the errors of one of the circles of the instrument, a metal plate, having a graduated arc of 15°, is attached to the opposite circle under a one-inch objective, to which is attached the interior illuminator for viewing opaque objects, invented by Mr. R. B. Tolles, of Boston, in 1867, and also invented independently by M. Tresca, in 1871. The lines under this objective are sharply defined. The value of one division of the micrometer is only $0''.12$ against $1''.0$ for the regular microscopes. It is the experience of the writer, that it is quite as easy to make every reading fall within one division in one case as in the other. With

the ordinary form of illumination, however, the advantage of a high power would not be so apparent.

The value of one division of the micrometer depends both on the magnifying power of the microscope and on the pitch of the micrometer-screw. Those who advocate a low magnifying power usually prefer a screw having a small pitch. Further observations are needed to determine the best relation between the pitch of the screw and the magnifying power of the objective. The following are the values of one revolution of a few of the micrometer-screws which have been used in the comparison of standards: —

Observer.	Value of 1 division. Inch.	Observer.	Value of 1 division. Inch.
Troughton, 1798,	.0001000	Chaney, 1880, new objectives,	.0000058
Kater, 1818,	.0000428	Rogers, 1880, Comparator, —	
Hassler, 1832,	.0001000	Microscope A. With 1 in. objective,	.0000197
Baily, 1832,	.0000500	Microscope A. With ½ in. objective,	.0000087
Baily, 1844,	.0000258	Microscope A. With ¼ in. objective,	.0000047
Bache, 1856, May,	.0000833	Microscope A. With ¼ in. and amplifier,	.0000028
Bache, 1856, October,	.0001000	Microscope B. With 1 in. objective,	.0000079
Hilgard, Saxton Comparator,	.0001000	Microscope B. With ½ in. objective,	.0000085
Clarke, 1866,	.0000286	Microscope B. With ¼ in. objective,	.0000019
Chaney, 1880, old objectives,	.0000319	Microscope B. With ¼ in. and amplifier,	.0000011
Tresca, 1880, 0.301 μ =	.0000118	Internat. Comm., 1.0 μ =	.0000394

The writer is inclined to the opinion that one can measure with *certainty* only what one can see.

(*b.*) *The best method of illumination for opaque objects.*

I cannot better illustrate the necessity for a proper illumination in making exact measurements, than by saying that I have been obliged to reject a series of observations extending over a period of four months, for the simple reason that I finally discovered that during all this time I have never once seen the actual lines ruled, but only their image. I used a parabolic reflector, giving a beautiful *white* line on a black background. The lines were traced upon a steel surface nickel-plated, their width being about one ten-thousandth of an inch. Investigation showed that the positions of the lines could be changed by an amount more than half their width, by shifting the position of the parabolic reflector.

The method of illumination employed by Baily and Sheepshanks seems to me radically defective. With the microscopes used by Sheepshanks I found myself unable to separate lines ruled on a polished steel plate, though separated by an interval of only one-thousandth of a centimeter. As already stated, I have used with great satisfaction the

form of illumination described by Mr. Tolles in the Annual of Scientific Discovery for 1866–67. I found this form of illumination in use by M. Tresca since 1871. It has also been since described as an original invention by Professor Wild. Troughton and Simms also constructed microscopes with the same method of illumination as early as 1869, at the instance of Mr. Warner, a retired gentleman residing at Sussex Place, Brighton. According to the present evidence, the priority of publication must be assigned to Mr. Tolles. M. Tresca was without doubt the first to make an actual application of the method to exact measurements. The reader who is interested in pursuing this subject farther will find a full description of the method in the number of the Journal of the Royal Microscopical Society for August of the current year. It is sufficient to say here, that, as none of the light is lost by the reflection, it is easy to get all, and even more, than is needed. Diffused daylight falling upon the plane face of the prism inserted between the two front lenses affords an abundance of light for the most delicate tracings. With a one-inch objective of the form recently constructed by Mr. Tolles, lines 30,000 to the inch, ruled on a polished steel surface, are resolved with the greatest ease.

(*c.*) *The method of support which is best adapted to neutralize the effect of the flexure of the bars upon which the graduations are traced.*

In all the early measurements the standards were placed upon a planed surface of wood. Troughton, in comparing Shuckburgh's scale, fastened it to a bed of mahogany by means of three screws. Kater was the first to discover the variations due to the flexure of the bars on which the graduations were traced. He was also the first to suggest a neutral plane, in which the effect of flexure upon the length would be zero. At first he located this neutral plane in the middle of the bar, but from subsequent investigations he concluded that it was not quite one third of the thickness of the bar below the graduated surface. He reached the following conclusions*: —

(1.) "That in a standard of lineal measure, traced upon the surface of a bar, an error arises from the thickness of the bar when it is placed upon a table, the surface of which is not plane."

(2.) "That this error in bars of the same material and of unequal thickness is within certain limits as the thickness of the bar, and depends upon the extension of that surface of the bar which becomes convex, and the compression of the surface which is concave."

(3.) "That the error to which the scale is liable from this cause is

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* Phil. Trans., 1830.

directly as the versed sine of the curvature of the surface upon which the scale is placed."

(4.) "That this error very far exceeds that which would arise from the difference of length between the arc and its chord, under similar circumstances; so much so, that the sum of the errors from this cause in a bar one inch thick, with a versed sine of not one-hundredth of an inch, is nearly one-thousandth of an inch; whilst double the distance between the chord and the arc is not one fifty-thousandth."

In the early observations of Kater, he used a wood surface for a support, but later he seems to have preferred a marble slab, which, however, was not planed.

In 1844 Sir George Airy showed that, if n represents the number of supports of a bar, the distance between the supports should be

$$\frac{\text{Length of bar}}{\sqrt{n^2-1}}$$

in order to neutralize the effect of the flexure. Thus, in the case of the yard, if the defining lines are near the ends of the bar, each support should be placed 10.39 inches from the centre, and in the case of the meter they should be placed 28.87 centimeters from the centre.

This general form of support was used by Mr. Sheepshanks in all of his observations, and it is the form which is ordinarily employed at the present time. In the construction of the National Standards it was considered important that the bars should be supported at numerous points in order that they should be exposed to as little strain as possible. The particular form of support finally adopted will appear from the following description by Sir George Airy, to whose suggestion it is due.

"Great facility is given to the arrangements for supporting a bar with definite pressures applied at special points, by the use of levers. Thus, if any portion of the bar rest upon two rollers which are placed at the ends of a lever, and if the fulcrum of this lever (whether movable or not) be in its centre, the pressure upwards produced by these rollers will necessarily be equal. If there be another such lever, and if the fulcrum of this and the former be upon the extremities of a third lever, and if its fulcrum be at its centre, then the pressures upward produced by the four rollers will be equal. By this arrangement of the rollers and levers, one half of the bar may be supported. If another similar system be applied to support the other half of the bar, the pressures produced by its four rollers will also be equal among

themselves; and if the bar be laid symmetrically upon them, all the individual pressures will be equal." *

Mr. Baily decided upon the adoption of eight rollers for the support of the National Standards, requiring for the distance between each roller and the one next adjacent $\frac{36}{\sqrt{63}}$ inches = 4.54 inches. As a further precaution, the defining lines were traced upon gold plugs inserted in wells sunk to the plane of the neutral axis. This form of support is the one now employed in the Standards Office. The reader who desires to pursue this subject will find elaborate discussions by Bessel and by Clarke.

At the International Bureau only two supports are used, the distance between them being determined by Bessel's formula. At the Conservatory, the bars are placed directly upon a plane surface, which is nearly in the neutral axis of the support itself.

In 1876 Professor Wild proposed a form of support which seems to leave very little to be desired. The bars are placed *one above the other with the graduations in the same vertical plane.* Here we have conditions quite unlike those which occur with bars supported in the way already described, and under which it would seem that no flexure can occur which will affect the distance between the defining lines. This method, therefore, affords a rigorous test of the flexure formulæ of Bessel and Airy. It appears from the discussion of Professor Wild, that, while the mean effect of observed flexure upon the relative lengths of the separate decimeters of the same bar agrees nearly with the mean computed value, the numerical mean of the differences between the observed and the computed effects is no less than 0.0004 mm., or nearly one half of the whole mean effect. On the other hand, a rigorous comparison instituted by Clarke showed a substantial agreement between the computed and the observed effects of flexure. It is evident, therefore, that this subject requires further investigation. In the case where eight, or even four rollers are employed, it is mechanically impossible to make them so that planes tangent to each roller shall fall in a common plane, which shall be parallel with the plane of the defining lines. Unless this takes place, the upward pressures will not be equally distributed, and the formulæ will not hold. In the case of bars of the Tresca form, I am compelled to admit that, at least with a bar of copper, attention must be paid to the form of the support.

In the comparator which has been constructed from designs furnished by myself, I have dealt with the question of supports in the

* Astron. Trans., xv. 157, &c.

following way. The bed of the comparator is made of cast-iron, and is sixty inches long by fourteen inches wide. It has an extreme depth of two inches. In the centre, V-shaped ways run the entire length of the bed, upon which a plate carrying the microscopes, slides. The bed has at one end the means of bringing it into a horizontal plane, and at intermediate points screws for taking up the flexure. For this purpose, free vertical bolts, pressed upwards by means of levers and controlled by weights, are without doubt better than screws.

It is now necessary to provide for the movement of the microscope slide in a horizontal plane. This is accomplished in the following way. A shallow dish of mercury is placed upon the bed of the comparator, extending along its entire length. An arm projects from the microscope plate, to which is attached a plate sliding between guides, and carried by a micrometer-screw. To the lower part of this slide a platinum point is attached. One wire of a battery having a single cell is attached to the platinum. Another is placed in contact with the mercury. A sounder is placed in the circuit. The microscope plate is moved to one end, and the platinum point is brought into contact with the mercury, the contact being indicated by the "click" of the magnet. The slide is then moved to the other end, which is elevated or depressed by means of the adjusting screws, till the platinum point again makes contact with the mercury. After one, or at the most two trials, it will be found that the two ends of the bed-plate are in the same horizontal plane. The microscope plate is now set at the middle of the comparator, and the amount of the flexure is measured with the micrometer-screw. After about one third of the measured amount has been taken up by means of the flexure screws, the entire operation is repeated. In this way I find that the microscope plate can be made to move sensibly in a true plane. In practice, it is found that almost equally good results can be obtained by directly observing the surface of the mercury, with an objective of pretty high power, using the interior illuminator. The surface of the mercury admits of nearly as sharp a focus as the surface of a metal bar. Good results have also been obtained by dropping fine threads of spun glass upon the surface of the mercury. This method is rather more convenient than the method by contacts, but the latter admits of somewhat greater precision. From a limited number of trials, I conclude that contacts can be made with a probable error of a single contact not exceeding .00002 inch.

Knowing that the microscope plate moves in a true plane, the sur-

face of the bed-plate on either side can be brought to a plane which shall be parallel with that through which the microscope moves, by working it down till every part remains in the same focus. The bars to be compared are placed directly upon the surface thus prepared.

(*d.*) *The form and material of a bar which is best adapted to fulfil all the conditions which are essential to success in comparisons extending over a long period of time.*

In general, the form of a standard bar should be the same as that with which it is to be compared. For example, if one desires a standard yard which is to be compared with "Bronze 11," at Washington, the bar should be made of Baily's metal, and should be one inch square and about thirty-eight inches long. Kater preferred a thin bar. A width of one centimeter with a depth of three centimeters will be found to yield good results when the bar is placed upon a flat surface. Of all the forms proposed, that of M. Tresca, which has been adopted by the International Commission, seems to me the best designed to overcome all the difficulties of the problem. It is convenient to handle; it retains its form under its own weight, and quickly answers to a given change of temperature. I have had the pleasure of using a bar of this form for several months with the most satisfactory results. I began with considerable prejudice against it, influenced to some degree by a remark made by Professor Wild concerning it. It is undoubtedly somewhat difficult to manufacture, and will probably be found to be rather costly; but these are the only serious objections that can be urged against it. I express the opinion that it is well adapted to scientific work of the highest order. For use with my own bar, I have had constructed a special objective provided with an interior illuminator. The working distance is just sufficient to allow the passage of the bar under it.

(*e.*) *The investigation of the error due to the horizontal curvature of the ways of a longitudinal comparator.*

If the microscopes are stationary, and the bars to be compared are brought in succession under them, the curvature of the ways will produce no effect; but when the relation between the separate subdivisions of the given unit are to be investigated, or when a given length is to be transferred from one bar to another, the error arising from the curvature of the ways cannot be neglected. The comparisons of Troughton, of Hassler, and of Bache are subject to this class of errors, though of course it might have happened that in each case the curvature of the ways was insensible. I cannot find that any observations were made to determine the amount of the curvature. By reversing the position

of the bars to be compared, the error due to curvature will be eliminated in proportion to the ratio between the length of the chords described by the microscope for the two positions of the bars.

The necessity for taking into account the error due to curvature in any given case will clearly appear from the following provisional investigation of its magnitude in my own comparator: —

O A =	—3.8 cm.	A ——— *a*	
O B =	+3.8	O ——— *o*	
O C =	+23.8	B ——— *b*	
O D =	+26.8	C ——— *c*	
O E =	+30.6	D ——— *d*	
		E ——— *e*	

A steel meter by Froment was placed in the constant position O *o*. A copper meter of the new form by Tresca was placed successively in the positions A *a*, B *b*, C *c*, D *d*, and E *e*. The microscopes were attached firmly to the plate, moving freely upon the ways of the bed-plate, each having its own adjustment for focus. Microscope B was adjusted for coincidence with the end line of the bar O *o* at O, and microscope A was at the same time adjusted for coincidence with the end line of bar A *a* at A. The plate was then moved along the ways until microscope B was adjusted on the terminal line at *o*, and for this position the micrometer of microscope A was read. Since the two microscopes remain in the same position with respect to each other, the difference between the two readings of A will indicate the difference between the length of the bars. By reading B for the positions O *o*, — A being a constant, — the relation between the two bars will be expressed in terms of the micrometer of B also.

Differences in the Apparent Length of the Bars compared, varying with the Position of Bar A with respect to Bar B.

	Divisions of Micrometer. From Microscope A.		Divisions of Micrometer. From Microscope B.	
O–A =	—17.7 =	—9.0 μ	—102.0 =	—9.1 μ
O–B	+13.3	+6.7	+80.3	+7.1
O–C	+83.3	+42.1	+474.0	+42.2
O–D	+96.4	+48.8	+553 7	+49.3
O–E	+112 4	+56.9	+635.0	+56.5

For the radius of curvature, we have approximately,

$$.00658 : 34.4 = 100 : x.$$

$$\therefore\ x + = 5228 \text{ meters.}$$

It is apparent, therefore, that, though the radius of curvature exceeds a distance of three miles, a correction of 3.8 divisions must be applied to the reading of microscope A, and of 21.4 divisions to the reading of microscope B, for each centimeter of the distance between the two bars.

The radius of curvature can also be found in the following way. If a tracing apparatus is attached to the microscope plate, a line traced upon the plane surface of a bar, by the motion of the plate from one end of the bed-plate to the other, will have the curvature due to the distance of the ruling diamond from the centre of the slide. If the bar is reversed, and a second line is drawn upon the same surface as nearly parallel to the first line as possible, then the ver-sin of the curvature will be equal to one half the difference between the distance of the lines at the middle point and the half sum of the distances at the two ends.

Finally, the deviation of the microscope plate both from a horizontal and from a vertical plane can be detected by means of a telescope mounted upon the sliding plate. If the telescope is pointed either at the cross wires of a collimator, or at a distant object, and the point remains fixed with respect to the cross wires of the telescope during the motion of the slide from one end of the comparator to the other, it may be assumed that it moves in an invariable plane. The longitudinal comparator at the Conservatory is provided with an attachment of this kind.

(*f.*) *The relative advantages of comparisons in air and comparisons in a liquid.*

Air temperatures are employed both at the Conservatory and at the International Bureau. At the Conservatory, the bars to be compared remain at a constant air temperature for forty-eight hours before the comparisons are made. The arrangements for maintaining a constant temperature are most admirable and effective. The means of controlling the temperature employed at the International Bureau are somewhat different, but they give most excellent results. Nevertheless, it is yet an an open question whether the absolute relation between two bars compared in air at a given temperature can be made to agree with the absolute relation determined by submerging them in a liquid at the same temperature. The writer has met with many difficulties in this direction.

(*g.*) *The variation of the absolute length of a bar by a change in its molecular structure or otherwise.*

There are some evidences that certain standards have undergone a

change of length since their original construction, but in no case does the evidence seem to me to be conclusive. It is understood that Colonel Clarke finds a well-defined change in some of the standards originally measured in 1842–55. The platinum meters of the Royal Society present some evidences of a change of length *inter se*.

According to the Fifth Report of the Standards Commission (Appendix), we have the Royal Society end-meter equals: —

Royal Society line-meter	+0.01759 mm.	(Arago, 1818.)
" "	+0.01881	(Kater, 1818.)
" "	+0.00940	(Baily, 1835.)
" "	+0.00837	(Standard's Office, 1869.)

We have here an appearance of a change. In deciding whether it is a real or an apparent change, it should be remembered that in 1818 there was no defining cross-line on the line-meter, and that there is no existing data by which the accuracy of the constants of the contact pieces used with the end-meter can be estimated.

The Russian standard of length used in the geodetic surveys previous to the work done by Prazmowski and Wagner presents the most authentic instance of a well-defined change of length which has come under my notice. This bar is made of iron, and has a length of seven feet. I am not certain whether it was forged or drawn through dies. Conical end-pieces of tempered steel were inserted in each end. In the course of two or three years, this bar was transported a distance exceeding 8,000 miles, being supported in a packing of feathers. At the end of this time it was found by Prazmowski to be one thirteenth of a line, or about .006 inch shorter than at the commencement of the expedition, the two sets of comparisons having been made at the same temperature.

If it can be established that no permanent flexure of the bar took place, we have here an authenticated instance of an actual change of length. Upon the discovery of this change, from whatever cause produced, a new standard was constructed, also made of iron. It was allowed to anneal for eight days after being forged. It is understood that no change of length has ever been detected in this bar. Against this somewhat doubtful evidence we have the positive evidence by Chisholm, that Bronze No. 6 showed no evidence of a change in length in 14 years, and of Baeyer that the precise mean length of Bessel's standard bars at 13° Reaumur had not altered in the 20 years from 1834 to 1854. The change in the mean length is to be distinguished from a change in the coefficient of expansion. The evidence of Baeyer

seems tolerably conclusive that bars of iron and zinc are liable to suffer a change in their coefficients of expansion, either by an actual change in their molecular structure or by the action of external causes.

The continuous measurements of the Conservatory and of the International Bureau will, in the course of the next decade, furnish data which will add much to our knowledge of this subject.

(*h.*) *The law which governs the expansion of bars of different materials, and having different masses, under a varying temperature.*

It is well known that Mr. Sheepshanks, in the comparison of "Bronze bar 28 with Cast-steel bar D," near the close of his labors on the National Standards, found deviations which he could not explain. The importance of this matter justifies me in quoting in full the statement of Sir George Airy in his account of the construction of the national standards.

"I proceed now to allude to a discordance which was a source of great anxiety to Mr. Sheepshanks.

"In April, 1855, Mr. Sheepshanks was engaged in measuring the bar Cast-steel D. By comparisons with four iron bars, (as stated in the table above,) whose results agreed very closely, the excess of Cast-steel D above Bronze 28 was found to be —$3^d.61$. But a direct comparison of Cast-steel D with Bronze 28 immediately preceding had given —$0^d.46$. This comparison was made at the temperature 45°.54, or 16°.46 below the standard temperature. A trifling error of expansion might account for part of the discordance, and the ordinary errors of observation might account for part. But in the opinion of Mr. Sheepshanks, though the whole discordance scarcely exceeded the effect of the thermometric expansion of Bronze 28 for 0°.3 Fahrenheit, it was impossible so to explain away the whole or a large part of it; and he was convinced that Bronze 28 had sensibly shortened. And so deeply and so painfully was this impression fixed in his mind, that he actually contemplated the rejection of all the results which had cost so many years of labor, and the commencing the work *de novo*."

Mr. Sheepshanks first disproved, by observation, the first conjecture on the possible cause of the apparent change; viz. "that Bronze 28, still covered with gold-beater's skin and cement (as in the earlier comparisons), might have been so constrained by that covering that it could not shrink down to its natural length; but that in the last comparisons with Cast-steel D, when that covering had been removed, it had contracted itself."

He then compared Bronze 28 with twenty-seven different bronze bars, and by comparing the old and the new measures found, with only

one exception, a very close accordance. Mr. Airy concludes this part of his report as follows: —

"First, there is no evidence whatever of a general preponderance of excess from the New Measures above the excess from the Old Measures; the signs + and — being intermixed, in the differences, in all possible ways, and the mean of the whole being less than $0^d.50$. Secondly, the only instance which fairly supports the conclusion deduced from Cast-steel D is the first of all, namely, Bronze 12. Cast-steel D was compared on April 13, 14, and 16; Bronze 12, on April 26, 27, 28, and May 1; Bronze 39 (the next), on April 30. The conclusion, I think, is inevitable, that Bronze 28 really was shortened at the beginning of April; that it recovered its exact length before April 30; but that this recovery took place with some fluctuations, so that on May 1 it was subject to nearly the same error as before. Bronze 21, observed June 26, exhibits a similar discordance. What circumstances can have produced these changes, or how far the later fluctuations are apparent rather than real, I am wholly unable to conjecture."

I believe that the explanation of the phenomena observed by Mr. Sheepshanks will be found to fall under the following: —

First, *two bars of different materials, having different shapes and different masses, have a variable coefficient of expansion with respect to each other, which is a function of the time of exposure to a given temperature.*

Second, *the more violent the change of temperature, the greater will be the variation in the length of the bars before they assume their normal condition under a constant temperature.*

All of the bronze bars had the same mass. The iron bars and the steel bars, on the other hand, not only had a different mass, but they were subject to a different degree of specific heat. Their conductive power was also different. They also had a different absorptive power. The difficulty with the observations of Mr. Sheepshanks was, that they were not sufficiently continuous. They did not extend over a sufficient length of time to enable him to discover the slow changes which were going on in the length of the bars through the heat already absorbed, and *which was not indicated by his thermometers.*

This paper has been already extended so far beyond the limits proposed that it is inexpedient to give even a *résumé* of all the observations which I have made bearing upon this point. It will be seen from the following brief account, that it is absolutely necessary to investigate the performance of two standards which are to be compared un-

der the action of a varying temperature, in order to decide how long they must remain at nearly a constant temperature before comparisons can be safely made. The comparing-room is a small triangular space partitioned off from the cellar of the west wing of the Observatory. It has two windows, one facing south and the other west. All the heat of a furnace can be turned into the room through a pipe entering it near the ceiling. The comparator is mounted on brick piers insulated from the building. The temperature of the room can be considerably reduced by means of two refrigerators, supported near the ceiling. Centigrade thermometer No. 1 is imbedded in the base of the comparator, and packed with iron filings taken from the bed-plate. Centigrade thermometer No. 2 is suspended by a fine wire about half-way between the point where the heat from the furnace enters the room and the upper surface of the comparator.

Series I. consists of partial records of a comparison of a line-measure steel yard by Troughton and Simms with a yard laid off on platinum-iridium plugs inserted in the bed of the comparator. Series II. consists of partial records of a comparison between two line-measure steel bars, of which one is nickel-plated, the graduations being upon the nickel surface. These two bars were made at the same time, and of the same material. The bars have the following dimensions: —

	Length.	Breadth.	Depth.
Nickel-plated bar,	40.4 in.	0.5 in.	2.0 in.
Steel bar,	39.37	0.6	1.6

A thin vertical lamina of platinum is inserted in the shorter bar. In other respects, it differs from the longer bar only in its dimensions, and in not being nickel-plated.

After the bars were placed in position, they were not disturbed till the close of the observations. Whatever changes took place, therefore, were due entirely to the action of temperature upon the bars. In Series I. the value of one division of the micrometer employed was .0000035 inch. In Series II. it was .0000197 inch, as stated on page 291.

SERIES I.

	Time. h.	m.	Thermometers. No. 1.	No. 2.	Diff. in Length in Divisions of Micrometer.
1880, Mar. 22,	9	45 A.M.	5.7	—	+35.6
	9	48	5.9	—	+35.3
	9	50	6.4	—	+40.7

Closed windows, and turned on heat.

	Time. h.	m.	Thermometers. No. 1.	No. 2.	Diff. in Length in Divisions of Micrometer.
1880, Mar. 22,	9	54 A.M.	6.7	—	+54.2
	9	57	6.8	—	+69.1
	9	59	7.6	—	+77.0
	10	2	8.3	—	+97.8
	10	4	9.0	—	+116.3
	10	7	9.4	—	+136.4
	10	11	10.0	16.7	+158.1
	10	15	11.0	19.4	+197.4
	10	18	11.4	20.6	+218.5
	10	20	11.7	21.1	+238.5
	10	23	12.2	22.2	+260.0
	10	26	12.5	22.5	+280.8
	10	28	12.8	23.3	+300.8
	10	30	13.1	23.9	+328.8
	10	32	13.4	24.0	+347.1
	10	35	13.9	24.9	+369.6
	10	38	14.2	25.0	+389.8
	10	41	14.7	25.6	+404.0
	10	43	15.1	26.2	+423.8
	10	46	15.3	26.7	+443.0
	12	0	20.3	33.3	+660.1
	0	4 P.M.	20.4	33.2	+659.7
	0	46	25.5	34.4	+615.9
	0	49	25.7	—	+611.7
	1	46	29.2	35.6	+550.6
	1	49	28.9	35.6	+537.9
	2	5	29.6	—	+499.9
	5	53	38.8	—	+50.2
	6	1	38.3	—	+31.1
	6	3	38.2	—	+23.0
	6	7	38.3	—	17.0
	7	18	38.8	—	+2.4
	7	50	38.8	—	—1.6
	8	21	38.8	—	—1.2
	8	37	38.8	—	—4.8
	8	39	38.8	—	—9.1
Mar. 23,	6	28 A.M.	5.0	—	—167.1
	6	30	5.4	—	—165.8
	6	50	7.3	—	—132.2

	Time. h.	m.		Thermometers. No. 1.	No. 2.	Diff. in Length in Divisions of Micrometer.
1880, Mar. 23,	9	7	A.M.	9.9	—	+62.9
	10	47		11.6	—	+19.4
	7	58	P.M.	19.0	—	—86.5
	8	1		19.1	—	—81.8
Mar. 24,	6	44	A.M.	17.5	—	—147.2
	7	51		17.2	—	—150.4
	8	54		17.2	—	—149.9
	8	57		17.2	—	—151.8

Series II.

	Time. h.	m.		Thermometers. No. 1.	No. 2.	Diff. in Length in Divisions of Micrometer.
1880, April 11,	4	33	P.M.	47.7	51.1	—297.3
	4	35		47.7	—	—296.5
	4	37		47.7	50.8	—295.0
	Shut off heat, and opened windows.					
	4	43		—	—	—288.1
	4	47		46.8	23.0	—327.0
	4	49		45.0	19.2	—420.3
	4	52		44.4	19.1	—464.1
	4	55		43.8	18.9	—535.7
	4	59		42.8	19.0	—580.7
	5	1		41.0	19.0	—638.1
	5	5		40.2	19.0	—669.5
	5	9		—	—	—702.1
	5	10		—	—	—707.7
	5	13		38.3	20.0	—716.0
	5	27		36.0	21.0	—742.5
	5	32		—	—	—746.7
	5	36		34.0	20.3	—741.2
	5	41		—	—	—730.7
	5	46		33.6	19.2	—694.5
	7	35		24.1	17.8	—497.1
	10	5		8.2	4.4	—453.9
April 12.	Windows open all night.					
	6	46	A.M.	1.2	1.3	—374.3
	7	18		2.7	1.1	—369.6
	8	7		1.8	1.7	—363.9
	2	34	P.M.	4.6	6.2	—311.6

	Time. h.	m.		Thermometers. No. 1.	No. 2.	Diff. in Length in Divisions of Micrometer.
1880, April 12,	4	46	P.M.	6.7	9.8	—283.0
	8	30		8.2	7.9	—360.7
	8	33		8.2	7.9	—364.9
	10	20		7.2	6.3	—386.5
	10	35		7.1	6.9	—376.5
	10	56		7.1	7.0	—376.2
	11	25		6.6	6.1	—378.0
	12	16		6.2	5.2	—372.0
April 13.	Windows open all night.					
	6	53	A.M.	3.1	3.9	—329.4
	6	59		3.1	3.9	—326.9
	8	23		4.1	6.0	—302.0
	8	26		4.1	6.0	—296.1
	8	33		4.3	6.9	—290.2
	9	13		5.2	8.2	—266.7
	9	15		5.2	8.2	—268.5
	10	6		6.9	10.2	—243.5
	10	8		6.9	10.2	—246.3
	11	37		9.7	13.0	—247.9

If the changes of temperature are gradual, the two steel bars will reach a state of rest under a constant temperature in about twelve hours; but if they are subjected to an abrupt and violent change of temperature, it is not safe to make the comparisons till after the lapse of from forty-eight to sixty hours.

Baily's comparisons of the yard of the Royal Society certainly show traces of an error of the kind I have here described. On the other hand, Clarke's comparisons do not show any marked evidence of their existence. But in the former case, the comparing-room was not well adapted to the maintenance of a steady temperature, while in the latter it appears to have been admirably constructed. As observations are still being made with the apparatus at Southampton, it would be interesting to see if results corresponding in a general way with those I have found could be obtained from a special series of comparisons arranged for this specific investigation.

An investigation of the character which I have indicated is especially necessary in the measurement of base lines in which the standard unit is necessarily subject to greater and more rapid variations of temperature than take place in a well arranged and protected comparing-room. Still further observations are neccssary in order to determine

whether end-measures behave in the same way as line-measures under a varying temperature.

(*i.*) *The exact relation between the length of the Imperial Yard and the Meter of the Archives.*

The Imperial Yard has never been directly compared with the Meter of the Archives. Our knowledge of the value of the equation between these standard units depends solely upon, —

First. The indirect comparison of the platinum meters of the Royal Society with the Meter of the Archives by Kater, Baily, Clarke, and Chisholm. All these determinations rest upon the relation of the Royal Society platinum meters to the Meter of the Archives given by Arago in 1818.

Second. The comparison of the iron meter of the United States Coast Survey with Troughton's scale, by Hassler ($27^{\text{in.}}$ —$63^{\text{in.}}$) in 1832.

The following are the relations given by the authorities named above. I have subtracted .00087 inch from the value given by Hassler, this being the amount by which the yard on Troughton's scale exceeds the length of "Bronze 11" when reduced to 62°.

Kater, 1818,	"Mètre à Traits"	= 39.37076 inches.
	"Mètre à Bouts"	39.37081
Kater, 1820,	Dolland's Scale	39.37045
Baily, 1836,	"Mètre à Traits"	39.36968
	"Mètre à Bouts"	39.36937
Clarke, 1866,	"Mètre à Traits"	39.37048
Chisholm, 1870,	Standard Meter on Baily's metal	39.37112
Hassler, 1832,	Original Iron Meter	39.38005

Mr. Hassler compared several other standards with Troughton's yard, but they are not included in the above table; first, because, with the exception of the brass meter by Lenoir, there is no evidence that the standards compared are authentic copies of the original; and, second, because the certificate of the meter by Lenoir refers to the Meter of the Observatory, and not to the Meter of the Archives.

It is obvious from this table, that our present knowledge of the relation between the length of the yard and the meter is subject to great uncertainty. In deciding upon the weight which should be assigned to any particular value, it is to be remembered, —

(*a.*) That in the comparisons by Kater, Baily, Clarke, and Chisholm, the constant relation given by Arago is subject to great doubt.

(*b.*) That the meters of the Royal Society do not admit of exact measurements in their present state.

(*c.*) That Kater has himself declared his own determination to be erroneous.

(*d.*) That the values given by Baily are expressed in terms of the Scale of the Astronomical Society, which for 39.37 inches is .00076 inch *longer* than the Imperial Yard.

(*e.*) That the value given by Mr. Chisholm involves the errors of transfer to the bar of Baily's metal.

(*f.*) That the lines on the bar of Baily's metal do not admit of great precision in comparisons with moderately high powers.

(*g.*) That the original iron meter has never been compared with the Meter of the Archives, but only with the Meter of the Conservatory.

In view of these facts, it does not seem too much to say that it is at present impossible to assign any value to the equation between the yard and the meter, which is not liable to an error as great as .005 inch.

Finally, the confusion with regard to this relation has become so great, that, by an act of Parliament passed in 1878, the relation found by Kater was declared to be the legal relation, without regard to the various determinations which have since been made.

I will close this article with an abstract from the Philosophical Transactions for 1798, p. 180, giving the errors of the six-inch spaces of Troughton's scale as determined by Shuckburgh. As this scale is understood to be still in existence, it would be interesting to have a remeasurement of these spaces with a comparator of modern construction. I know of no modern graduations which have a much greater apparent accuracy than is here indicated.

Inches.	Error, or Difference from the Mean.
0 to 6	+.00012 inch.
6 to 12	+.00000
12 to 18	+.00007
18 to 24	—.00013
24 to 30	—.00006
30 to 36	+.00020
36 to 42	—.00033
42 to 48	+.00007
48 to 54	—.00003
54 to 60	+.00010

HARVARD COLLEGE OBSERVATORY, April 14, 1880.

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In Catalogue

DIMENSIONS

OF

THE FIXED STARS,

WITH ESPECIAL REFERENCE TO

BINARIES AND VARIABLES OF THE ALGOL TYPE.

BY

EDWARD C. PICKERING.

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PAPERS READ BEFORE THE ACADEMY.

I.

DIMENSIONS OF THE FIXED STARS, WITH ESPECIAL REFERENCE TO BINARIES AND VARIABLES OF THE ALGOL TYPE.

BY EDWARD C. PICKERING.

Presented May 25, 1880.

SINCE direct measurements cannot at present be made of the disks of the fixed stars, any information with regard to their dimensions derived from the amount and character of their light will have a value. This is the course ordinarily taken in the case of a satellite or small planet, and appears to deserve a more extended trial beyond the limits of the solar system than it has yet received. The principal objection to this method is the uncertainty in the numerical value of the intrinsic brightness or other constants involved, which cannot at present be measured with accuracy. The exact formulæ will therefore be first given, and hypotheses then introduced regarding the values of these constants.

Let B, $b =$ the diameters of the Sun and of any given star, as seen from the Earth, expressed in seconds of arc.

Let $l =$ the intrinsic brightness of the star, that of the sun being taken as unity; in other words, let l denote the ratio borne by the quantity of light emitted by the star to that emitted by the Sun from the same superficial area.

Let S, s = the light of the Sun and of the star expressed in stellar magnitudes by means of the scale of Pogson, in which a difference of one magnitude corresponds to the logarithmic ratio, 0.4. This ratio, expressed in numbers, is approximately 2.512.

Let p = the parallax of the star in seconds of arc.

The observed light of the star will be to that of the Sun as $l\,b^2$ is to B^2; the difference in their stellar magnitudes, or

$$s - S = 2.5 \log \frac{B^2}{l\,b^2} = 5 \log B - 5 \log b - 2.5 \log l.$$

Hence, $\log b = \log B + 0.2\,S - 0.2\,s - 0.5 \log l.$

The radius of the Sun equals 16′ 2″, and accordingly $B = 1924''$. The value of S is more uncertain. Various determinations of the ratio of the light of the Sun to that of Sirius have been made by different observers. In 1698, Huyghens found the value 756,000,000 by reducing the light of the Sun by a minute hole.* Wollaston, in 1829, compared the image of the Sun and of a lamp reflected in a silvered bulb of glass, and deduced the ratio 20,000,000,000.† Steinheil, in 1836, using the Moon as an intermediate standard of comparison, gave the value 3,840,000,000.‡ In 1861, Bond determined the relative light of the Sun and Moon by comparing their reflections in a glass globe with that of a Bengola light. Combining his measures with the comparisons of the Moon and Sirius by Herschel and Seidel, he deduced the value 5,970,500,000.§ . In 1863, Clark found that, if the Sun was removed to 1,200,000 times its present distance and Sirius to 20 times its distance, they would appear equally bright, and equal to a sixth-magnitude star. Their ratio, consequently, equals 3,600,000,000.‖ Reducing these measures to magnitudes, we obtain the values, Huyghens, 22.20; Wollaston, 25.75; Steinheil, 23.96; Bond, 24.44; and Clark, 23.89. The mean of all of these is 24.05, with an average deviation of 0.84. The last three agree well, and give 24.10, with an average deviation of 0.23. Probably 24.0 is not far from the truth, and may be assumed to represent this ratio as closely as it is at present known. If we adopt —1.5 for the magnitude of Sirius, from the measures of Herschel and Seidel, we obtain for the stellar magnitude of the Sun —25.5.

* Cosmotheoros, La Haye, 1698.

† Phil. Trans., cxix. 28.

‡ Elemente der Helligkeits-Messungen, Munich, p. 24.

§ Mem. Amer. Acad., viii. N. S., p. 298.

‖ Amer. Jour. Sci., xxxvi. 76.

Substituting in the formula for log b given above, $B = 1924''$ and $S = -25.5$, we obtain $\log b = 3.284 - 5.100 - 0.2\,s - 0.5 \log l = 8.184 - 0.2\,s - 0.5 \log l$. This formula is exact, and would give the true diameter of any star if l was known.

An approximate value of l might be determined by the following method. Suppose that an electric current be passed through a platinum-iridium wire heating it to incandescence, and that the brightness of a short portion of it be compared with an artificial star when the current is varied by a known amount. As the current increases, the color of the light changes, the amount of the blue light increasing more rapidly than that of the red. The ratio of the two may be determined by inserting a double-image prism in the collimator of a spectroscope and viewing the wire through it. The two images may be made to overlap by any desired amount by varying the distance of the double-image prism from the slit of the collimator. The blue rays may thus be combined with the red, yellow, or green, as desired. The relative brightness of the two images may be varied by a Nicol placed in the eyepiece and turned through a known angle. We may thus combine any portion of the spectrum with any other part in such a proportion as to produce a tint to which the eye is especially sensitive. From the readings of the Nicol when different currents are passed through the wire, we may determine the varying proportion of any two rays, as the red and blue, when the wire is emitting a given amount of light. Observing in the same way the spectra of the Sun and star, and applying to them the law deduced from the observations of the wire, we obtain an approximate value of the comparative light emitted by equal areas of the two bodies. This will not be exact, since the effect of absorption is not allowed for, a difference of temperature being assumed to be the only cause of the observed difference in color. Probably the error will not be large, except perhaps in the case of the red stars. Until these measurements are made, we can do no better than to assume that $l = 1$, or that the emissive power is the same for the Sun and star. As a large portion of the stars have nearly the same color as the Sun, and a similar spectrum, this assumption will probably not be far from the truth. The term equivalent diameters may be conveniently applied to the quantities thus computed. They may be defined as the diameters the Sun would have if removed successively to such distances that it would equal in light stars of the given magnitudes. The expressions, equivalent densities and equivalent masses, will be used in the same manner to denote the densities or masses of bodies in their other properties resembling the Sun.

Table I. gives the equivalent diameters of stars of various magnitudes, assuming $l = 1$.

TABLE I. — EQUIVALENT DIAMETERS OF STARS OF VARIOUS MAGNITUDES.

Magn.	Diam.	Magn.	Diam.	Magn.	Diam.
0	0″.01528	5	0″.00153	10	0″.00015
1	.00964	6	.00096	11	.00010
2	.00608	7	.00061	12	.00006
3	.00384	8	.00038	13	.00004
4	.00242	9	.00024	14	.00002

The diameters corresponding to the intermediate magnitudes may be found from Table II., which gives the diameters for every tenth of a magnitude from 0.0 to 4.9.

TABLE II. — EQUIVALENT DIAMETERS OF STARS FOR EACH TENTH OF A MAGNITUDE.

Magn.	Diam.	Magn.	Diam.	Magn.	Diam.	Magn.	Diam.	Magn.	Diam.
0.0	0″.01528	1.0	0″.00964	2.0	0″.00608	3.0	0″.00384	4.0	0″.00242
0.1	.01459	1.1	.00920	2.1	.00581	3.1	.00366	4.1	.00231
0.2	.01393	1.2	.00879	2.2	.00555	3.2	.00350	4.2	.00221
0.3	.01330	1.3	.00840	2.3	.00530	3.3	.00334	4.3	.00211
0.4	.01271	1.4	.00802	2.4	.00506	3.4	.00319	4.4	.00201
0.5	.01213	1.5	.00766	2.5	.00483	3.5	.00305	4.5	.00192
0.6	.01159	1.6	.00731	2.6	.00461	3.6	.00291	4.6	.00184
0.7	.01107	1.7	.00698	2.7	.00441	3.7	.00278	4.7	.00175
0.8	.01057	1.8	.00667	2.8	.00421	3.8	.00266	4.8	.00168
0.9	.01009	1.9	.00637	2.9	.00402	3.9	.00254	4.9	.00160

When the magnitude is increased by five, the diameter will be reduced ten times, and the decimal point should accordingly be moved one place to the left. Thus, if a star of the 3.5 magnitude has a diameter of 0″.003, one of the 8.5 magnitude will have a diameter of 0″.0003 and one of the 13.5 magnitude, 0″.00003. The diameter of Sirius would be that corresponding to —1.5 magnitudes, or 0″.03, were it not that l is probably greater than 1 owing to the blue color of the star, and the diameter consequently less.

Should future measurements render some other value of S more probable, Tables I. and II. can still be used, merely changing s by the same amount that S is altered.

The smallest star that can be seen in the 15-inch telescope of the Harvard College Observatory has a magnitude of about 15.5, and a corresponding equivalent diameter of 0.″000012.

When the parallax of a star is known, these principles may be applied to determining its linear diameter. If the Sun was removed to the distance of the star its diameter would have the same ratio to the parallax that the chord of the Sun's diameter, as seen from the Earth, has to unity. It would therefore equal

$$2p \sin 16' 2'' = 0.00933p.$$

Table III. gives the light in stellar magnitudes which would be emitted by the Sun if removed to such a distance that its parallax would have the value given in the first column.

TABLE III. — PARALLAX.

Par.	Magn.	Par.	Magn.
0''.1	6.07	0''.6	2.18
0.2	4.57	0.7	1.84
0.3	3.68	0.8	1.56
0.4	3.06	0.9	1.30
0.5	2.58	1.0	1.07

If the parallax of *α Centauri* is assumed to be $0''.9$, the Sun as seen from it will appear as a star of the 1.3 magnitude. The light of *α Centauri* is not known with much certainty, as we have to depend upon eye estimates. Assuming the magnitude of the two components to equal 0.0 and 3.0, we find that if $l = 1$ for both of them, their diameters will be 1.82 and 0.46 times that of the Sun. The parallax of 61 *Cygni* may in like manner be assumed to be $0''.3$, and the magnitude of its components 5.0 and 6.0. The Sun would then appear, from this distance, as a star of the 3.7 magnitude, and the diameter of the two components, compared with that of the Sun, if their emissive powers are the same, will be 0.55 and 0.35.

I. BINARY STARS.

In the case of a binary star, another equation of condition may be introduced from Kepler's third law. Let N denote the mass of the binary in terms of that of the Sun, P the period of revolution in years, a the semi-axis major, or mean distance of the components, and b the equivalent diameter, or the diameter of a star having the same mass as the binary, and the same density and intrinsic brightness as the Sun.

Comparing the binary with the system formed by the Sun and Earth seen at the same distance, we see that the two systems have masses in

the ratio of N to 1, mean distances in the proportion of a to p, and periods of revolution as P to 1. Accordingly, by Kepler's law, $N : 1 = \frac{a^3}{P^2} : \frac{p^3}{1}$, or $N = \frac{a^3}{p^3 P^2}$. But $N = \frac{b^3}{(0.00933\,p)^3}$, since $0.00933\,p$ will equal the diameter of the Sun at the distance of the binary. Hence, equating these two values of N, p is eliminated, and we have $b = 0.00933\ a\ P^{-\frac{2}{3}}$. The stellar magnitude corresponding to the diameter, b, may now be found from Tables I. and II. So far, no hypothesis has been introduced, and the errors in these quantities will depend only on the errors in the photometric measurements and in the micrometric determination of the elements of the orbit.

If now we could find the value of l for each of the components, as suggested above, we could determine the true diameter of the two stars, and from their orbits, and the mass of the binary, deduce their average densities. Until these measures are made, we can do no better than assume that both stars have the same density, and that $l = 1$ for each. On this hypothesis, if b_1, b_2 are the equivalent diameters of the two components, and b the equivalent diameter of the binary as computed from the time of revolution and mean distance, the density will equal $\frac{b^3}{b_1^3 + b_2^3}$.

Since the value of the parallax is eliminated, it follows that these considerations will not aid the determination of the distance of a binary. The time of revolution of a binary would remain unchanged if removed to double the distance, provided that the linear distance of the components and their diameters were increased in the same proportion, or that the angular dimensions of the system remained unchanged. In other words, the observed time of revolution of a binary system is wholly independent of its distance from the observer.

The relative masses of the two components could be determined micrometrically and independently of the above methods, by measuring the position of each component from the adjacent stars. If this was repeated at intervals during an entire revolution of the binary, the components would be found to have described similar ellipses whose dimensions would be inversely proportioned to the masses. From the *Proc. Roy. Astron. Soc.*, xl. 235, it would appear that Mr. Gill will apply this test to the components of *α Centauri*. If the difference in light is three magnitudes, and the intrinsic brightness and densities the same for the two components, the ratio of the masses would be as 63 to 1. The semi-axis major of the ellipse described by the larger star would therefore be, according to the elements given by Hind,

$\frac{21.80}{64} = 0''.34$. Owing to the inclination of the plane of the orbit the apparent ellipse would be much less than this. Some other stars would appear better adapted to this test. The smaller difference in light more than compensates for the smaller orbit. From the data given in Table V., the semi-axis major of the ellipse described by several stars has been computed. The name of the star is followed by the time of revolution in years, and the semi-axis of the ellipse described by the larger component; γ *Coronæ Australis*, 45, 1″.2; ξ *Ursæ Majoris*, 60, 0″.8; 70 *Ophiuchi*, 94, 0″.4; ξ *Boötis*, 127, 0″.2; γ *Virginis*, 185, 2″.0. Some others might give a larger apparent orbit, but a very long time would be required to detect the motion. When the inclination of the orbit is not zero, the apparent ellipse will be less than that computed in this manner in the same proportion that the apparent orbit is less than the real orbit described by the companion. Similar observations might be made on any double stars whose components appear to be physically connected. The proper motion, however, complicates the phenomenon, and cannot be distinguished from the orbital motion as long as the latter appears to be rectilinear.

So many large telescopes are now devoted to the measurement of double stars that there is great danger of an unnecessary duplication of work. A valuable contribution might be made to our knowledge of stellar motion by determining the positions of the components of a double star with regard to several adjacent stars. Even if the masses of the components could not thus be determined, we should at least provide the material for an accurate measurement of their proper motions in the future. The same may be said of the determination of the proper motions of other stars, which could be observed in this way with much greater precision than by the usual meridian observations. Useful work could be done by an observer unprovided with means for measurements by simply examining a large number of double stars and stars having a large proper motion, and noting the approximate position and distances of any adjacent stars near enough and bright enough for accurate measurement. A list would thus be formed from which the selection of suitable objects would be easy.

The spectroscope, which has opened so rich a field for work in astronomy, may be applied also to the study of the binary stars. If measurements could be obtained of the approach or recession of the two components, several interesting conclusions could be derived from them. A single measurement would not give the relative masses of the components, since the effect of the proper motion cannot be dis-

tinguished from that caused by the inequality of the masses. The proper motion may be eliminated if the observations are repeated in different parts of the orbit of the binary, since its effect would be always the same, while that due to the inequality of the masses would be continually altering, becoming zero and altering its sign twice during each revolution. If the ratio of the masses could be determined micrometrically as described above, the measures with the spectroscope would determine the component of the proper motion in the direction of the line of sight. The principal use of the measures with the spectroscope would be to determine the true dimensions of the orbit, and consequently the distance of the binary.

Let Ω denote the position angle of the node of the binary, i the inclination of the plane of its true to that of its apparent orbit, s the distance, and p the position angle at the time of observation; let ds and dp represent the annual changes in these quantities. Let us make a transformation to a system of rectangular co-ordinates in which the axis of X shall coincide with the line of nodes, the axis of Z coincide with the line of sight, and the axis of Y be perpendicular to both of them. Then dz will equal the annual change in the distances of the two components from the observer, or will measure in seconds of arc the same quantity that the spectroscope measures by the difference in velocity of the two components. But

$$dz = dy \tan i \text{ and } y = s \sin (p - \Omega);$$

hence $$dz = \tan i \sin (p - \Omega)\, ds + s \tan i \cos (p - \Omega)\, dp.$$

Substituting the proper numerical values we obtain dz in seconds of arc; it should be remembered that dp must be expressed in terms of the radius, or 57°.3 must be taken as the unit. This method may be employed if we have an ephemeris of the star, the inclination of the orbit, and the position angle of the line of nodes. If the elements of the orbit are given without an ephemeris, a different formula must be used. Let ρ denote the real distance of the components, and u the angle from the node measured in the plane of the orbit. If a system of co-ordinates is employed such that X' lies in the line of nodes, Y' perpendicular to it in the plane of the orbit, and Z' in the line of sight, we have

$$y' = \rho \sin u, \text{ and } dz' = dy' \sin i = \sin i \sin u\, d\rho + \rho \sin i \cos u\, du.$$

If the orbit is circular, u increases uniformly with the time, and ρ is constant and equals a; hence $dz' = a \sin i \cos u\, du$. If in this expression $du = \frac{2\pi}{P}$, or denotes the fraction of the orbit

traversed in one year, $d z' = \frac{2 \pi a \sin i}{P} \cos u$. The maximum value of this expression occurs when $u = 0°$ or π, and is $\frac{2 \pi a \sin i}{P}$. If the orbit is elliptical, ρ and u may be deduced from the elements, and $d z$ may be expressed as a function of the eccentricity, node, and time, multiplied by the factor, which is constant for each orbit, $\frac{a \sin i}{P}$.

Let V denote the velocity of light, v the velocity of approach of a star, λ the wave-length of a given ray of light, and l the corresponding change it undergoes, due to the velocity. Then $V + v : V = \lambda + l : \lambda$ or $v = V \frac{l}{\lambda}$; v and V are commonly expressed in kilometers per second, l and λ in ten-millionths of a millimeter; $V = 300000$. The line F is frequently used in these measures, and for it $\lambda = 4865$. Substituting these values, $v = 62\, l$. For the D line, $\lambda = 5900$, and since the interval between the two components equals 6, a velocity of 305 kilometers per second will be required to produce a deviation equal to the interval between these lines. It will be more convenient to measure the velocity of a star in terms of m, the annual motion, taking the distance from the Earth to the Sun as a unit. This may then be reduced to seconds of arc, if the distance of the star is known, by multiplying by the parallax p. Light traverses the distance from the Earth to the Sun in about 498 seconds, or would traverse 63300 times this distance in a year. Accordingly, $v = 63300 \frac{l}{\lambda}$; for the F line $v = 13\, l$, for the interval of the D lines, $v = 64\, l$. If l is positive or the line moves toward the red end, it denotes that the star is receding from the observer. We have thus two values of the relative motion of the stars in the line of sight; one, $d z$, deduced by computation from the micrometer measurements; the other, $v\, p$, or $13\, l\, p$, if the F line is observed, found by the spectroscope. Equating these values, since p is the only unknown quantity, $p = \frac{d z}{13\, l}$. The dimensions of the orbit are now found directly, since $\frac{a}{p}$ will equal the semi-axis major in terms of the distance of the Sun from the Earth.

It not unfrequently happens that we have an estimate of the difference in magnitude of the two components of a double star by one observer using a telescope, and also an estimate of their combined light by another observer viewing them with the unassisted eye. From these data we wish to determine the brightness of either component alone. Sometimes we have the opposite problem, given the magnitude of the separate stars to find that of both, as seen by the eye or in a

telescope not capable of separating them. Let l denote the light of the fainter star in terms of the brighter, and m the magnitude of the fainter minus the magnitude of the brighter. Then, on Pogson's system, $m = -2.5 \log l$. If M is the magnitude of the brighter star minus that of a star equivalent to the two combined, or having the light $(1 + l)$, then $M = -2.5 \log (1 + l)$. From these formulæ we can always find the corresponding values of M and m. The maximum value of $M = 0.75$ when m is zero or the stars are equal. Table IV. enables us to determine M to the nearest tenth of a magnitude for any value of m. As an example, suppose two stars have magnitudes 2.0 and 3.0; then $m = 3.0 - 2.0 = 1.0$, and M, from the table, lies between 0.35 and 0.45 or equals 0.4. The light of both combined will therefore equal $2.0 - 0.4 = 1.6$.

TABLE IV. — Combination of Two Stars.

M.	m.	m'.
0.05	3.32	1.90
0.15	2.07•	1.06
0.25	1.47	0.64
0.35	1.05	0.34
0.45	0.72	0.11
0.55	0.45	—
0.65	0.22	—
0.75	0.00	—

It is sometimes convenient to know what would be the magnitude of a star whose mass was equal to that of the two components of a double star of the same density and brightness. Let m' equal the difference in magnitudes of the two components, and l and n, the light and mass of the fainter in terms of the brighter. Then

$$m' = -2.5 \log l = -2.5 \log n^{\frac{2}{3}} = -1.67 \log n,$$

since the light is proportional to the square, and the mass to the cube, of the diameter. If then M equals the magnitude of the brighter component minus that of both combined, we shall have $M = 1.67 \log (1 + n)$, from which M is determined as before from any given value of m'. The third column of Table IV. gives the value of m' corresponding to every odd twentieth of a magnitude of M. The value of the latter may thus always be determined to the nearest tenth of a magnitude. The maximum value of M is 0.50, when $m' = 0$. Adopting the same magnitudes as in the last example, if two stars have the magnitudes of 2.0 and 3.0, m' will equal 1.0. This value from the third column of Table IV. will correspond to a value of M lying between 0.15 and 0.25, or will equal 0.2. The magnitude

of a star having the same mass as the binary will therefore have a magnitude 2.0 — 0.2 = 1.8.

Most of the binary stars whose orbits have been computed are compared in Table V. The successive columns give a current number, the name of the star, the number of the Dorpat Catalogue, the right ascension and declination for 1880, the semi-axis major in seconds, the eccentricity, the period in years, and the inclination of the plane of the orbit in degrees. The next two columns give the magnitudes of the components as estimated by Struve. Three of the stars are not contained in the Dorpat Catalogue, and for them the magnitudes given have been assumed. The next column gives the equivalent diameter 0.00933 $a\ P^{-\frac{2}{3}}$, or the magnitude of a star having the mass of the binary and the density and brightness of the Sun. From the magnitudes of the components we may compute, by the third column of Table IV., the brightness of a star having the same mass as the binary and the same brightness and density as its components. Subtracting from this quantity that given in the preceding column gives the next column. If these quantities were small, we might assume that they were due to errors in the assumed magnitudes of the stars. Their variations are, however, far too large to be explained in this way. As they are almost all negative, we may infer that the assumed light of the Sun is too small, or that a larger value should have been given on page 2 to S. A great part of the difference must be ascribed to variations in the density or brightness of the stars. We have at present no way of discriminating between these causes. Such a method as has been proposed on page 3 for determining l would serve to distinguish them. Until then, it will be convenient to reduce this difference from magnitudes to the relative diameters of two stars of equal density and brightness, one having a mass, the other emitting a light equal to that of the binary. Assuming the diameter of the first of these stars as a unit, the diameter of the other is given in the next column, and may be denoted by C. In almost all cases this quantity is greater than unity, from which we should infer that most of the stars enumerated are either much brighter or much less dense than the Sun, unless, as suggested above, the measurements of the light of the Sun are largely in error. Let d denote the density, b the brightness of the components of the binary, and D the equivalent diameter of the binary in terms of the same unit as C. Then $D^2 : C^2 = 1 : b$, and $D^3 : 1^3 = 1 : d$; eliminating D, $C = \frac{b^{\frac{1}{2}}}{d^{\frac{1}{3}}}$, or the brightness is proportional to the square of C and the density inversely as its cube. If

TABLE V.—BINARY STARS.

No.	Name.	Σ	R. A. 1880. h.	m.	Dec. 1880. °	′	a	e	P.	i	A.	B.	Equiv. Magn.	O.—C.	Relat Diam.	$\frac{a \sin i}{P}$	Computer.
1	42 Comae Berenicis	1728	13	4.1	+18	10	0.66″	0.48	25.7	90.0°	6.0	6.0	6.7	−1.2	1.74	0.026″	O. Struve.
2	ζ Herculis	2084	16	36.8	+31	49	1.36	0.41	34.6	51.1	3.0	6.5	5.5	−2.5	3.16	0.031	Flammarion.
3	Anon. Leonis	3121	9	10.7	+29	7	0.71	0.26	37.0	74.2	7.5	7.8	7.0	+0.1	0.96	0.019	Doberck.
4	η Coronae Borealis	1937	15	18.2	+30	43	0.99	0.29	40.2	60.4	5.2	5.7	6.4	−1.5	1.90	0.021	Flammarion.
5	221 B Ophiuchi	2173	17	24.7	−0	58	1.01	0.13	45.4	80.5	5.6	6.1	6.6	−1.2	1.74	0.023	Dunér.
6	α Canis Majoris	....	6	39.7	−16	32	7.11	0.61	49.4	47.1	−1.6	8.5	2.5	−4.0	6.31	0.108	Auwers.
7	γ Coronae Australis	....	18	58.3	−37	14	2.40	0.70	55.6	68.6	5.5	5.5	5.0	0.0	1.00	0.040	Schiaparelli.
8	ξ Ursae Majoris	1523	11	11.6	+32	13	2.58	0.38	60.7	56.3	4.4	5.3	5.0	−0.8	1.44	0.037	Hind.
9	ζ Cancri	1196	8	5.3	+18	1	0.91	0.36	62.4	20.7	5.0	5.7	7.2	−2.4	3.02	0.005	O. Struve.
10	α Centauri	...	14	31.3	−60	20	21.30	0.67	85.0	82.3	0.0	3.0	0.8	−0.8	1.44	0.264	Hind.
11	"	....	"	"	"	"	18.45	0.58	88.5	79.4	"	"	1.2	−1.2	1.74	0.205	Doberck.
12	70 Ophiuchi	2272	17	59.4	+2	33	4.88	0.39	92.8	62.1	4.0	6.8	4.2	−0.2	1.10	0.046	Flammarion.
13	γ Coronae Borealis	1967	15	37.7	+26	41	0.70	0.35	95.5	85.2	4.0	7.0	8.4	−4.4	7.59	0.007	Doberck.
14	ξ Scorpii	1998	15	57.8	−11	2	1.26	0.06	95.9	68.7	4.9	5.2	7.2	−2.7	3.47	0.012	"
15	Anon. Cassiopeiae	3062	23	59.9	+57	46	1.27	0.46	104.4	33.2	6.9	8.0	7.8	−0.5	1.26	0.006	"
16	ω Leonis	1356	9	22.0	+9	35	0.89	0.54	110.8	64.1	6.2	7.0	8.1	−2.1	2.63	0.007	"
17	p Eridani	407	8	24.8	−11	33	3.82	0.38	117.5	44.7	8.2	10.7	5.1	+3.1	0.24	0.028	"
18	25 Canum Venaticorum	1768	13	32.1	+36	54	0.75	0.66	124.5	51.5	5.7	7.6	8.7	−3.1	4.17	0.005	"
19	ξ Boötis	1888	14	46.3	+19	36	4.86	0.71	127.4	36.9	4.6	6.9	4.7	−0.1	1.06	0.028	"
20	γ Virginis	1670	12	35.6	−0	47	3.39	0.87	176.0	0.0	3.0	3.0	5.9	−3.4	4.79	0.000	Flammarion.
21	"	"	"	"	"	"	3.97	0.90	185.0	36.1	"	"	5.6	−3.1	4.17	0.012	Thiele.
22	τ Ophiuchi	2262	17	56.5	−8	11	1.40	0.61	217.9	58.7	5.0	5.7	8.1	−3.3	4.57	0.005	Doberck.
23	η Cassiopeiae	60	0	41.8	+57	11	9.83	0.58	222.4	58.5	4.0	7.6	3.9	+0.1	0.96	0.036	"
24	λ Ophiuchi	2055	16	24.9	+2	16	1.19	0.49	233.9	44.7	4.0	6.1	8.6	−4.6	8.32	0.004	"
25	44 Boötis	1909	14	59.8	+48	7	3.09	0.71	261.1	70.1	5.2	6.2	6.7	−1.7	2.19	0.011	"
26	μ² Boötis	1938	15	20.0	+37	46	1.47	0.60	290.3	40.0	6.7	7.8	8.4	−2.0	3.51	0.003	"
27	36 Andromedae	73	0	48.0	+23	56	1.54	0.65	349.1	41.6	6.2	6.8	8.6	−2.7	3.47	0.003	"
28	γ Leonis	1424	10	13.4	+20	27	1.98	0.73	407.0	43.1	2.2	3.6	8.8	−6.3	17.88	0.003	"
29	δ Cygni	2579	19	41.2	+44	50	2.31	0.29	415.1	37.7	3.0	7.9	8.0	−5.0	10.00	0.003	Behrmann.
30	61 Cygni	2758	21	1.4	+38	7	15.40		452.0	.	5.1	6.0	4.0	+0.9	0.66	—	——
31	σ Coronae Borealis	2032	16	10.2	+34	10	5.88	0.75	845.9	31.9	5.1	6.4	7.0	−2.0	2.51	0.004	Doberck.
32	α Geminorum	1110	7	27.0	+32	9	7.43	0.38	1001.2	44.0	2.7	3.7	6.7	−4.2	6.93	0.006	"
33	ζ Aquarii	2909	22	22.8	−0	33	7.64	0.65	1578.3	44.7	4.0	4.3	7.3	−3.7	5.50	0.008	"

then the star has the same density as the Sun, the square of C will give its brightness. Again, if the star has the same brightness as the Sun, its density will equal one divided by the cube of C.

The product of the semi-axis major by the sine of the inclination and divided by the period is given in the last column but one. It serves as a measure of the annual approach or recession of the two components. Neglecting the eccentricity, the maximum motion in seconds will equal this quantity multiplied by $2\pi = 6.28$.

The last column gives the name of the astronomer by whom the orbit was computed, which is adopted in this discussion.

An inspection of the last column but one shows that the value of $\frac{a \sin i}{P}$ in several cases amounts to $0''.03$ or even more. Neglecting the eccentricity, the maximum motion would therefore equal 2π times this quantity, or nearly $0''.2$. The eccentricity in some cases would diminish the motion, but in other cases it would increase it. An eccentricity of 0.5 might vary it from $0''.1$ to $0''.4$, according to the position of the peri-astron. This value of $\frac{a \sin i}{P}$ would probably be even larger for some of the recently discovered stars, in which P is still smaller than in the stars given in the table. It is commonly supposed that the parallax of an average first-magnitude star does not much exceed $0''.1$. That of a sixth-magnitude star would then be about $0''.01$ unless the fainter stars are really smaller than the brighter, or unless there is a perceptible absorption of light in space. Substituting the values $dz = 0''.2$, $p = 0''.01$, in the formula for the F line, $p = \frac{dz}{18\,l}$, given on page 9, we deduce $l = \frac{dz}{18\,p} = 1.5$. Accordingly the difference in the positions of the F line would be 1.5 times as great as the deviation observed in the case of Sirius. As the spectra of the two components could be observed in turn (or perhaps simultaneously) without disturbing the spectroscope, many of the causes of uncertainty present in similar measures of single stars would be removed.

In any case, if the F line could be seen in both components, we could assign a limit within which we could be certain that it was the same for both, and this would give a value of the parallax which must be less than the true parallax. A determination of the outside limit of distance of a star would appear to have nearly the same importance as the inside limit of distance found by micrometric distance. Moreover it does not seem probable that a star will be found whose parallax is very large, or previous observation might have detected it.

The search for a star with a very small parallax seems more hopeful, since it could not have been detected by other measures.

The observation would have value if we could determine the direction of the motion, even if we could not measure its amount, since it would show which portion of the orbit was turned towards the observer. This cannot be found from the micrometric measurements, since, although we can obtain from them the amount of the inclination, we cannot determine its sign.

It is also possible that some method of greater delicacy may be discovered, so that the spectroscope may be replaced by a more sensitive instrument, as it has been by the interferential refractometer in measuring the index of refraction of gases.

The semi-axes major of Σ 3121, 1768, 2262, and 2055 are not given in the original publications of the orbits. The values inserted in Table V. are those given in the Handbook of Double Stars, by Messrs. Crossley, Gledhill, and Wilson. This work has also proved most useful in various ways in the preparation of this paper. The value of a given by Dr. Auwers for *α Canis Majoris* is 2.33. This relates to the ellipse described by the bright star. As the companion is assumed to have a mass $\frac{1}{2.05}$ times as great as this, the value of a must be multiplied by 3.05, and therefore $2.33 \times 3.05 = 7.11$ is the value adopted. It is obvious that for this star the intrinsic brightness of the two components is by no means the same. If the density is the same, the diameter of the companion would be 0.79 that of the primary. The area of its disk would be 0.62, while its light* is only 0.0001 of that of its primary. The very large relative diameter of *γ Leonis* is remarkable. Its brightness must be about three hundred times that of the Sun, if its density is the same. On the other hand, if no brighter than the Sun, its density would be only one seventh of that of atmospheric air at the standard density and pressure, to give it a sufficient bulk to emit its observed light. If the other binaries have the same density as the Sun, their brightness must vary from 100 in the case of *δ Cygni* to 0.06 in the case of *p Eridani*, the brightness of the Sun being taken as the unit. The semi-axis major and period of 61 *Cygni* are taken from Newcomb and Holden's Astronomy. Although this star is commonly regarded as a binary, the evidence in favor of this view seems to depend upon the large proper motion of both components, and the fact that both appear to be comparatively near the Sun. It is doubt-

* Ann. Harv. Coll. Observ., xi. 177.

ful whether the observations yet made are sufficiently exact to prove a connection between the components. To establish this proposition, and also as an example of a convenient means of distinguishing a binary star from one which is optically double, the following investigation is given of the more important observations of 61 *Cygni*. We cannot conclude that a star is binary unless the path described by one of its components appears to be concave with respect to the other. If the motion appears to be rectilinear, it is more probably that due to the proper motion of one of them, or rather to the combined effect of the proper motions of both. On the other hand, if the path is convex, it is extremely probable that there is a real connection between the two, as there is no instance known of a star describing a curved path due to proper motion alone. The motion, if rectilinear, should also be uniform, while, if curved, the motion should be most rapid when nearest the other star. The law that the area described by the radius vector is proportional to the time, cannot be used to distinguish between those motions, since it will apply to both.

Suppose that the measures are transformed to a system of rectangular co-ordinates, having one component as the origin, and the axis of X nearly parallel to the path of the other component. Except for the accidental errors, the value of y, if the motion is rectilinear, should be the same for all the observations from the beginning to the end of the series. If the axis of X is not exactly parallel to the line of motion the values of y should increase slowly from one end of the series to the other. If they are corrected by an amount which will be proportional to the time, this variation should disappear. If the star is binary, however, the value of y will vary, in general having its greatest value during the middle of the period, and being smaller at the beginning and end.

The values of x, if the motion is rectilinear, will vary uniformly with the time, and, if corrected by a constant, plus another constant multiplied by the time, will leave residuals that are very small. If the motion is curved, on the other hand, this condition will not be fulfilled.

A reduction of the observations of 61 *Cygni* is given in Table VI. Of the measurements made during the last half-century only those made by the Struves and by Dembowski have been employed. The position angles are first corrected for precession and reduced to the epoch of 1880 by the formula

$$0^\circ.00557 \sin \alpha \sec \delta \, (t - 1880) = -0^\circ.005 \, (t - 1880).$$

A simple computation shows that the direction of the motion is nearly that of the position angle 256°. We have accordingly, $y = s \cos (p - 166°)$, and $x = s \sin (p - 166°)$. In the successive

TABLE VI. — PATH OF 61 CYGNI.

No.	Date.	Obs.	Cor. p.	s.	x	y	Δx	Δy
1	1753.8	Bradley.	34.8	19.68	−12.98	14.77	−0.20	−0.43
2	1778.0	Mayer.	50.4	15.24	− 6.59	13.74	+0.97	−1.46
3	1781.9	Herschel.	58.3	16.83	− 6.30	15.07	+0.44	−0.13
4	1793.6	Lalande.	62.3	14.87	− 5.98	13.61	−1.70	−1.59
5	1800.0	Piazzi.	69.8	19.27	− 2.08	19.16	+0.86	+3.96
6	1805.0	"	78.1	14.50	+ 0.53	14.49	+2.42	−0.71
7	1812.3	Bessel.	78.8	16.74	+ 0.82	16.72	+1.18	+1.52
8	1813.8	Lindenau.	68.8	16.56	− 2.08	16.43	−2.04	+1.23
9	1814.5	W. Struve.	68.6	15.20	− 1.96	15.08	−2.06	−0.12
10	1820.5	"	83.2	15.11	+ 1.89	14.99	+0.53	−0.21
11	1822.7	"	85.5	14.93	+ 2.46	14.73	+0.63	−0.47
12	1828.7	"	89.1	15.31	+ 3.47	14.91	+0.38	−0.29
13	1831.7	"	90.9	15.63	+ 4.02	15.11	+0.80	−0.09
14	1832.8	"	91.8	15.79	+ 4.30	15.20	+0.85	0.00
15	1835.6	"	93.6	15.97	+ 4.83	15.22	+0.29	+0.02
16	1836.6	"	94.2	16.08	+ 5.02	15.27	+0.37	+0.07
17	1837.7	"	95.2	15.93	+ 5.24	15.04	+0.26	−0.16
18	1843.5	O. Struve.	98.8	16.67	+ 6.46	15.44	+0.26	+0.24
19	1847.5	"	100.7	17.02	+ 7.11	15.46	+0.07	+0.26
20	1850.3	"	102.3	17.18	+ 7.61	15.40	+0.01	+0.20
21	1851.8	"	103.5	17.34	+ 8.02	15.38	+0.08	+0.18
22	1852.7	"	104.4	17.46	+ 8.30	15.36	+0.17	+0.16
23	1854.2	"	105.1	17.57	+ 8.54	15.35	+0.10	+0.15
24	1857.2	"	106.4	18.02	+ 9.12	15.55	+0.05	+0.36
25	1860.8	"	108.6	18.22	+ 9.82	15.35	−0.01	+0.15
26	1868.5	"	112.4	18.81	+11.16	15.14	−0.28	−0.06
27	1874.7	"	116.1	19.42	+12.61	14.85	−0.24	−0.35
28	1854.7	Dembowski.	105.4	17.29	+ 8.49	15.06	−0.06	−0.14
29	1855.8	"	106.0	17.34	+ 8.67	15.02	−0.11	−0.18
30	1856.8	"	106.3	17.45	+ 8.80	15.07	−0.15	−0.13
31	1857.6	"	107.2	17.73	+ 9.19	15.17	+0.03	−0.08
32	1858.5	"	107.7	17.78	+ 9.32	15.08	−0.02	−0.12
33	1862.8	"	109.3	18.36	+10.08	15.35	−0.17	+0.15
34	1863.4	"	109.5	18.37	+10.14	15.32	−0.23	+0.12
35	1864.7	"	110.6	18.58	+10.44	15.31	−0.21	+0.11
36	1865.6	"	110.8	18.57	+10.60	15.25	−0.24	−0.05
37	1867.2	"	111.6	18.72	+10.90	15.22	−0.27	+0.02
38	1868.7	"	112.7	18.83	+11.25	15.10	−0.24	−0.10
39	1869.7	"	113.4	18.96	+11.52	15.06	−0.18	−0.14
40	1870.6	"	113.9	19.16	+11.77	15.12	−0.12	−0.08
41	1871.6	"	114.2	19.28	+11.69	15.10	−0.21	−0.10
42	1872.6	"	114.8	19.33	+11.98	15.11	−0.33	−0.09
43	1873.6	"	114.8	19.44	+12.18	15.15	−0.34	−0.05
44	1874.5	"	115.3	19.50	+12.35	15.05	−0.35	−0.15
45	1875.6	"	115.9	19.58	+12.56	15.02	−0.38	−0.18

columns of Table VI. are given a current number, the date, the name of the observer, the corrected position angle, the distance, and

the values of x and y. The value of y is approximately $15''.20$; that of x, $0''.21$ $(t - 1814)$. Residuals are accordingly given in the last two columns by subtracting the values of y and x thus obtained from those observed.

The residuals in the last two columns are evidently not due to accidental errors, but whether they are caused by curvature of the path or systematic errors of the observer is less evident. The first nine sets are so discordant, that little dependence can be placed upon them. The values of Δy show a very slight increase, followed by a diminution in the later values. Δx seems to diminish slowly, the later values of the Struves and of Dembowski being somewhat less than the earlier. The curvature is so slight, that it has been thought to indicate an hyperbolic orbit. The observations so far made will however be very nearly satisfied by a large circular orbit seen obliquely, so that the part described during the last century has been that near the end of the minor axis of the apparent ellipse.

If we take the mean of the residuals, we find the values for Δx of $0''.25$ and for Δy of $0''.15$. As these include all kinds of systematic errors, the deviations from a straight line can scarcely be regarded as certain.

II. Variable Stars of the Algol Type.

Variable stars may be divided into several classes, according to the nature of the fluctuations of their light. First, temporary stars, which appear suddenly, and gradually fade away during the next few months. The most famous star of this class is that observed in 1572, by Tycho Brahe. The new stars in *Corona Borealis* in 1866 and in *Cygnus* in 1876, are recent examples of this class. Second, a large part of the variable stars pass from their maximum to their minimum and back again, in from six months to two years, the period and the brightness at the maximum and minimum being somewhat variable. The change in light is generally very great, amounting to several hundred, or even thousand times. The most striking examples of this class are *o Ceti* and χ *Cygni*. Thirdly, we have the slight changes to which many (or, according to Dr. Gould, most) stars are liable. These changes seem to be irregular in many cases; at least, their law is not yet known. Examples of this class are furnished in α *Orionis* and α *Cassiopeiæ*. Fourthly, certain stars continually vary, going through a series of changes in the course of a few days, which appears to be repeated exactly. Two causes seem here to be superimposed, one producing one

maximum and one minimum in each period, the other two maxima and two minima in the same time. As examples, β *Lyræ* and δ *Cephei* may be noted. Fifthly, we have a class of stars which during the greater part of the time remain unchanged in brightness, but at regular intervals lose in the course of a few hours a large part of their light, and regain it with equal rapidity. These changes appear to be repeated with the greatest regularity, so that the interval can be computed in some cases within a fraction of a second. *Algol*, or β *Persei*, is the most striking example of this class to which δ *Cancri* and δ *Libræ* also belong.

Various theories have been advanced to account for these phenomena. Probably different causes act in the case of the different classes. One theory would assume that by a collision, or by the liberation and ignition of a vast amount of hydrogen, the star was suddenly heated to incandescence, and gradually lost its light by cooling. This explanation would apply only to stars of the first class; it is strengthened in the case of the new star in *Cygnus* by the observations with the spectroscope. The spectrum gave at first the lines of incandescent hydrogen which disappeared as the light faded. It has been urged that, to account for the rapid cooling, the star must have been small, perhaps only a few miles in diameter, and consequently not very distant. This view is contradicted by the absence of perceptible parallax. If we consider how quickly a meteorite becomes heated, and again gives up its heat, this argument loses its force. The star may be large and distant, the surface only being heated, and soon losing its heat by radiation and conduction. This explanation appears more probable than that the light is cut off by clouds of smoke or steam, as has been suggested by some astronomers.

Stars constituted like our Sun, but in which the variations in size of the spots would be far greater, might undergo considerable changes in light. While it is difficult to account for the great changes in class two in this way, those in class three may be thus explained. A popular theory for the variation of stars of short period is that it is due to the revolution of the star upon its axis, when the different portions are of unequal brightness. The variation in light of Iapetus, the outer satellite of Saturn, is commonly explained in this way. A similar effect would be produced if the star was not spherical, and in revolving exposed a disk of varying area. A great variation could not thus be produced without the revolving body assuming a condition of unstable equilibrium. For the application of these principles to Iapetus, see *Annals of Harvard College Observatory*, xi. 264. This theory may ex

plain the variations of stars of the fourth class. Another theory would account for the changes of light by an opaque body or satellite passing between the star and the observer. It will be the object of the following discussion to show how fully this explanation will account for the variations of stars of the fifth class. A modification of this theory would replace the single eclipsing body by a cloud of meteorites. Such a theory will account for almost anything by suitably modifying the distribution of the meteors. If we can show that all the effects may be explained by a single body, or what amounts to the same thing, a spherical cloud of meteors so dense as to be opaque, there seems to be no reason for assuming a cloud of another form. All that can be claimed for any theory is that it explains all the facts. If then the computed variations of light agree with the observations within the limits of errors of observation, that is all that can be asked, and the theory should be accepted as the most probable explanation until some new fact is discovered which it will not explain, or some new theory which agrees equally well with observation and appears to be less improbable. The diminution in light might be caused by the interposition of a body which was self-luminous, instead of dark. We should then have a close double-star, one component of which passed in front of the other. If the orbit was circular, we should have two minima during each revolution, and at these times the star would appear of unequal brightness, unless the intrinsic brightness of the two bodies was the same. When the darker body passed in front of the brighter, the light would be less than when the brighter passed in front of the darker. If the orbit was elliptical there might be only one minimum. In the case of Algol more than half the light is cut off at the minimum; consequently one body must be darker than the other. As no second minimum has ever been observed, it is probable that the eclipsing body is not self-luminous.

We must now show that neither of the other theories named above will explain the variations of Algol and of other stars of the fifth class. The regularity of the variation disposes of the theory of a volcanic eruption, a collision, or a system of sun-spots. These effects also could scarcely be repeated so frequently without exhausting the source of energy from which they were derived. The theory that the variation is due to the revolution of the star appears more probable, and the regularity and shortness of the period add weight to it. On the other hand, it is difficult to account by this theory for the sudden changes in the light. If the light was reduced by a dark portion of the star being turned towards the observer, the minimum should last until, by the

revolution of the star, this part had been turned around so as to disappear on the other side. The short minimum observed could only be caused, according to this theory, by supposing a large dark star with a small bright spot near its polar regions, and that the pole was directed at such an angle from the observer that a large part of the spot would disappear for a short time during each revolution. Even then we have still the apparently insurmountable difficulty that the bright spot would change its apparent size and the angle at which it emitted its light to the observer, and therefore vary in brightness during the whole period of revolution. No such variation has been established in the light of Algol.

Before showing how far the theory of an eclipsing body will account for the observed phenomena, we must see what knowledge we have of these variations in light.

Only five stars are at present known to belong to the Algol class of variables. These are β *Persei*, S *Cancri*, λ *Tauri*, δ *Libræ*, and U *Coronæ*. Of these, the first is the only one whose variations are known with sufficient precision to justify a discussion in the present article. The variations of β *Persei*, or Algol, have been carefully studied by three observers, Argelander, Schmidt, and Schönfeld. Argelander's observations extend from 1840 to 1866, and are nearly two thousand in number. He compared Algol from time to time with the adjacent stars of nearly equal brightness, and noted the apparent difference in steps or grades (*Stufen*). Arranging his comparison stars in the order of brightness, and determining the number of grades between each from all his measures, he was then able to denote them all in grades. Thus, suppose at a given time he observed that Algol was slightly, if at all, brighter than star A, or that the difference was one grade; again, that it was perceptibly fainter than B, or differed from it by two grades; if then he found in his final discussion that $A = 12.5$ grades and $B = 14.9$, the first observation would make Algol 13.5 grades, and the second 12.9. These comparisons are all given in the *Bonn Observations*, vii. 315, but, unfortunately, they have not been reduced, so that at present no use can be made of them. I undertook their reduction, but was informed that this had been done at Bonn. No answer has, however, been received to letters of inquiry on this point.

The observations of Dr. Schmidt extend from 1846 to the present time, and the results up to 1875 are published in the *Astronomische Nachrichten*, lxxxvii. 193. His object was only to determine the time of the minima, and accordingly these only are given, without the comparisons. He also generally used a single comparison star, which

has the advantage of eliminating an error in estimating its brightness, but does not give a good determination of the light curve. Dr. Schönfeld observed Algol, according to the method of Argelander, from 1859 to the present time, and has given the results up to 1870, in the *Sechsunddreissigster Jahresbericht des Mannheimer Vereins für Naturkunde*, p. 70. He has not published his comparisons, but has given his resulting light curves, which will be made the basis of the following discussion. We must first reduce his grades to absolute measures, which is done in Table VII. The successive columns give the name of the comparison stars, the light in grades adopted by Schönfeld, the logarithm of the light as measured by Seidel (*Resultate photometrischer Messungen*, München, 1862), the logarithms of the light as measured by Wolff (*Photometrische Beobachtungen an Fixsternen*, Leipzig, 1877), after subtracting 0.232 to eliminate the constant difference between his measures and those of Seidel. The next column gives the difference between the measures of Seidel and Wolff, and shows that on the average they differ only .040, or a tenth of a magnitude. If L denotes the logarithm of the light, and g the corresponding number of grades, we may assume $L = a + g\,b$. This is only equivalent to admitting Fechner's law or assuming that Schönfeld's grades correspond to equal ratios of light. A solution by least squares gives $a = 8.446$ and $b = 0.025$, or $L = 8.446 + 0.025\,g$. The sixth column gives the value of L computed by this formula for the various values of g assumed by Schönfeld. The last two columns give the errors of Seidel and Wolff, assuming the estimates of Schönfeld to be exact. The average value of these differences is but little more than a tenth of a magnitude. Apparently, ι *Aurigæ* was estimated by Schönfeld about three tenths of a magnitude too bright, and δ *Persei* about two tenths too faint. Omitting these stars the errors would be reduced about one half.

TABLE VII. — COMPARISON STARS FOR β PERSEI.

Name.	Grades.	S	W — 0.232	W — 0.232 — S	Comp.	S — C	W — C
γ Androm.	23.4	9.038	9.021	—.017	9.031	+.007	—.010
ι Aurigæ	17.3	8.697	8.803	+.106	8.878	—.181	—.075
β Arietis	16.7	8.897	8.862	—.035	8.864	+.033	—.002
ϵ Persei	12.8	8.800	8.746	—.054	8.766	+.034	—.020
γ Persei	10.9	8.699	8.691	—.008	8.718	—.019	—.027
β Trianguli	9.1	8.716	8.716	.000	8.674	+.042	+.042
δ Persei	7.8	8.741	8.604	—.047	8.641	+.100	+.053
α Trianguli	3.5	8.531	8.588	+.057	8.534	—.003	+.054
ν Persei	0.9	—	8.435	—	8.468	—	—.033
				±.040		±.052	±.045

Schönfeld has given on page 84 of his memoir a table of his mean results, arranged in seventy-nine groups, seventy-two of them occurring within about four hours and a half of the minimum, and sixty-two within three hours of the minimum. He then drew an empirical curve through these points, and gives their residuals, which vary from +0.73 to —0.58 grades, and have an average value of 0.17 grades. Reducing this to logarithms, by multiplying by 0.025, gives 0.004, or only one hundredth of a magnitude. There are thirty-five changes of sign in the residuals, out of a possible seventy-one. There is, therefore, no reason to doubt that the curve represents the observations as nearly as possible.

The light in grades for intervals of every half-hour before and after the minimum is given in Table VIII. The successive columns give the time in hours, the corresponding light in grades before and after the minimum, the difference between these two, their mean, and the corresponding light expressed in logarithms. This is found by subtracting the light in grades from 20.8, which is assumed by Schönfeld as the full brightness of Algol, multiplying the result by 0.025 to reduce to logarithms, and taking the arithmetical complement. The number corresponding to this logarithm is given in the last column. It gives the light of Algol, its maximum light being assumed as 1.000.

TABLE VIII.—LIGHT CURVE OF β PERSEI.

Hours.	Dec.	Inc.	D—I	$\frac{D+I}{2}$	Log L	Light.
0.0	5.56	5.56	0.00	5.56	9.619	0.416
0.5	6.26	6.20	+0.06	6.23	9.636	0.433
1.0	8.48	7.60	+0.88	8.04	9.681	0.480
1.5	12.05	9.81	+2.24	10.93	9.753	0.566
2.0	15.28	13.17	+2.11	14.22	9.836	0.685
2.5	17.35	15.78	+1.57	16.06	9.882	0.762
3.0	18.68	17.71	+0.97	18.20	9.935	0.861
3.5	19.59	19.19	+0.40	19.39	9.964	0.920
4.0	20.24	20.23	+0.01	20.24	9.986	0.968
4.5	20.70	20.75	—0.05	20.72	9.998	0.995
4.6	20.8	20.8	0.00	20.80	0.000	1.000

From this table it appears that the law respecting the increase of light is not the same as that of its diminution. At a given interval of time from the minimum, the light is greater when decreasing than when increasing. The mean value will first be considered, and the cause of this difference then discussed.

We shall first assume that the star and satellite present circular

disks, one uniformly bright, the other dark, and that the form of orbit is circular. Three cases may occur, corresponding to a total, an annular, and a partial eclipse of the star. In the first case, all the light would be cut off for a longer or shorter time; in the second, the minimum light would be maintained during the transit of the satellite across the face of the star; and in the third case the light would diminish until the minimum was attained and then immediately begin to increase. Algol appears to belong to the last of the classes. We must next determine the relative diameters of the satellite and star. A minimum diameter of the satellite may be computed from the minimum light. To reduce the light to 0.416, or to cut off 0.584 of the light, the diameter of the satellite must be at least $\sqrt{0.584} = 0.764$ times that of the star. In this case it would just pass completely on to the disk before it began to pass off. No maximum can be determined in this way, so that the diameter is only limited between 0.764 and infinity. A change in diameter will, however, produce a change in the law of variation of the light. We may deduce the diameter from the values agreeing most nearly with observation. We must now determine the amount of light remaining when the star is partially eclipsed by a satellite of radius r. The radius of the star is taken as the unit. The area of the segment of a circle of radius unity whose versed sine is z, is equal to $\text{versin}^{-1} z - (1 - z) \sqrt{2z - z^2}$. A table is given in the eighth edition of the *Encyclopædia Britannica*, xiv. 525, Art. *Mensuration*, which gives this quantity for values of z varying by hundreds from 0.00 to 1.00. The portion of the disk cut off will always be composed of two segments having the radii 1 and r, and having a common chord which may be computed when we know the distance of the centres. The area of each may be taken from the table, multiplied by the square of the radius of its circle, and the two areas added. This will give the required diminution in light.

If now we assume r the radius of the satellite, several of the elements may be computed.

The period of revolution of the satellite is given with much precision from the observations of the minima. It appears to undergo slight changes, but may be assumed for the present time to equal 2 days 20 hours 48.9 minutes. Calling w the longitude of the satellite in its orbit reckoned from its minimum, the mean change in w per hour will equal $5^\circ.023$. Since the beginning and ending of the obscuration precede and follow the minimum by $4^h\ 35^m$, the corresponding values of w will be $337^\circ.0$ and $23^\circ.0$. At these points the centre of the satellite will be at a distance $(1 + r)$ from the centre

of the star, or the disks will touch each other. They correspond to the first and last contacts of an eclipse. The orbit is projected into an ellipse whose major axis, a, equals the true distance of the centres, and whose minor axis, b, equals the distance at the time of greatest obscuration. When $r = 0.746$, $b = 1 - 0.746 = 0.254$. For other values of r, b must be determined from a computation of the area eclipsed, by successive approximations, until such a value is found as will reduce the light to 0.416. If x and y are the co-ordinates of the point in the orbit reached by the satellite at the time of first contact, by the properties of the ellipse $x = a \sin w$, and $y = b \cos w$. The square of the distance of the centres, or D^2, may be written

$$\begin{aligned} D^2 &= (1 + r)^2 = (x^2 + y^2) = a^2 \sin^2 w + b^2 \cos^2 w \\ &= a^2 - (a^2 - b^2) \cos^2 w. \end{aligned}$$

Since $w = 23°.0$,

$$(1 + r)^2 = 0.153\ a^2 + 0.847\ b^2.$$

Substituting the proper values of r and b, a may be deduced. The cosine of the inclination, i, of the orbit will equal $\frac{b}{a}$. The three lines of Table IX. give the values of a, b, and i computed by these formulas for the minimum value of $r = 0.764$, for $r = 1.000$, and for $r = 2.000$. There is no maximum value of r, which may be indefinitely large. Let R be any large value of r, and let $a = R + A$, $b = R + B$, and $D = R + d$; substituting these values in the formula, $D^2 = a^2 \sin^2 w + b^2 \cos^2 w$, the terms containing R^2 cancel each other, and we have $2\,R\,d = 2\,RA \sin^2 w + 2\,RB \cos^2 w$, omitting the terms not containing R, since when R is very large they may be neglected. Dividing both sides by $2\,R$ gives $d = A \sin^2 w + B \cos^2 w$. When $w = 23°$, d must equal 1, and when $w = 0°$, B will equal -0.132, since the arc of the large circle becomes sensibly a straight line, and the segment whose versed sine is $1.000 - 0.132$ has an area of 0.416, or the minimum area of the uneclipsed portion. From these values, we may deduce $A = 7.300$. The two axes, therefore, become $R - 0.132$ and $R + 7.300$. The inclination in this case continually diminishes as R increases, and would equal zero if R became infinite.

The residuals which will be deduced below at first led to the belief that the phenomenon might be that of an annular eclipse. This case has therefore been included to show the change effected in the variation of the light, although the residuals are not materially reduced. If the eclipse is annular, the value of r must be 0.764.

The value of b cannot be determined directly, but must be deduced from the times of internal and external contact. The interval between the internal contacts is assumed to be 24 minutes, or that during which the satellite moves through 2° of longitude. In the equation $D^2 = a^2 \sin^2 w + b^2 \cos^2 w$, we have for $w = 1°$,

$$D = (1 - r) = 0.236,$$

and as before for $w = 23°$,

$$D = 1 + r = 1.764.$$

From these conditions the values of a and b given in the last column of Table IX. are deduced.

TABLE IX. — ELEMENTS OF ORBITS.

Elements.	$r = 0.764$	$r = 1.000$	$r = 2.000$	$r = R$	Ann.
Minor semi-axis, b	0.236	0.666	1.788	$R - 0.182$	0.223
Major semi-axis, a	4.480	4.872	6.427	$R + 7.300$	4.482
Inclination, i	87°.0	82°.1	73°.9	Small.	87°.1

We must next compute the amount of obscuration at the end of each half-hour, for the various values of r. The distance between the centres is first computed by the equation $D^2 = a^2 - (a^2 - b^2) \cos^2 w$, substituting successively, w = 2°.5, 5°.0, 7°.5, 10.°0, 12°.6, 15°.1, 17°.6, and 20°.1. The first part of Table X. gives the values of D corresponding to those assigned to r at the head of each column. The triangles formed by the centres of the two bodies and one end of the segment now become known, since their three sides equal 1, r, and D. Calling the angle at the centre of the luminous body α, we have $r^2 = 1^2 + D^2 - 2\, D \cos \alpha$. From this we deduce $\cos \alpha$ and versin α, or the height of the segment bounded by the circle having a radius unity. The height of the other segment will equal $R - D + \cos \alpha$, from which the areas of the segment, and consequently of the uneclipsed portion, may be deduced. This area is given in the second portion of the table. For comparison the observed light is repeated in the last column from the last column of Table VIII. The residuals, or the observed values minus those computed with each value of r, are given in the third part of Table X. The residuals are all zero when the time equals 0.0 or 4.6, and are therefore omitted. The average residuals are given in the last line.

TABLE X.—Distances of Centres.

Hours.	0.764	1.00	2.00	$R-D$	0.764
0.0	0.236	0.666	1.783	—0.132	0.223
0.5	0.307	0.700	1.806	—0.116	0.298
1.0	0.468	0.794	1.865	—0.072	0.457
1.5	0.629	0.917	1.957	—0.005	0.625
2.0	0.811	1.072	2.061	+0.092	0.807
2.5	0.988	1.235	2.223	+0.211	0.986
3.0	1.191	1.425	2.404	+0.375	1.190
3.5	1.373	1.606	2.583	+0.548	1.372
4.0	1.556	1.789	2.773	+0.748	1.555
4.6	1.764	2.000	3.000	+1.000	1.764

Light of Uneclipsed Portion.

Hours.	0.764	1.00	2.00	R	0.764	Obs.
0.0	0.416	0.416	0.416	0.416	0.416	0.416
0.5	0.434	[illegible]	0.432	0.427	0.430	[illegible]
1.0	[illegible]	0.491	[illegible]	0.454	0.497	[illegible]
1.5	0.579	0.562	0.527	0.497	[illegible]	0.566
2.0	0.668	0.648	0.608	0.559	0.667	0.685
2.5	0.751	0.731	0.686	0.633	0.750	0.762
3.0	0.838	0.822	0.765	0.738	0.838	0.861
3.5	0.907	0.898	0.874	0.831	0.907	0.920
4.0	0.968	[illegible]	0.949	0.927	0.968	0.968
4.6	1.000	1.000	1.000	1.000	1.000	1.000

Residuals.

Hours.	0.764	1.00	2.00	R	0.764
0.5	— .001	— .008	+ .001	+ .008	+ .008
1.0	— .020	— .011	+ .011	+ .026	— .017
1.5	— .018	+ .004	+ .039	+ .069	— .012
2.0	+ .017	+ .037	+ .062	+ .126	+ .018
2.5	+ .011	+ .031	+ .076	+ .129	+ .012
3.0	+ .028	+ .039	+ .076	+ .128	+ .023
3.5	+ .018	+ .022	+ .046	+ .089	+ .018
4.0	.000	+ .009	+ .019	+ .041	.000
	± .012	± .020	± .044	± .077	± .012

The residuals are all expressed in terms of the full light of the star. They therefore represent a larger error expressed in logarithms, or stellar magnitudes, when the star is faint than when it is bright. If reduced to logarithms their mean values become .008, .012, .027, .049, .008. Dividing these quantities by 0.4 to reduce them to magnitudes,

we see that while a large value of r would give an average residual of over one tenth of a magnitude, the value of $r = 0.764$ would make this quantity less than two hundredths of a magnitude. In all of them, however, there is a distinct systematic variation, the computed light being too small when t is large, and sometimes becoming too large when t is small. It appeared that this error might be reduced by assuming that the eclipse was annular, or that the light retained its minimum value for a short time. The corresponding residuals are given in the last column. They reduce the positive residuals when the star is faint, but do not sensibly affect the others, although the time between the internal contacts is assumed to be twenty-four minutes. The observations scarcely admit so great an interval, and certainly would not justify its increase. As the average residual is not diminished by the assumption of an annular eclipse, and as the observations do not indicate that the light remains constant during the minimum, we cannot do better than to assume the value of $r = 0.764$, and adopt the values of the second column of the table.

Several explanations may be offered of the small systematic error that remains. The most plausible seems to be that derived from the residuals given in the last column of Table VII. They show that, from a comparison of the estimated grades of Schönfeld with the measures of Wolff, that Schönfeld estimated the light too faint when the star was faint, and too bright when the star was bright. In other words, that a grade did not have the same values when expressed in logarithms for a faint as for a bright star. Assuming the photometric measures of Wolff to be free from systematic error, we should therefore increase the estimates of Schönfeld when the star was faint, and diminish them when it was bright, without affecting the actual maximum and minimum values. Such a correction would make the systematic error noted above disappear, or even give it an opposite sign. This view receives a slight confirmation from the measures of Seidel, but the accidental discrepancies far exceed this small systematic error. We may therefore conclude that the computed light agrees with observation as closely as the brightness of the fundamental stars is at present known, and there is no evidence of a real systematic difference between the two.

Another explanation of the residuals in Table X. has suggested itself. The presence of lines in stellar spectra leads to the belief that the stars, like our Sun, are surrounded by an absorbing atmosphere. They also, therefore, probably resemble it in presenting a disk brighter in the centre than at the edges, owing to the greater thickness of the

atmosphere and consequent greater absorption at the edges. The effect of such an absorption is best determined by the consideration that if, owing to absorption, the average light of the eclipsed portion is less than that of the whole disk, the effect of the atmosphere will be to diminish the proportion of the light cut off; in the opposite case, it will increase it. Now when a small portion only of the star is eclipsed, evidently the average light of this portion, since it lies near the edge, must be less than that of the whole. The atmosphere, although then diminishing the light of the remaining portion, will not reduce it as much as it does that of the entire disk; the relative light will therefore be increased. On the other hand, when a large part of the eclipsed portion is from the central and brightest portion the opposite effect will be produced. We should therefore expect, when t is large, that the computed light should be increased. When t is small, it may be diminished. In the case of the Sun the effect is so slight, except close to the borders, that the previous explanation seems more probable.

We return now to the consideration of differences in the rate of diminution and increase of the light. The observations ought to give this quantity with much accuracy. An error in estimating the light of the standard stars will not sensibly affect it, since the same stars are used in measuring the increase and dimiuution. The effect of atmospheric absorption is reduced, since some of the comparison stars are always above and others below the variable, and besides, although, when observed before passing the meridian, the star is brighter when increasing than when diminishing, yet the opposite effect is produced when the star is west of the meridian. Nevertheless this difference is doubted by many astronomers, and if it exists it is evident that an important correction should be applied to the observed minima of Algol. If the curve found by Schönfeld is correct, an error of ten minutes in the time of the minimum might be caused by comparing with a star like *ε Persei*, having a brightness of about twelve grades, and taking the mean of the times when the two stars appeared equal.

Three explanations may be offered for this phenomena. First, that the satellite is not spherical, but egg-shaped, and that the large end is turned forwards; or that the satellite is of unequal density, and that the heaviest portion is forward. In this case the centre of gravity of the disk would follow that of the satellite, or for a given distance of the centres the interposed area would be greater when the satellite was passing off, than when coming on. So great a deviation from the spherical shape would be needed to produce the observed difference

that this theory does not seem very probable. We should also, in this case, assume that the time of revolution was exactly equal to that of rotation of the satellite. A second explanation would assume that one portion of the disk of Algol was darker than the rest, so that when the satellite entered the disk it would cut off the dark portion, or affect the light less than when passing off and obscuring the brighter parts. In this case we must assume that Algol does not rotate, or it would show a variation independent of the eclipse by its satellite. Its axis of rotation might be parallel to the path of the satellite and the variations in light on its surface be distributed in zones, but such a theory seems improbable. The third explanation is that the orbit of the satellite is elliptical, and that the difference is due to the varying velocity of the satellite.

An analytical solution of this problem may be found by reducing the observed light to distances of centres, either by interpolation from the values computed above, or by successive approximations. The case then becomes that of a binary star, in which we have given the period and a number of distances, but no position angles. It is of course impossible to deduce the position angles of the peri-astron or other point of the orbit, but its other dimensions may be determined. The solution of this problem will be undertaken at another time should the accumulation of observations of Algol and other similar stars render it desirable. For the present, it will be sufficient to obtain an approximate solution. The nature of the variation is not so simple as would appear at first sight; since the observed time of increase equals that of diminution, we must assume that the apparent motion, when compared with that in a circular orbit, is less at the beginning and end, and greater in the middle of its path. The satellite must therefore either pass its peri-astron during the eclipse, or it must be approaching this point, so that the increased obliquity of its path to the line of sight will produce the apparent diminution in its motion. An ellipse was constructed, having an eccentricity of 0.5 and divided into thirty-two parts, corresponding to the position of the satellite at the end of each thirty-second of its time of revolution. The eccentric anomaly was derived from the mean anomaly by the tables of Dr. Doberck, *Astronomische Nachrichten*, cxii. 275.

As the time of eclipse is very nearly one eighth of that of revolution, four of these divisions correspond to the passage of the satellite over the star. Laying this ellipse on a sheet of rectangular paper and turning it around its focus, the effect of a change in the position of the peri-astron could be determined. The problem is greatly sim-

plified by the fact that the apparent path of the satellite during the eclipse is nearly rectilinear. It was found that, if the longitude of the line of nodes was made equal to 17°, the periods of ingress and egress would be nearly equal. The peri-astron then happens to coincide with the point of egress. The variation in light due to this orbit is compared with observation in Table XI. The successive columns give the time, the observed light in grades, the logarithm of this light, and its value compared with the full light of the star. The next column gives the light already found in the second column of the second part of Table X., and which may be called *A*. The next column gives the variation in light for the elliptical orbit assumed above, which will be denoted as *B*. The second part of the table gives the residuals found by subtracting these values of the light from those observed. The last columns give the residuals found by subtracting the logarithms of these quantities. Although the residuals even of *A* are not very large, they are systematic, being positive when the light diminishes, and negative when it increases. The residuals *B* are much smaller than those of *A* during ingress, but they are larger during egress. In other words, while the systematic error of ingress has been nearly eliminated, a nearly equal error has been introduced during egress. Accordingly the average residual is not diminished. We have so far adopted the times of first and last contact given by Schönfeld. An inspection of the table from which he derived his curve shows that the weight he assigns to his observations when more than three hours from the minimum is small, and that consequently the times of contact must be somewhat uncertain. The exact time of minimum must also be uncertain, although to a less degree than that of the two points just mentioned. An approximate solution by least squares was therefore made, with the times of contact and of minimum as unknown quantities. One half weight was given to the equations of condition formed from the observed terms of contact. From this a correction to the observed minimum was found of 5 minutes, or the true minimum appears to occur nearly one tenth of an hour later than that given by the curve. The time of first contact should also be diminished by about 2 minutes, and the time of last contact increased by about 13 minutes. The columns *C* give the values of the light and of the residuals corresponding to this orbit. The third place of decimals is not always exact, as this would have involved a great increase in the labor of computation and the accuracy attained appears to be all that is at present justified by the observations.

The residuals thus obtained are quite satisfactory as regards their

magnitudes and the number of changes of sign, but the orbit is open to a criticism of a wholly different kind. Its semi-axis major is only 3.55, and as the eccentricity is 0.500, the distance of the centres at peri-astron is 1.775. Now as the radius of the star is 1.000 and of the satellite .764, it is evident that, although they would not actually

TABLE XI. — COMPARISON OF ORBITS.

Hours.	Grades.	Log *L*	*L*	*A*	*B*	*C*	*D*
4.6	20.8	1.000	1.000	1.000	0.999	1.000	1.000
4.0	20.24	0.986	0.968	0.968	0.982	0.986	0.987
3.5	19.59	0.970	0.933	0.907	0.945	0.949	0.987
3.0	18.68	0.947	0.885	0.838	0.883	0.890	0.866
2.5	17.35	0.914	0.820	0.751	0.809	0.815	0.788
2.0	15.28	0.862	0.728	0.668	0.714	0.725	0.697
1.5	12.05	0.781	0.604	0.579	0.618	0.626	0.601
1.0	8.48	0.692	0.492	0.500	0.517	0.534	0.518
0.5	6.26	0.636	0.432	0.434	0.440	0.450	0.446
0.0	5.56	0.619	0.416	0.416	0.416	0.416	0.416
0.5	6.20	0.635	0.432	0.434	0.440	0.429	0.426
1.0	7.60	0.670	0.468	0.500	0.515	0.494	0.486
1.5	9.81	0.725	0.531	0.579	0.603	0.576	0.570
2.0	13.17	0.809	0.614	0.668	0.695	0.665	0.660
2.5	15.78	0 874	0.748	0.751	0.785	0.754	0.753
3.0	17.71	0.923	0.838	0.838	0.858	0.830	0.839
3.5	19.19	0.960	0.912	0.907	0.926	0.899	0.909
4.0	20.23	0.986	0.968	0.968	0.975	0.953	0.970
4.6	20.8	1.000	1.000	1.000	1.000	0.995	1.000

Hours.	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
4.6	.000	—.001	.000	.000	.000	.000	.000	.000
4.0	.000	—.014	—.018	—.019	.000	—.006	—.008	—.006
3.5	+.026	—.012	—.016	—.004	+.012	—.005	—.007	—.002
3.0	+.047	+.002	—.005	+.019	+.024	+.001	—.002	+.009
2.5	+.069	+.011	+.005	+.032	+.038	+.006	+.003	+.018
2.0	+.060	+.014	+.003	+.031	+.037	+.008	+.002	+.019
1.5	+.025	—.009	—.022	+.003	—.018	—.007	—.016	+.002
1.0	—.008	—.025	—.042	—.026	—.007	—.022	—.036	—.022
0.5	—.002	—.008	—.018	—.014	—.002	—.008	—.017	—.013
0.0	.000	.000	.000	.000	.000	.000	.000	.000
0.5	—.002	—.008	+.003	+.006	—.003	—.009	+.003	+.006
1.0	—.032	—.047	—.026	—.018	—.029	—.042	—.024	—.017
1.5	—.048	—.072	—.045	—.039	—.038	—.055	—.035	—.031
2.0	—.024	—.051	—.021	—.016	—.016	—.033	—.014	—.011
2.5	—.003	—.037	—.006	—.005	—.002	—.021	—.003	—.003
3.0	.000	—.020	+.008	—.001	.000	—.011	+ 004	—.001
3.5	+.005	—.014	+.013	+.003	+.002	—.007	+.006	+.001
4.0	.000	—.007	+.015	—.002	.000	—.003	+.007	—.001
4.6	.000	.000	—.005	.000	.000	.000	—.002	.000
	±.018	±.019	±.014	±.012	±.012	±.013	±.010	±.009

touch, yet they would come so near that the least disturbance would at once produce a catastrophe. This, therefore, gives the limiting value to the eccentricity. A computation with a smaller eccentricity gave less satisfactory residuals. The question now arises, will it not be possible to satisfy the observations by returning to the circular elements, since we have permitted a change in the times of contact and of minimum. Columns *D* give the residuals for a circular orbit with a diminution of 0.1 hour in the time of minimum, and assuming that the periods of ingress and egress are each equal to 4.45 hours instead of 4.6 hours. In other words the ingress occurs about fourteen minutes later, and the egress two minutes earlier, than was assumed by Schönfeld.

The errors which remain, even in the last orbit, are not wholly accidental; but their values are so small, and the changes of sign so frequent, that it is not safe to base important conclusions upon them. Their average value is only .012, or expressed in logarithms .009, and in magnitudes .02. Accordingly, we may compute the variation in the light of Algol, which shall not differ from observation on an average more than a fiftieth of a magnitude. If then this is not the true cause of the variation of the light, it at least satisfies it well within the errors of observation. The orbit *D* may therefore be adopted as representing the law of variation as well as it is at present known.

The stellar magnitude of Algol is about 2.0, so that by Table II., if its brightness equals that of the Sun, its diameter will equal 0″.006. The diameter of the orbit of the satellite will be about 0″.028. The motion of the bright star, if its density is the same as that of its satellite, will equal 0″.009, since its mass in this case will be to that of its satellite as 1.000 is to 0.446. It would therefore be useless to attempt to observe the motion micrometrically. For the same reason, there seems to be no means by which we can determine the position angle of the satellite, or the direction of the axes of the ellipse into which the orbit is projected. Even if future observations should render a larger value of the radius probable, the motion would be scarcely perceptible micrometrically. If $r = 2.000$, the diameter of the orbit becomes 0″.08 and the motion of Algol about 0″.07. It would be difficult to measure so small a quantity, although, as it is traversed in less than a day and a half, many sources of systematic error would be eliminated.

Below are given, in successive columns, the corresponding values of several elements of the orbits *A*, *B*, *C*, and *D*. The diameter of Algol is assumed to be 0″.006. The times are given in minutes from the

minimum adopted by Schönfeld. A negative sign denotes that the time precedes the minimum; a positive, that it follows it.

Elements.	*A.*	*B.*	*C.*	*D.*
Eccentricity	0.00	0.50	0.50	0.00
Semi-axis major	0″.0184	0″.0109	0.″0106	0″.0138
Inclination	87°.0	84°.3	84°.2	87°.1
Longitude of nodes	—	17°	17°	—
Time of first contact	—276	—279	—278	—261
" minimum	0	— 1	+ 5	+ 6
" last contact	+276	+273	+289	+278

The elliptical orbits *B* and *C* are much smaller than the others. Since the eclipse takes place near the peri-astron, the angular motion is so great that the radius vector must be reduced to maintain the same duration of eclipse.

To give a more tangible idea of the dimensions of this system a projection is given of the orbit denoted by *D* in its own plane in Fig. 1,

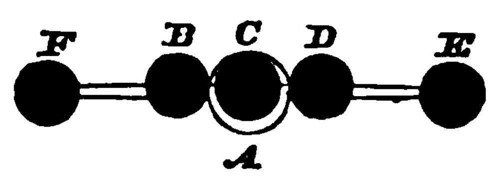

Fig. 1.

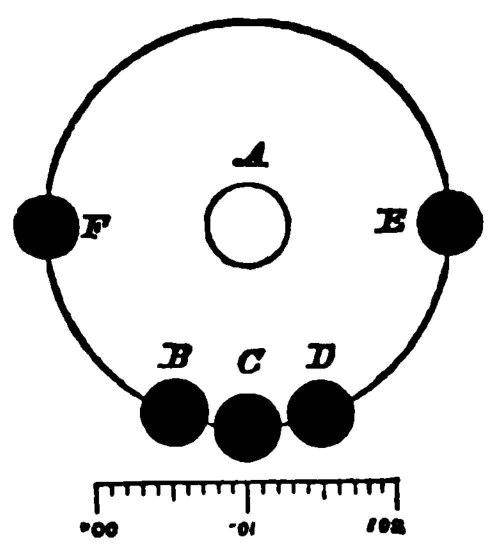

Fig. 2.

and as seen from the Earth in Fig. 2. In both projections *A* denotes the primary, *B* the satellite at first contact, *C* when half across the disk, *D* at the last contact, and *E* and *F* at its elongations. The scale is one hundredth of a second to a centimeter. Accordingly, if Fig. 2 is removed to a distance of 206 kilometers, or about 120 miles, it would appear of the same size as Algol when seen from the Earth.

The application of the spectroscope to this binary star offers a most interesting field for work. Assuming the same data as before, we find the circumference of the orbit equals $2\,\pi \times 0''.0138 = 0''.087$; or

multiplying by 0.446 and dividing by 1.446 will give a motion of Algol of 0″.027 in each revolution. This corresponds to 3″.43 annually. If the parallax of Algol is 0″.1, this would correspond to a velocity of about 160 kilometers (100 miles) per second. Substituting the values in the equation on page 9, $v = 13\ l$, we have $l = 2.6$, or the *F* line would be deviated through an interval equal to nearly half the space between the *D* lines. Moreover, as this quantity would be alternately positive and negative every thirty-five hours, the systematic errors which are so troublesome in such measures could be eliminated, and the quantity to be observed would be doubled. If the parallax of the star is more than 0″.1 the motion would be less, but on the other hand the parallax would then become a suitable object for micrometric measurement. If the parallax is much less than 0″.1 the motion would be so large that its variations might be determined with some accuracy, and the form of the orbit computed from the varying velocity along the line of sight. These measures would also determine the dimensions of the orbit, and if we assume the value of the brightness, l, they would give the distance and parallax of the star. The spectrum of Algol has already been examined without the detection in it of any peculiarity. The time selected for observation would be more likely to be near its minimum, to detect any changes in the spectrum accompanying its variation in light. But this is the very time when the motion along the line of sight is zero, which may be the reason why this phenomenon has as yet escaped detection.

Two objections have been offered to the theory that the variation in light was due to the interposition of a non-luminous satellite. First, the large size of the satellite; and, secondly, the rapidity of its motion. It has been said that, according to the prevalent theories regarding the formation of the stars, so large a body could not well have lost all its heat while the luminous star is still so bright. This argument would have some force, if we were sure of the true origin of the stars, and also if we knew that both bodies are of the same age. They may, however, have had a wholly independent origin, and have come together through their proper motion, under the influence of a resisting medium or other disturbing force.

The objection to the rapidity of motion cannot be defended in that form. By the law of gravitation we can compute what should be the velocity with a given density, and the only proper criticism would be that to produce the observed velocity an improbable density would be required. To determine this density we may use the formula of

page 6 for the equivalent diameter of the system, using as a unit the radius of the star. We thus find, $b = 0.00933\, a\, P^{-\frac{2}{3}} = 0.00933 \times 4.60\, (0.00785)^{-\frac{2}{3}} = 1.087$. Accordingly, a body having the density of the Sun, and a diameter but little more than half that of Algol, would give the observed time of revolution to the satellite. If, therefore, the velocity is remarkable, it is remarkable that it is not greater. If the satellite of Algol has a diameter of 0.764, and its density equals that of the primary, its relative mass will be 0.446. The two bodies combined would form a sphere having a radius of 1.130 and a diameter of 2.260. This is 2.08 times that of the equivalent diameter, and shows that the average density can be only 0.11 of that of the Sun, or about one seventh of that of water.

It may be noted that the density affords a means of distinguishing between a satellite and a spherical cloud of meteors. If the individual meteors were very minute, they might completely cut off the light, and yet bear a very small ratio in volume to the space between them. Accordingly, if the density of the eclipsing body could be shown to be very small, we might infer that it was composed of meteorites. In this case the motion of Algol would be insensible, as seen in the spectroscope.

The observed times of minima of Algol seem to show that its period has undergone a diminution during the last century. Such a change is easily explained on the theory of a secondary satellite. The disturbance caused by a third body, or by a resisting medium, might very sensibly vary the period from year to year. The law of this change is not yet known, but its nature is shown in Table XII. The minima are distinguished by successive numbers, E, that occurring on Jan. 1, 1800, being designated as 0. Those preceding 9000 have been arranged in groups of 500 each. Since 1870 the observations of each year are grouped together. The successive columns of the table give a current number, the mean of the numbers of the minima, the corresponding year and tenth and the number of minima included in the group. In the last nine groups, which relate to a single year, the minimum corresponding to opposition is used, instead of the mean of those observed.

The first eleven sets were observed by various astronomers; sets 12 to 18 were made by Argelander; sets 19 to 21 by Schönfeld; and sets 22 to 30, by Schönfeld and Schmidt. Sets 18 and 19 relate to the same period of 500 revolutions from 7500 to 8000. The fifth column is found by subtracting from the observed time that given by the formula of Schönfeld on page 94 of his memoir, —

Ep. E = 1867, Jan. $0^{d}\ 11^{h}\ 1^{m}.2$ M. Z. Paris + $2^{d}\ 20^{h}\ 48^{m}.9$ (E — 8534).

For the earlier observations the reduction given by Argelander (*Bonn Observations*, p. 347) are used, after reducing them to the above formula by subtracting $355^{m.}$ — 0.0749 E. The sixth column gives the ordinates of a smooth curve without points of inflection drawn through the points whose abscissas and ordinates are respectively given in the third and fifth columns. The last column gives the difference between the fifth and sixth columns.

TABLE XII. — MINIMA OF ALGOL.

No.	Mean Epoch.	Date.	No. Min.	Obs.	Curve.	O. — C.
1	— 2101	1783.5	27	— 510	— 507	— 3
2	— 1860	1785.4	17	— 488	— 489	+ 1
3	— 1308	1789.8	17	— 450	— 446	— 4
4	— 706	1794.5	6	— 400	— 401	+ 1
5	— 250	1798.0	10	— 368	— 367	— 1
6	+ 214	1801.7	2	— 308	— 332	+ 24
7	+ 734	1805.7	2	— 280	— 293	+ 13
8	+ 1831	1814.4	2	— 211	— 210	— 1
9	+ 2282	1817.9	6	— 180	— 177	— 3
10	+ 2574	1820.2	5	— 155	— 155	0
11	+ 3212	1825.2	3	— 114	— 108	— 6
12	+ 4081	1832.0	2	— 44	— 46	+ 2
13	+ 5259	1841.3	16	+ 25	+ 24	+ 1
14	+ 5741	1845.1	4	+ 37	+ 25	+ 12
15	+ 6154	1848.4	6	+ 24	+ 24	0
16	+ 6838	1853.7	16	+ 21	+ 20	+ 1
17	+ 7308	1857.4	17	+ 10	+ 15	— 5
18	+ 7688	1860.4	4	— 1	+ 11	— 12
19	+ 7799	1861.3	5	— 5	+ 9	— 14
20	+ 8374	1865.9	12	+ 2	+ 2	0
21	+ 8791	1869.0	15	— 1	— 5	+ 4
22	+ 9026	1870.9	13	+ 1	— 8	+ 9
23	+ 9153	1871.9	12	— 4	— 10	+ 6
24	+ 9281	1872.9	19	— 7	— 12	+ 5
25	+ 9408	1873.9	16	— 1	— 14	+ 13
26	+ 9535	1874.9	4	— 8	— 17	+ 9
27	+ 9662	1875.9	9	— 7	— 20	+ 13
28	+ 9789	1876.9	13	— 22	— 23	+ 1
29	+ 9916	1877.9	9	— 46	— 26	— 20
30	+ 10043	1878.9	4	— 29	— 29	0

The numbers in the last column nearly equal the accidental errors of observation. There is a slight grouping of negative signs about 1860, and of positive signs soon after 1870. This could not be avoided without giving to the curve a point of inflection. The average value of these residuals is about six minutes, which shows the accordance to be expected from any assumed formula.

Adopting the curve described above as representing the true variation, its ordinates for every ten years have been read off, and are given in the third column of Table XIII. The direction of its tangent has also been determined, and the seconds of the resulting period is entered in the fourth column. To this is to be added $2^{d.}\ 20^{h.}\ 48^{m.}$ The second column gives approximately the corresponding value of *E*.

TABLE XIII. — VARIATION OF PERIOD.

Year.	*E*.	Curve.	Period.
		m.	s.
1780	− 2545	− 541	58.6
1790	− 1273	− 444	58.6
1800	0	− 348	58.5
1810	+ 1273	− 252	58.5
1820	+ 2545	− 157	58.4
1830	+ 3818	− 64	58.3
1840	+ 5090	+ 16	56.6
1850	+ 6363	+ 24	53.7
1860	+ 7635	+ 11	53.3
1870	+ 8908	+ 7	53.0
1880	+10180	+ 31	52.7

An inspection of the curve of variation of the times of minimum shows that a curious change took place between 1830 and 1850. Before then, the period given by Wurm of $2^{d.}\ 20^{h.}\ 48^{m.}\ 58.5^{s.}$ represents the observations well; after 1850, the formula of Schönfeld appears to be more nearly correct. There seems, during this interval, to have been a change of four or five seconds in the period, and that besides this there has been a small but gradually increasing diminution in the period throughout the century.

HARVARD COLLEGE OBSERVATORY,
CAMBRIDGE, U. S.

[FROM THE AMERICAN JOURNAL OF SCIENCE, VOL. XX, OCTOBER, 1880.]

New Planetary Nebulæ; by EDWARD C. PICKERING.

MEASUREMENTS of the light of the planetary nebulæ have been made during the past year with the fifteen-inch telescope of the Harvard College Observatory. In connection with these observations the spectrum of each nebula has also been examined. A spectroscope of the usual form would be open to many objections for this work, especially as it must be frequently removed, and replaced by the photometer. Accordingly a direct-vision prism was placed between the eye-piece and objective of the telescope, thus forming a spectroscope without a slit. When a star was brought into any part of the field it was spread out into a colored line of light, the rays of each wave-length forming an image of the star in a different place. A nebula, on the other hand, being mainly monochromatic, would form a point or small disk of light, while a minute cluster would give a spectrum like that of a star. The difference in these appearances is so marked that the idea suggested itself that this device might serve to detect any minute planetary nebulæ, which could not otherwise be distinguished from stars. Accordingly a systematic search for such bodies was undertaken. A power of about 140 is employed with a field 12′ in diameter. The telescope is clamped in right ascension and moved through 5° in declination. This is repeated so frequently that the successive sweeps shall overlap, the region continually varying by the diurnal motion. Great numbers of stars pass through the field and are spread out into lines. The position of any object presenting a different appearance is at once determined by observing the declination and time, The position of bright stars are also observed to furnish corrections for the limits of the zone. Various precautions must be taken; for instance, if the spectra run north and south, the lines cannot be distinguished from points, when the telescope is moved, owing to the persistence of vision. The prism is therefore always turned so that the direction of the spectra shall be perpendicular to the line of motion. Even then the eye is constantly deceived and an object thought to be a nebula is seen to be a star when the telescope is stopped. The retina appears to be especially sensitive to rays of particular wave-lengths. The strain upon the eye and mind in examining so many objects, several a second, renders this work very fatiguing and I have found it best not to continue it for more than half an hour without an intermission. A count of the number of stars to be seen at a time in fields taken at random shows

that the spectra of over ten thousand stars are often examined in this time.

The first sweep was made on July 13, and revealed in a few minutes a bright point of light wholly unlike the lines formed by the stars. This proved to be a new planetary nebula having the position for 1880 R. A. 18^h $25{\cdot}2^m$ and Dec. $-25°$ $13'$. Its disk is so small that it can scarcely be distinguished from a star and would not probably have been detected with an ordinary eye-piece even if brought into the field of view. Measures of its light show that it is about eight magnitudes fainter than λ *Sagittarii* or of about the eleventh magnitude on the scale of Pogson. The next evening another new nebula was found somewhat fainter than this, but with a larger disk. Its position for 1880 is R. A. 18^h $4{\cdot}3^m$ and Dec. $-28°$ $12'$. This region was selected since it contains four of the fifty previously known planetary nebulæ. Sweeps on several subsequent evenings in this vicinity and elsewhere revealed nothing new.

On August 28 an object entered the field having a very singular spectrum. Two bright bands were seen near the ends of a faint continuous spectrum. The position of this object for 1880 was found to be R. A. 18^h 1^m 17^s, Dec. $-21°$ $16'$. It therefore is identical with the star Oeltzen No. 17681. Its position was observed once by Argelander and twice in the Washington zones. It must therefore have had nearly its present position and brightness over thirty years ago. It appears to be slightly fainter than Oeltzen No. 17648 which precedes it about a minute and is $4'$ north, so that even a small change in its light can be easily detected hereafter. A careful examination of the spectrum shows that the bright portions are longer than they are wide, and accordingly that they are bands and not lines. This view was confirmed by attaching a spectroscope of the usual form to the telescope. The less refrangible band extends from the wave-length 5800 to 5850, the other from 4670 to 4730. A third band was suspected at about 5400. All these measures are only approximate, and should be repeated at some observatory where spectroscopy is made a special study. A large telescope is needed, since at best the spectrum of so faint a star will not be easily measured. It will be noticed that the first of these bands is in the yellow not far from the D line, but of somewhat less wave-length. The other band is in the blue between the F and G lines. This spectrum is unlike that of any other source of light so far as is yet known. It is difficult to know in what class to place this body. From its spectrum of bright bands on a faint continuous back-ground, we might place it with the nebulæ, since most of the planetary nebulæ seem to have a faint, continuous

spectrum not due to the presence of stars in their vicinity. The material of which this object is composed must, however, be different. On the other hand, it resembles a star in other respects, showing no disk and having a much greater intrinsic brightness than other nebulæ.

The fourth new object was discovered on September 2, and consists of a very minute nebula in R. A., 18^h $14{\cdot}3^m$ and Dec. $-26°$ $53'$. This is the smallest planetary nebula known and could not be distinguished from a thirteenth magnitude star in an ordinary telescope. The difference between it and a star is, however, very marked in the prism, and had it been a magnitude fainter its peculiar character would probably have been detected.

It is estimated that the spectra of about a hundred thousand stars have so far been examined, although only about one hundredth part of the heavens has as yet been explored. A more rapid survey of the whole heavens is also being made with a comet-seeker of about four-inches aperture, to show the presence or absence of peculiarities in the spectra of the brightest stars.

Cambridge, U. S., Sept. 7, 1880.

12.

THIRTY-FIFTH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

PRESENTED TO THE VISITING COMMITTEE
DECEMBER 6, 1880,

BY

EDWARD C. PICKERING.

CAMBRIDGE:
UNIVERSITY PRESS: JOHN WILSON AND SON.
1881.

THIRTY-FIFTH

ANNUAL REPORT

OF THE

DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF HARVARD COLLEGE.

1879–80.

To the President of the University: —

Sir, — The past year has been one of unusual activity at the Observatory. The funds which were recently subscribed for the temporary support of its scientific work have, as was expected, removed all present apprehension of the necessity of restricting either of the principal instruments to merely occasional observations. In fact, while both the Equatorial and the Meridian Circle have continued in regular employment, the increase of our means has permitted many important researches to be conducted with the smaller instruments without interference with the activity of the chief telescopes. I trust that it will appear hereafter that the entire work possesses an interest, and has been pursued with an economy of time and labor, which will do credit to the liberality of those who have provided the necessary means for its continuance. Many of the subscribers have paid in one sum the entire amounts promised during five years. This furnishes the Observatory for the present with an additional fund, the income of which will increase the total amount of the subscription. Of the Annals of the Observatory, the second part of Vol. XI. and Vol. XII. have been completed and distributed during the past year. Good progress has been made with the reductions of recent observations; and many of those made in earlier times have been prepared for publication. The present report will describe successively the work done with the large Equatorial and its subsidiary instruments, with the Meridian Circle, and with the

Meridian Photometer; the work connected with the distribution of the standard time signals; the publications of the past year, the volumes now in preparation, the condition of the library, and the improvements in the buildings and grounds.

Large Equatorial. — The observations with this instrument and with others subsidiary to it may be classified by subjects, as follows: —

Satellites of Mars. — While, during the opposition of 1877, the principal observations of these satellites made here were photometric, during that of 1879 their position with respect to Mars was made the chief subject of inquiry. A longer series of observations was obtained here during the past year than was made elsewhere; and, so far as I am aware, Deimos was last seen at this Observatory. The method of reducing the light of Mars by colored glass — which was first employed here during the micrometric measures in 1877 — was again used. It was also adopted during the recent opposition at Washington, and at various foreign observatories. The observations were made almost exclusively by Mr. Wendell and myself. They principally consisted of measurements of position angle; but many measures of distance were also made after the work had been continued for some time. The number of observed position angles of Deimos was 825, of Phobos 278, and that of observed distances 245. The probable error of one setting in the three cases was 0.°6, 0.°9, and 0.″6.

Besides the micrometric work just mentioned, many photometric measures were made, the results of which have been compared with those obtained in 1877, and published in Vol. XI. of the Annals of the Observatory. These measures appear to show that, if we assume the satellites of Mars to have a capacity for reflecting sunlight equal to that of Mars, Deimos has a diameter of about six, and Phobos of about seven miles. Deimos, as is shown by experience at various places as well as here, appeared somewhat brighter in 1879 than in 1877; and in both years it seems to have been brighter measured photometrically, and to have been more readily seen when it followed than when it preceded Mars.

Eclipses of Jupiter's Satellites. — The photometric determination of the times of these eclipses, begun in the summer of 1878, has been regularly continued during the past year. The total

number of eclipses thus observed up to Nov. 1, 1880, is 119, of which 60 are of Satellite I., 26 of II., 22 of III., and 11 of IV. The results thus far obtained confirm the hope with which the work was undertaken, — that it would lead to a more precise knowledge of the times of the eclipses than had formerly been attainable. Two eclipses of Satellite I. have been compared by plotting the observations of each, and drawing smooth curves through them. The respective intervals in seconds, from the computed time of the eclipse, at which the satellite had diminished in brightness 1.5, 2.0, 2.5, 3.0, and 3.5 magnitudes, were in the first case +33, +16, +4, —6, —13; in the second case, +40, +18, +6, —2, —9. Regarding these as two sets, each consisting of five independent and equally precise observations, we find that they indicate a change of 3.′8, with a probable error of 0.′6. It, therefore, seems reasonable to hope that in the final reduction, in which proper weight will be assigned to each observation, the time of each eclipse will be determined with a probable error of less than a second. The probable error in observations of the customary kind was found by M. Glasenapp, some years ago, to be between nine and ten seconds. If further experience confirms the expectations now entertained with regard to the photometric method, the eclipses of Jupiter's satellites may again come into use as a means of determining the longitudes of remote stations. The eclipses of the three outer satellites furnish nearly as good results as those of the first, since the greater number of observations attainable compensates a considerable extent for the less rapid variation in brightness.

A new photometer, frequently used in the more recent of these observations, is mentioned under the heading "Variable Stars."

Planetary Nebulæ. — The observations of these objects described in my last report are now nearly completed. Four sets of measures of the light of each nebula are made, the work on each not being confined to one night, and being conducted by different observers, so that the results may be more trustworthy. The spectra of the nebulæ are also observed through a direct-vision prism placed between the object-glass and the eyepiece of the telescope. The planetary nebulæ retain their shape under these circumstances, which obviously indicates that their light is mainly monochromatic. The presence of a continuous spectrum, too

faint for observation by a spectroscope of the usual form, is also frequently detected by the method above described.

The difference in the appearance of monochromatic objects and of ordinary stars is so marked when thus examined as to offer a promising method of discovering some small bright nebulæ, which would appear as stars with an ordinary eyepiece. A region where planetary nebulæ were known to be comparatively frequent was accordingly examined with the prism, and three objects of the expected kind were found. Their positions for 1880 are as follows: R. A. $18^h\,4^m\,19^s$, Dec. —28° 12′; R. A. $18^h\,14^m\,23^s$, Dec. — 26° 53′; R. A. $18^h\,25^m\,10^s$, Dec. —25° 13′.

A systematic search was accordingly undertaken, and has been completed for about six hundred square degrees. A more rapid survey of the whole heavens has also been begun with the Bowditch Comet-seeker. About an eighth of the visible heavens has thus far been explored. No additional monochromatic objects have been found; but two of the stars examined probably belong to Secchi's fourth type, of which only about fifty were previously known. Their positions for 1880 are in R. A. $19^h\,55^m\,16^s$, Dec. —28° 3′; R. A. $19^h\,59^m\,35^s$, Dec. —27° 34′. R Aquarii seems to be of the same class. Its place for 1880 is in R. A. $23^h\,37^m\,37^s$, Dec. —15° 57′.

The most remarkable discovery, however, is that the spectrum of the star Oeltzen 17681 (the place of which for 1880 is in R. A. $18^h\,1^m\,17^s$, Dec. —21° 1′), possesses a peculiar character. The light of this star is principally concentrated in two points of the spectrum, one in the blue, the other in the yellow, a little more refrangible than the D line. A faint, continuous spectrum is also seen.

By sweeping the telescope over the region, the stars may be examined very rapidly. Over a hundred thousand spectra have already been viewed in this way, and it is expected that by the end of the next year the number observed will exceed half a million.

Variable Stars. — Last summer a remarkable variable star was discovered by M. Ceraski of the Moscow Observatory. This star belongs to the Algol type, of which only five others are as yet known. Its period was announced as about ten days. Dr. Schmidt showed that this time should be divided by two, and gave the period as a little less than five days. Observations made

at this Observatory showed that the period should be again divided by two, so that the true period is only two days and a half. The star has also been watched to see that no further subdivision will be made. Photometric measures have been made of the light of the comparison stars, and preparations are now in progress for determining the light curve of the variable photometrically.

The new star in Cygnus was found by Lord Lindsay to consist of monochromatic light, or apparently to have changed into a planetary nebula. This object has now diminished greatly in light, and appears to have lost this peculiarity, and to give a continuous spectrum like that of an ordinary star. Regarding it as a nebula, we seem to have, in this case at least, undoubted evidence of variability.

A new photometer has been constructed for the comparison of stars moderately near each other, but too distant to be brought into the same field by any of the photometers previously in use. The polarized images of the two stars are brought to equality by turning a Nicol prism, and both images are seen under the same conditions. The range of positions in which the double-image prism may be placed is greater in this than in previous photometers made on the same general principle. The instrument has hitherto been used, when attached to the equatorial, for the observation of eclipses of Jupiter's satellites. Its most extensive employment has been in comparisons of β with ω *Persei*, for which purpose it is provided with a small object-glass, and mounted as a separate instrument. The results of these comparisons are, in general, remarkably accordant, and seem to promise a determination of the light curve of β *Persei* with increased precision. Three observers take part in the work, each taking in turn three sets of four settings each, reversing the images in the middle of each set. The average deviation of the sets amounts only to about 0.06 magnitudes. Accordingly, while photometric observations in general give results somewhat less accordant than those obtained by the naked eye under favorable circumstances, this instrument appears to surpass the unaided eye in precision, without losing the advantage common to all good photometers of supplying results independent of each other and reducible to a definite standard.

Miscellaneous. — The places of Hartwig's comet (1880 *d*), and of Iphigenia (112), have been observed by Mr. Wendell. The

four first asteroids — Ceres, Pallas, Juno, and Vesta — have likewise been photometrically observed.

The investigation of suitable methods for the detection of stars having a considerable parallax or proper motion, which was proposed by Mr. Searle, has been carried on by him, with the aid of Mr. Wendell as in the previous year, during the few hours, generally towards the end of the night, which could be made available for the purpose; and will perhaps be continued, as opportunities may occur, with apparatus improved in accordance with the indications afforded by the observations already made.

Meridian Circle. — The present Report covers the period from September 24, 1879, to November 1, 1880. During this time observations have been made on 277 days. This number of days is distributed by months as follows: —

	Month.	No. Days Observed.		Month.	No. Days Observed.
1879.	September, . . .	7	1880.	April	27
,,	October	26	,,	May	27
,,	November, . . .	18	,,	June	28
,,	December, . . .	23	,,	July	22
1880.	January	2	,,	August . . .	21
,,	February	0	,,	September . .	26
,,	March	23	,,	October . . .	27

During the present year, work with the Meridian Circle has been for the most part limited to the continuation of the observations for the determination of the absolute co-ordinates of the selected list of 109 fundamental stars. The only miscellaneous work, has been a series of observations for the determination of the difference of longitude between this Observatory and the Winchester Observatory of Yale College.

The series of observations which was begun February 15, 1879, was completed January 3, 1880.

Before the commencement of the series for the current year, the following instrumental changes were made: —

(*a*) The Standard Clock, Frodsham, No. 1327, was removed from its exposed position in the East Computing Room to the pier formerly occupied by the Howard Clock in the Clock-room of the Time Service.

(*b*) In order to furnish the means of measuring the variation of the instrumental constants during the interval from one culmination of Polaris to the next, Messrs. Alvan Clark & Sons were employed to construct a collimator of long focus. The marble

cap-stone of the south collimator pier was removed, and four upright rods were inserted in the brick pier, by which the bed-plate carrying the Y's of the collimator-tube were firmly secured. The object-glass of the new collimator, which has a focus of 206 feet, was set in a frame which was securely fastened to the under side of the bed-plate holding the object-glass of the regular collimator. An opening through the pier and the wall of the building allows the free passage of light.

The experience of 1879 has shown that it is impossible for one observer to carry on the regular system of observations, and at the same time make, in a systematic way, the requisite number of observations for the investigation of the differential refraction by means of a large and miscellaneous list of stars. The observations of this class made during the present year were therefore limited to a few stars observed, as nearly as possible, at successive culminations.

It has been found impracticable to limit the observations to stars which have nearly the same declination as the sun, and at the same time to obtain an equal number of observations at opposite seasons of the year. Hence, during the present year, the entire working-list has been arranged in the order of right ascension. The advantage gained by making the observations differential with respect to the sun will be secured in their reduction.

Inasmuch as the report of the last year closed in September, the tabular statement of the observations for the entire series is included in the series for 1880.

The limiting dates for the different classes of observations are as follows: —

Entire Series I. extends from February 15, 1879, to January 2, 1880.
Entire Series II. extends from February 29, 1880, to November 1, 1880.
Series at Vernal Equinox extends from February 15, 1880, to April 27, 1880.
Series at Autumnal Equinox extends from August 15, 1880, to November 1, 1880.
Series at North Solstice extends from June 6, 1880, to July 6, 1880.
Series at South Solstice extends from December 7, 1880, to January 4, 1880.

	Polaris. No. Obs.			Sun. No. Obs.					Observations for Refraction.			Fundamental Stars.
	U. C.	L. C.	Total.	Vernal Eq.	Autumn Eq.	North Solstice.	South Solstice.	Total.	No. Stars.	No. Obs. U. C.	No. Obs. L. C.	No. Obs.
1879.	102	112	214	36	42	18	10	148	62	282	143	1744
1880.	80	103	183	83	42	24		131	10	181	70	1760

Of the entire number of observations of fundamental stars made during the present year, 1,010 were made previous to August 15; and 750 observations of the same stars were made between August 15 and November 1. At the conclusion of the series in January, these numbers will be nearly equal.

The number of observations between noon and midnight is nearly equal to that between midnight and the following noon.

Reductions. — The chronograph sheets for 1879–80 have been read off to August of the current year. Beyond this but little work has been done towards the reduction of the observations.

Since the completion of Vol. XII. the time of the computing force has been devoted exclusively to the reduction of the zone observations. In order to give a symmetrical character to the entire series of zone observations, it was considered better to make the instrumental constants and clock errors depend directly upon the places of the fundamental stars given in Publication XIV. of the Astronomische Gesellschaft. The labor of recomputation for the years 1871, 1872, 1874 and 1875 has not been very great.

The present state of the zone reductions is as follows: —

(*a*) The means of the right-ascension wires, of the declination wires, and of the circle-readings, have been taken in duplicate for the entire series.

(*b*) The instrumental errors in right-ascension, and the clock errors, have been recomputed from May, 1872, to January, 1879.

(*c*) The declination constants have been recomputed for the same period except for the years 1876 and 1877.

(*d*) Considerable progress has been made with the reduction of the fundamental stars between November 10, 1870, and May, 1872.

The investigation of the variability of the flexure of the telescope has been so far completed that it is safe to say that for the zenith distance 53° the value of the flexure for January of any year is about 0.″6 greater than for July of the same year. This variability was suspected as early as 1876; but it was impossible to determine at that time whether it might not be due either to the refraction constants employed, or to the systematic errors of the provisional system of fundamental stars. Beginning with September, 1878, observations for flexure have been made with

the collimators whenever the conditions for its determination have been favorable. The agreement of the values derived in this way with those derived from the new system of fundamental stars seems to indicate that the Pulkowa constants for refraction nearly satisfy observations made under the atmospheric conditions common in this latitude.

Meridian Photometer. — Much progress has been made with the undertaking announced in my last report of measuring the light of all the stars visible to the naked eye between the north pole and —30°. The photometer consists of a transit instrument in which polarized images of the star to be measured and of the pole-star are placed side by side in the field, and brought to equality by turning a Nicol prism inserted in the eye-piece. The instrument is so constructed that the two objects are viewed under precisely the same conditions, with the same magnifying power, the same aperture, the same background, and the same emergent pencil. Moreover, their positions are reversed during each observation. The observer remains in comparative darkness and confines his attention to the settings, all the readings of circles and the recording being done by an assistant. A large number of preliminary measures were made, during which various sources of error were detected. These were finally eliminated, and the first zone was taken on Oct. 25, 1879. Owing to bad weather and other causes, only twenty zones were taken before December. Since then two or three zones of about an hour each have been taken by different observers on almost every clear evening. Before Nov. 1, 1880, 298 zones were taken on 137 nights. The working-list contains about four thousand stars, each of which is to be observed on three different nights by different observers. Four settings are made each night. To determine the atmospheric absorption, a hundred circumpolar stars are observed at their upper and lower culminations. Each of these stars will be observed on about ten or twelve nights. The stars of the first and second magnitude will also be observed on about twelve nights. Except in a few of the early zones, two images of the pole-star are compared at the beginning, middle, and end of each zone, thus eliminating differences in the two objectives and prisms. The number of separate settings is as follows: 30,076 of ordinary stars, 2,996 upper culminations, 3,168

lower culminations, 1,268 of bright stars, and 2,848 of the pole-star. Total, 40,356. Besides these, many measures have been made of the brighter planets, of Vesta, and of the brighter variables and their comparison stars.

More than half of the work is now done, and it is probable that the observations will be completed during next October, unless the weather is very unfavorable. It is possible, however, that additional observations will be made of the brighter stars and of the variables, and the work thus extended through another year. In this case the total number of settings will doubtless exceed one hundred thousand. The average difference in the three measures of the pole-star taken in each zone is somewhat less than 0.08 magnitudes. The probable error in the mean of the three measures of a star is also about 0.08 magnitudes. When the three measures differ so much that the probable error exceeds 0.2, another observation is taken, and this is repeated until the error is reduced to 0.2, or until one of the observations is shown to be erroneous. Although the reduction of a single observation is very simple, yet the great number taken renders the clerical work very laborious, the manuscript already filling about seven reams of letter paper.

Time-Signals. — The distribution of time-signals from the Observatory has been efficiently maintained during the year by Mr. F. Waldo, the assistant in charge of this service. The error of the signals at 10 A.M. has very seldom exceeded one-tenth of a second, as determined by comparison with the standard sidereal clock. The clock-room, built in the cellar of the west wing of the Observatory, continues to give satisfaction. The extreme variation in its temperature during the year was 13.2 degrees Fahrenheit, and the variation from week to week is only about four degrees.

The Observatory and the Waltham Watch Factory are in telegraphic communication; and an arrangement has been made with the Watch Company which permits the use of an excellent clock at the factory for comparison with the clocks of the Observatory. This is chiefly serviceable as an additional security in the observations made with the meridian circle, but it may be of occasional use also in the distribution of time-signals in case of accident or during a long continuance of cloudy weather.

The signals sent from this Observatory are used in New York,

in connection with those of the United States Naval Observatory and of the Allegheny Observatory for the regulation of the New York time-service. This Observatory receives no payment for the assistance thus rendered.

The time-ball in Boston has been dropped at noon with great regularity and precision, owing largely to the skill and care, and especially to the experience, of Mr. Purssell of the United States Signal Service, who is in charge of the ball. On 355 days, the ball was dropped exactly at noon, and on four other days at five minutes past noon, according to the rule adopted; on four days it was not dropped, — leaving only three cases of inaccuracy of dropping. From May 1 to Nov. 1, 1880, no failure has occurred; and in only one case has the descent of the ball been postponed five minutes; on one other occasion it was dropped at noon by hand, instead of by telegraph.

The telegraph lines and batteries have continued in charge of Messrs. Stearns and George, whose care and watchfulness have largely contributed to the efficiency of the work.

Publication. — The second part of Volume XI. of the Annals of the Observatory has been completed and distributed during the past year. It contains a discussion of a part of 25,000 photometric observations made with the large equatorial in 1877, 1878, and 1879. This discussion relates chiefly to the fainter objects observed, such as the satellites of planets and minute stars situated near brighter ones. No photometric observations of most of these objects were previously in existence, and these must accordingly have a considerable value, although they cannot be expected to be as accurate as the observations of brighter objects, made with photometers of a different kind, and discussed in Part I. of the same volume. The faint objects should, if possible, be re-observed by some method wholly different from that first employed, and this work will probably be undertaken here at some future time.

Volume XII., which contains the results of observations made in 1874 and 1875 by Professor W. A. Rogers with the Meridian Circle, has also been completed and distributed. This volume includes a discussion of the proper motion of 618 stars, and a comparison of their places as found here and at other observatories. The values of the probable errors show that the work compares favorably with the best done elsewhere. The list of stars is sub-

stantially the same as that of Volume X. In that volume the times of transit over the separate wires were given in full; but in Volume XII. only the means have been published; and the expense of publication makes it probable that still further abridgment will be necessary in future volumes of the same class.

The following publications have also been made by officers of the Observatory: —

On the Present State of the Question of Standards of Length. By Wm. A. Rogers. Proc. Amer. Acad., xv. 273.

On Tolles' Interior Illuminator for Opaque Objects. By Wm. A. Rogers. Journ. Roy. Micros. Soc., iii. 754.

Observations of Comet *c*, 1879 (Swift), made with the 15-inch Equatorial of the Harvard College Observatory. O. C. Wendell. Astron. Nach., xcvi. 21, No. 2282. Observations on sixteen nights.

Observations of the Satellites of Mars made at the Harvard College Observatory. Edward C. Pickering and O. C. Wendell. Astron. Nach., xcvii. 115, 145. Nos. 2312, 2314. Contains the results of 1,348 observations.

Dimensions of the Fixed Stars with Especial Reference to Binaries and Variables of the Algol Type. Edward C. Pickering. Proc. Amer. Acad., xvi. 1. pp. 37. Contains a discussion of the conclusions that might be derived if all stars had equal intrinsic brightness; also determines the orbit which a dark satellite must have to produce the observed variations in light of β *Persei.*

New Planetary Nebulæ. Edward C. Pickering. Amer. Journ. of Science, xx. 303.

The following papers were read by Professor Rogers before the American Association, and will appear in their forthcoming volume of Proceedings: —

On the Progress made at the Observatory of Harvard College in the Determination of the Absolute Co-ordinates of 109 Fundamental Stars.

On a Simple and Expeditious Method of Investigating all the Division Errors of a Meridian Circle.

On the Systematic Errors of the Greenwich Right Ascensions of Southern Stars observed between 1816 and 1831.

On a Preliminary Determination of the Equation between the British Imperial Standard Yard and the Metre of the Archives.

Besides some shorter notices, the following reviews of the Annals have been published: Vol. IX., by Mr. Th. Wolff, in Vierteljahr. der Astron. Gesell., xv. 193; Vol. XI., Part I., by Mr. E. Lindemann, Vierteljahr. der Astron. Gesell., xv. 208. Vol. XI. has also been reviewed in Nature, xxi. 23; Observatory, iii. 387, 515; and the Astronomical Register, xvii. 290, xviii. 110.

A circular inquiring to what extent the series of Annals of this Observatory was complete as far as Volume XI. inclusive, was prepared last winter, and over two hundred copies of it were sent in February to the institutions and to some of the astronomers among whom our publications are distributed. At the time of the distribution of Volume XII., in July, the deficiencies reported in answer to the circular were supplied so far as the stock of the older volumes would permit; and a second copy of the circular was sent to each library from which no reply to the first copy had been received. From the reports which have thus far been made it appears that fifty-nine sets of the Annals were already complete; and this number has been increased to eighty-nine by the recent distribution. Forty-six sets were also partially completed, and notice sent of the scarcity of the volumes not supplied. From thirty-five institutions no reply to the circular has as yet been received. The volumes for which the demand relatively to the stock on hand is greatest are: II. Part I. (observations of Saturn), III. (Comet of 1858), and V. (Nebula in Orion). These can be supplied only in exceptional cases. Any duplicate copies will be thankfully received, and will be used in completing broken sets. The following statement shows that few copies remain of some other early volumes, duplicates of which would also be welcome: —

Vol. I. Part I. 242, Part II. 98; Vol. II. Part I. 39, Part II. 45; Vol. III. 43; Vol. IV. Part I. 22, Part II. 157; Vol. V. 39; Vol. VI. 191; Vol. VII. 124; Vol. VIII. 30; Vol. IX. 300; Vol. X. 115; Vol. XI. Part I. 189, Part II. 206; Vol. XII. 180.

A new edition of Vol. VIII. was published by Messrs. Ginn & Heath; but a large portion of it was destroyed by fire some months ago, and the remainder was purchased by the Observatory in order to complete sets.

A re-examination has been made of the equatorial observations

taken in 1866–76. A second reduction of the measurements of double stars is now nearly complete. The observations of nebulæ and of occultations are also nearly ready for the press. The observations of asteroids and comets have in most cases lost their value; but, as they may be much needed in special cases, it is proposed to publish an index to them, so that any which are wanted can be furnished on application to this Observatory.

A beginning has been made in the reduction of the meteorological observations from 1840 to 1880. Owing to their great bulk, it is probable that only the monthly means will be published. These, with the Meridian Circle observations, the recent photometric observations of the large telescope, the work with the Meridian Photometer, and the observations of Jupiter's satellites, will fill at least six volumes, which are in a more or less advanced state of preparation for publication.

Library. — Notwithstanding the additional shelves placed in the Library, more room is now much needed, and it will probably be necessary soon to devote the room opposite to library purposes. Some binding has been done, especially in collecting the pamphlets, but much more is required. One or two hundred volumes could be bound to advantage. The press of other work has also prevented cataloguing the later additions to the Library, and marking the books so as to define their precise location. The Library, however, is not yet so large as to render this omission a serious inconvenience.

Buildings, Grounds, &c. — Various improvements have been made in the west wing of the Observatory. The wooden pier of the west Equatorial had become somewhat unsteady; it obstructed and darkened the entry through the building, and was a source of much danger in case of fire. The entry was accordingly carried through it, over a structure of brick in the basement, where a pier suitable for a clock has been placed. Additional storage room for volumes of the Annals was attained by this alteration. Hard-pine floors were laid in the entries and in the Library. The entries were painted and the upper story repapered. A handsome carpet has also been purchased for the Computing Room in the east wing.

The lower end of the sunken road, formerly opening on Concord Avenue, has been filled up, and the appearance of this part of the grounds has been greatly improved. Another important

improvement has been made by regrading and sowing with grass-seed the portion of the grounds near Madison Street.

Last spring an unusually favorable opportunity offered itself to Professor Rogers to procure copies of the standard yard and metre, and to determine the relation between them. He accordingly made a brief visit to France and England for this purpose, and collected the material for what promises to be a valuable contribution to this subject.

It is my painful duty to announce the death of Mr. Joseph F. McCormack, who aided in the work of the Observatory from 1872 to 1880. He was a faithful and conscientious assistant, and attained great skill in his special work, that of reading the microscopes of the Meridian Circle. He died of typhoid fever on Feb. 2, 1880, at the age of twenty-five years and six months.

The principal want of the Observatory at the present time is means for the publication of its Annals. The large number of volumes (Vols. IV. Part II., VIII., IX., X., XI. Parts I. and II., and XII.) issued during the past five years has exhausted the accumulated interest of the Sturgis Fund; only one more volume can be paid for at present from the Quincy Fund; and no other funds are especially intended to defray the expenses of publication. A part of the annual subscriptions might be used for printing; but this seems unadvisable while the whole can be so usefully and economically employed in providing additional assistance for carrying on and reducing the work of the Equatorial and other instruments.

The cost of each volume will be about two thousand dollars, or where the two parts are published separately, about a thousand dollars for each part. Some volumes, especially the results of the zone observations and of the Meridian Photometer, are likely to be standard works of reference for many years. It is hoped that persons may be found willing to further the cause of astronomical science by contributing to this object.

EDWARD C. PICKERING.

[illegible]

VARIABLE STARS

OF

SHORT PERIOD

BY

EDWARD C. PICKERING.

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1881.

XIII.

VARIABLE STARS OF SHORT PERIOD.

BY EDWARD C. PICKERING.

Presented February 9, 1881.

IN a recent communication to this Academy * the following classification of the variable stars was proposed: —

I. Temporary stars. Examples, Tycho Brahé's star of 1572, new star in Corona, 1866.

II. Stars undergoing great variations in light in periods of several months or years. Examples, *o Ceti* and χ *Cygni*.

III. Stars undergoing slight changes according to laws as yet unknown. Examples, *a Orionis* and *a Cassiopeiæ*.

IV. Stars whose light is continually varying, but the changes are repeated with great regularity in a period not exceeding a few days. Examples, β *Lyræ* and δ *Cephei*.

V. Stars which every few days undergo for a few hours a remarkable diminution in light, this phenomenon recurring with great regularity. Examples, β *Persei* and *S Cancri*.

A discussion was given, in the article referred to, of the stars of the last class. It was shown that in the case of β *Persei* at least, the observed variations could be very satisfactorily explained by the theory that the reduction in light was caused by a dark eclipsing satellite. The dimensions of this satellite and of its orbit were then computed. The variations of the stars of the fourth class will be considered in the present paper. Both of these papers must be regarded as preliminary, rather than final, discussions. Observations are now in progress at the Harvard College Observatory which greatly increase the precision of our knowledge of many of the constants involved. When these are completed, a revision of the whole investigation is much to be desired. To avoid all prejudice, the present papers are made to

* Proc. Amer. Acad., xvi. 1.

depend entirely on the work of previous observers. Approximate methods are depended upon throughout, where a rigorous computation would have been employed, if the results were to be regarded as final. In Table I. is given a list of all the known variable stars whose periods are less than three months. The successive columns give a current number, the number in Schönfeld's Second Catalogue,* the name of the star, and the class to which it belongs, when this is known with certainty. Then follow the right ascension and declination for 1880, the period in days, and the magnitudes at maximum and minimum. The data for the southern stars which are not given by Schönfeld are taken from the Uranometria Argentina. The last columns give the name of the discoverer and the year in which the variability was detected.

TABLE I. — VARIABLE STARS OF SHORT PERIODS.

Number.	Schön.	Name.	Cl.	R. A. 1880.	Dec. 1880.	Period.	Max.	Min.	Discoverer.	Year.
				h. m.	° ′					
1	..	— Cephei	V	0 51.7	+81 14	2.49	7	10	Ceraski	1880
2	17	*β* Persei	V	3 0.4	+40 30	2.87	2.2	3.7	Montanari	1669
3	19	*λ* Tauri	V	3 54.0	+12 9	3.95	3.4	4.2	Baxendell	1848
4	32	*T* Monocerotis	IV	6 18.7	+ 7 9	27.00	6.2	7.6	Gould	1871
5	34	*S* Monocerotis	..	6 34.4	+10 0	3.40	4.9	5.4	Winnecke	1867
6	36	*ζ* Geminorum	IV	6 57.0	+20 45	10.16	3.7	4.5	Schmidt	1844
7	..	*U* Monocerotis	..	7 25.1	— 9 32	46.00	6.0	7.2	Gould	1873
8	47	*S* Cancri	V	8 37.1	+19 28	9.48	8.2	9.8	Hind	1848
9	..	*N* Velorum	..	9 27.6	—56 30	4.25	3.4	4.4	Gould	1871
10	..	*l* Carinæ	..	9 42.0	—61 57	31.25	3.7	5.2	Gould	1871
11	..	*R* Muscæ	..	12 34.8	—68 45	0.89	6.6	7.4	Gould	1871
12	66	*W* Virginis	..	13 19.8	— 2 45	17.27	8.7–9.2	9.8–10.4	Schönfeld	1866
13	74	*δ* Libræ	V	14 54.6	— 8 2	2.32	4.9	6.1	Schmidt	1859
14	..	*T* Triang. Aust.	..	14 58.6	—68 15	1.00	7.0	7.4	Gould	1871
15	..	*R* Triang. Aust.	..	15 9.1	—66 3	3.40	6.6	7.5	Gould	1871
16	75	*U* Coronæ	V	15 13.3	+32 5	3.45	7.6	8.8	Winnecke	1869
17	96	*u* Herculis	..	17 12.9	+33 14	38.50	4.6	5.4	Schmidt	1869
18	98	*X* Sagittarii	..	17 40.0	—27 47	7.01	4	6	Schmidt	1866
19	99	*W* Sagittarii	..	17 57.4	—29 35	7.59	5	6.5	Schmidt	1866
20	..	— Sagittarii	..	18 9.8	—34 9	2.42	6.2	7.4	Gould	1871
21	103	*U* Sagittarii	..	18 24.8	—19 13	6.75	7.0	8.3	Schmidt	1866
22	105	*R* Scuti	..	18 41.1	— 5 50	71.10	4.7–5.7	6.0–8.5	Pigott	1795
23	..	*κ* Pavonis	..	18 44.6	—67 23	9.10	4.0	5.5	Gould	1872
24	106	*β* Lyræ	IV	18 45.6	+33 13	12.91	3.4	4.5	Goodricke	1784
25	107	*R* Lyræ	..	18 51.7	+43 47	46.00	4.3	4.6	Baxendell	1856
26	108	*S* Coron. Aust.	..	18 53.1	—37 7	6.20	9.8	11.5?	Schmidt	1866
27	109	*R* Coron. Aust.	..	18 53.8	—37 7	54.00	10.5–11.5	<12.5	Schmidt	1866
28	116	*S* Vulpeculæ	..	19 43.5	+26 59	67.50	8.4–8.9	9.0–9.5	Rogerson	1837
29	118	*η* Aquilæ	IV	19 46.4	+ 0 42	7.18	3.5	4.7	Pigott	1784
30	122	*R* Sagittæ	..	20 8.6	+16 22	70.42	8.5–8.7	9.8–10.4	Baxendell	1859
31	137	*δ* Cephei	IV	22 24.7	+57 48	5.37	3.7	4.9	Goodricke	1784

* Zweiter Catalog von veränderlichen Sternen. Mannheim, 1875.

From this table it appears that only six stars of the fifth class are as yet known. Although the published observations of some of the others are insufficient to determine the nature of their variations, it is probable that most of them belong to the fourth class. The first star on the list, which is DM. 81°.25, has been designated as *T Cephei*,* but the use of this name has created much confusion. In 1863,† Argelander announced the variability of the star DM. 55°.2943, and this star is called *T Cephei* in Chambers' Astronomy, p. 586. In 1879, Ceraski ‡ announced that DM. 67°.1291 was variable. When correcting its position § he called it *T Cephei*. This correction is quoted in the "Astronomical Register," xviii. 322, under the heading "W. Ceraski's new Variable," apparently confounding it with Ceraski's last discovery, DM. 81°.25.

The most natural explanation of the variation of a star of short period is that it is due to its rotation around its axis.

In the Annals of the Harvard College Observatory, xi. 264, the variation in light of Iapetus, the outer satellite of Saturn, is discussed on this hypothesis. It is there shown that if the axis of revolution is perpendicular to the line of sight, the variation of light, L, may be approximately represented by the formula, $L = a + b \sin v + c \cos v + d \sin 2v + e \cos 2v$; a here denotes the mean light, v the angle of rotation, b and c are constants depending on the comparative brilliancy of the two hemispheres, each of which is supposed to be of uniform intensity, but one brighter than the other; d and e depend on a supposed deviation of the body from the form of a solid of revolution. This equation may also be written in the form $L = a + m \sin (v + \alpha) + n \sin (2v + \beta)$, in which α depends upon the angular position of the plane separating the two hemispheres from the line of sight at the epoch from which the variation in light is reckoned; β in like manner depends upon the positions in which the body subtends its largest and smallest discs. Our problem then is to see how far this equation will represent the variation in light of all the stars of the fourth class.

Both of these proposed causes of variation may be criticised as improbable; but what could be more improbable than the phenomenon itself, were it not verified by observation? With our present knowl-

* Science Observer, iii. 30, 38, 48. English Mechanic and World of Science, xxxii. 297. Astron. Nach. xcix. 87.

† Astron. Nach., lxi. 281.

‡ Astron. Nach., xciv. 175.

§ Astron. Nach., xcviii. 239.

edge of the constitution of the stars it would seem extremely unlikely that certain of them would lose half their light at regular intervals of from one to twelve days. If it can be shown that the hypothesis satisfies the observed facts, it seems unreasonable to deny it until some more probable explanation can be offered. The difference in brightness of the two sides of a star may be due to spots like those of our sun, to large dark patches, or to a difference in temperature. In the latter case, observations of the distribution in light in the spectrum at the maxima and minima might show a greater variation in the blue than in the red portions. If the body had the form of an oblate ellipsoid rotating around one of its longer axes, its condition of equilibrium would be unstable. If, however, it was a prolate ellipsoid it would be in stable equilibrium, and if sufficiently rigid might revolve in this way indefinitely. If, like our sun, it was in a fluid condition, we might anticipate a return to the form of a solid of revolution. Jacobi has however shown * that a fluid ellipsoid having three unequal axes may be in equilibrium when revolving around its shortest axis. An analogous case is found in Plateau's experiment, where a globule of oil suspended in alcohol and water is made to revolve. With a sufficient velocity the globule, if slightly eccentric, elongates before throwing off a satellite. We may also assume the existence of two nuclei, or that the two components of a binary star are so close together that both are enveloped in the incandescent gas or photosphere.

Another equation of condition would thus be furnished which might serve to determine the absolute diameter of the star in miles. Thus the observations discussed below give the relative dimensions of two of the axes, and the condition that the body shall be in equilibrium will determine the relative length of the axis of revolution. If the star was an ellipsoid of revolution we could compute the flattening at the poles from the diameter and the time of revolution; we could also compute the diameter if the other two constants were given. Although the problem is more complex, evidently the same principle may be applied to an ellipsoid with three unequal axes.

Four of these stars, ζ *Geminorum*, β *Lyræ*, η *Aquilæ*, and δ *Cephei*, have been observed with great care, so that their variations are known with much precision. Each will therefore be discussed in turn, according to the following method. As the variation is periodic, it will be convenient to denote the time by an angle, v, such that 360° shall correspond to one period or revolution of the star. We now wish the light

* Poggendorff's Annalen, xxxiii. 229; see also Journ. Frank. Inst. cx. 217.

corresponding to $v = 0°, 15°, 30°, 45°$, etc. The period is divided into twenty-four equal parts, and the number of grades corresponding to each is taken from the light curves. Argelànder and Schönfeld give the number of grades for each hour, and the number of grades was found from their tables by interpolation. Oudemans represents his results graphically, and the grades were taken from his curves by inspection. These curves were used rather than the original observations, in order to reduce the accidental errors. The results are free from prejudice, since they were drawn by the observers themselves without regard to any theory. They are, however, open to the objection that small systematic errors may be present which are not easily detected.

We must next pass from grades to the actual intensities of light. For this purpose we cannot rely on an assumed value of a grade, since we have no certainty that this will be the same for lights of different intensity. Accordingly, the comparison stars used by each observer are compared with the measures of Wolff.* Points were constructed whose abscissæ equal the assumed light of each comparison star in grades, and their ordinates, the logarithms of the light as measured by Wolff. A smooth curve was then drawn through these points, and served to convert the grades into logarithms. The readings are only made to hundredths, since one unit in this place corresponds to one-fortieth of a magnitude, and all the curves are uncertain by much more than this amount. The largest logarithm is then subtracted from all the others, and the number corresponding to the difference gives the intensity of the light. This is multiplied by one hundred, so that the results are given in percentages.

In the equation, $L = a + m \sin (v + \alpha) + n \sin (2v + \beta)$, a is found from the mean of all the observed values of L. The other constants might be found from a solution by least squares, forming twenty-four equations of condition with the twenty-four deduced values of L. Sufficient accuracy is, however, obtained by approximate graphical methods. By adding together the values of L, corresponding to values of v differing 180°, we eliminate the term $m \sin (v + \alpha)$. Their differences, in like manner, eliminate $n \sin (2v + \beta)$. Each then may thus be found independently of the others.

ζ Geminorum. In 1848, Argelander gave a light curve of this star.†

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* Photometrische Beobachtungen an Fixsternen, Leipzig, 1877.

† Astron. Nach., xxviii. 33.

Table II. gives the data for converting the grades into light ratios by means of the comparison stars. The successive columns give the name of the star, its light in logarithms as measured by Wolff, its light in grades assumed by Argelander, the corresponding ordinate of the curve, and the second column minus the fourth, or the assumed errors.

TABLE II. — COMPARISON STARS FOR ζ GEMINORUM.

Name.	Wolff.	Grades.	Curve.	$W - C$
ξ Geminorum	8.81	9.9	8.81	.00
δ Geminorum	8.73	8.8	8.74	—.01
λ Geminorum	8.68	8.0	8.70	—.02
ι Geminorum	8.66	6.0	8.64	+.02
υ Geminorum	8.58	8.3	8.58	.00
ν Geminorum	8.55	2.0	8.55	.00

The curve here agrees very well with the measurements, but its inclination is much greater for the brighter than for the fainter stars. In other words, a grade represents a much larger difference in magnitude when the star is bright than when faint. The change is, however, slight between the limits within which the curve is used.

Table III. gives a comparison of the light curve with theory. The successive columns give the angle v, the corresponding time from the minimum, and the observed light in grades, in logarithms, and in percentages. The next column gives the light computed by the formula, $L = 89.6 + 10.0 \sin v$; this is followed by the residuals found by subtracting the computed from the observed brightness. As they show a perceptible systematic error, two more columns are given corresponding to the formula $L = 89.6 + 10.2 \sin (v - 11°.3)$. This gives an entirely satisfactory agreement with observation, the average deviation amounting to less than one per cent. It cannot be reduced directly to magnitudes, since, when the light equals 100, one per cent equals .011 magnitudes, when 80, .014 magnitudes, and when 50, .022 magnitudes. The average deviation is accordingly only about one hundredth of a magnitude. Since two smooth curves are compared, the small irregular variations in the residuals are principally due to the neglected thousandths in the logarithm of the light. They are, however, probably far less than the real errors of the curves. The mean of the residuals is given in the last line of the table.

TABLE III. — VARIATION IN LIGHT OF ζ GEMINORUM.

v.	Time.		Gr.	Log.	Obs.	Comp.	$O-C$	Comp.	$O-C$
°	d.	h.							
0	0	0.0	3.4	8.58	79	80	—1	79	0
15	0	10.2	3.6	8.58	79	80	—1	80	—1
30	0	20.3	4.1	8.59	81	81	0	82	—1
45	1	6.5	4.7	8.61	85	82	+3	84	+1
60	1	16.6	5.2	8.62	87	85	+2	86	+1
75	2	2.8	5.7	8.63	89	87	+2	89	0
90	2	12.9	6.1	8.64	91	90	+1	92	—1
105	2	23.0	6.5	8.65	93	92	+1	94	—1
120	3	9.2	6.9	8.66	96	95	+1	96	0
135	3	19.4	7.2	8.67	98	97	+1	98	0
150	4	5.5	7.3	8.67	98	98	0	100	—2
165	4	15.7	7.4	8.68	100	99	+1	100	0
180	5	1.8	7.4	8.68	100	100	0	100	0
195	5	12.0	7.3	8.67	98	99	—1	99	—1
210	5	22.2	7.1	8.67	98	98	0	98	0
225	6	8.3	6.8	8.66	96	97	—1	96	0
240	6	18.5	6.4	8.65	93	95	—2	93	0
255	7	4.6	6.0	8.64	91	92	—1	90	+1
270	7	14.8	5.5	8.63	89	90	—1	88	+1
285	8	0.9	5.0	8.61	[illegible]	87	—2	85	0
300	8	11.1	4.5	8.60	83	85	—2	83	0
315	8	21.3	4.0	8.59	81	82	—1	81	0
330	9	7.4	3.7	8.59	81	81	0	80	+1
345	9	17.5	3.5	8.58	79	80	—1	79	0
					Mean . .		±1.1	..	±0.5

There seems to be no evidence of the term $n \sin (2v + \beta)$; in other words, the star appears to be a surface of revolution, one side being about four-fifths of the brightness of the other. It is also possible that the star may be elongated with axes in the ratio of four to five, but of equal brightness on all sides, and that its time of revolution is 20.32 days, or double the period commonly given. In this case there may be a slight difference in brightness at the alternate maxima or minima which has hitherto escaped detection, because not anticipated. From the second formula we may infer that the true maximum and minimum precede that adopted by Argelander, by the angular amount of 11.3°, or 7.6 hours. As this would only affect the light curve by about a fiftieth of a magnitude, it might readily escape detection. It will be noticed that in this case the interval from maximum to minimum is equal to that from minimum to maximum, instead of, as is generally the case, exceeding it. A more direct determination of the correction to the minimum may be found from the light curve, by comparing the times at which the light is equal.

In Table IV. are given the light in grades, the corresponding times

taken from the light curves, and the time of the minimum, assuming that it lies midway between them. This last column is found by adding to the second column $10^d\ 3.7^h$, or the period of the star; adding to this the third column, and dividing the result by two; finally subtracting the quotient from the period, $10^d\ 3.7^h$. The mean of this value, or 7.4, agrees closely with that given above.

TABLE IV. — MINIMUM OF ζ GEMINORUM.

Gr.	Increasing.	Decreasing.	Mean.
	d. h.	d. h.	h.
4.0	0 18.5	8 22.0	—5.6
5.0	1 12.0	8 0.5	—7.6
6.0	2 9.5	7 4.5	—6.9
7.0	3 13.5	6 2.0	—9.6

β Lyræ. Light curves of this star were given by Argelander in 1842, Astron. Nach., xix. 397, and in 1844, De stella β Lyræ variabili disquisitio. In 1859 he gave a more complete discussion of the problem.* He divided his previous observations into three periods, and derived a curve from each; concluding that they differed from each other only by their accidental errors, he gave a curve representing the entire series.

Oudemans † gives a light curve from his observations, reduced to the same system as that given in Argelander's second publication. This differs so little from the last system of Argelander that the same curve, for reduction to logarithms, has been used for both. In no case, within the limits used, would the difference of the logarithms exceed one or two hundredths.

In 1870, Schönfeld gave another curve, Astron. Nach., lxxv. 1. His grades represent a smaller variation in the light than Argelander's, and, like the latter, a grade is larger for the brighter stars than for the fainter, as in the case of ζ *Geminorum.* The relation of the grades to the logarithms of the light is shown in Table V. The columns have the same meaning as in Table II., except that three additional columns are given for the comparisons of Schönfeld.

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* De stella β Lyræ variabili commentatio altera.

† Zweijaehrige Beobachtungen der meisten jetzt bekannten veraenderlichen Sterne. Verhand. Akad. Amsterdam, 1856.

TABLE V.—COMPARISON STARS FOR β LYRÆ.

Name.	Wolff.	Argelander.			Schönfeld.		
		Grades.	Curve.	$W - C$	Grades.	Curve.	$W - C$
γ Lyræ . .	8.89	12.7	8.87	+.02	15.0	8.88	+.01
μ Herculis .	8.79	..	..	..	13.0	8.80	—.01
ξ Herculis .	8.70	10.8	8.75	—.05	10.8	8.71	—.01
o Herculis .	8.69	7.6	8.69	.00	7.8	8.65	+.04
ϵ Lyræ . .	[8.77]	4.9	8.60	[+.17]	4.2	8.58	[+.19]
ζ Lyræ . .	8.56	3.4	8.56	.00	2.9	8.56	.00
κ Lyræ . .	8.56	2.6	8.55	+.01	1.6	8.55	+.01

The estimate of ϵ *Lyræ* has not been included in drawing these curves. As the observations were made with the naked eye, in some cases aided by an opera-glass, ϵ and 5 *Lyræ* were treated as a single star. Wolff gives the logarithms of their light as 8.50 and 8.45. Their combined light would therefore equal 8.77, or nearly half a magnitude brighter than would be inferred from the estimate in grades, using the curves derived from the other stars. This may also be expressed by the statement that, together, they appear only a quarter of a magnitude brighter than either would alone, while a star of their combined brightness should appear about three quarters of a magnitude brighter than the separate components. It is possible that their proximity affected their measures by Wolff, but this seems less probable since they would be readily separated by the telescope of a Zöllner photometer. Evidently, ϵ *Lyræ* should not be used hereafter as a comparison star for this variable.

Table VI., like Table III., serves to compare the observations with theory. The first column gives the angle; the second, the corresponding time. Three sets of three columns each give the light in grades, in logarithms, and in percentages, for Argelander, Oudemans, and Schönfeld. Although the observations are not of equal value, it would be difficult to decide what weight should be given to each, and especially, to decide how large are the systematic errors to which each is subject. This last quantity should determine the weight, since the accidental errors are in a great measure eliminated by the smoothness of the light curves. Their mean, which is given in the next column, will accordingly be employed. The excess of the curve of each observer over the mean is given in the next three columns. An examination of the mean curve shows that it has two equal maxima symmetrically situated on each side of the point where $v = 180°$. The curve must therefore have the form $L = a + m \sin (v - 90°) + n \sin (2v - 90°)$.

The mean value of L or a is 81.1. When $v = 0°$, $L = a - m - n$; when $v = 180°$, $L = a + m - n$; $v = 90°$ or $270°$, gives $L = a + m - n$. Were there no accidental errors, either two of these three equations would determine m and n. After various trials the equation $L = 81.1 + 4.1 \sin (v - 90°) + 20.0 \sin (2v - 90°)$ was found to give the most satisfactory results. The brightness computed by this formula, and the residuals found by subtracting them from the mean of the observed values, are given in the last two columns.

TABLE VI. — VARIATION IN LIGHT OF β LYRÆ.

v.	Time.	[redacted]			Oudemans.			Schonfeld.			Mean.	$A - M$	$O - M$	$S - M$	Comp.	$O - C$
		Gr	Log.	Obs.	Gr.	Log.	Obs.	Gr.	Log.	Obs.						
°	d. h.															
0	0 0.0	3.4	[redacted]	49	4.0	8.67	49	3.6	8.57	51	60	−1	−1	+1	57	−7
15	0 12.9	5.0	8.60	56	5.0	8.60	52	4.4	8.58	58	55	+1	−3	+3	60	−5
30	1 1.8	9.2	8.71	72	8.0	8.67	62	9.2	8.71	71	68	+4	−6	+3	68	0
45	1 14.7	11.1	8.78	85	10.6	8.76	76	11.3	8.79	85	82	+3	−6	+3	78	+4
60	2 3.6	11.8	8.82	93	11.8	8.82	87	12.2	8.84	96	92	+1	−5	+4	89	+3
75	2 16.6	12.2	8.84	98	12.5	8.86	95	12.6	8.87	102	99	−1	−3	+3	97	+2
90	3 5.5	12.3	8.85	100	12.6	8.87	98	12.7	8.87	102	100	0	−2	+2	101	−1
105	3 18.4	12.1	8.84	98	12.4	[redacted]	93	12.5	8.86	100	97	+1	−4	+3	100	−3
120	4 7.3	11.8	8.83	93	11.7	8.81	85	11.9	8.82	91	90	+3	−5	+1	93	−3
135	4 20.2	11.2	8.79	87	10.7	8.76	76	10.9	8.77	81	81	+6	−5	0	84	−3
150	5 9.1	10.3	8.75	79	10.0	8.73	71	9.9	8.73	74	75	+4	−4	1	75	0
165	5 22.0	8.8	8.69	69	9.2	8.71	68	9.1	8.70	69	69	0	−1	0	68	+1
[redacted]	6 10.9	8.6	8.69	69	9.1	8.70	66	8.9	8.70	69	68	+1	−2	+1	65	+3
195	6 23.8	9.4	8.71	72	9.5	8.72	69	9.3	8.71	71	71	+1	−2	0	68	+3
210	7 12.7	10.8	8.77	83	10.7	8.76	76	10.2	8.74	76	78	+5	−2	−2	75	+3
225	8 1.6	11.6	8.81	91	11.5	8.82	87	11.2	8.79	85	88	+3	−1	−3	84	+4
240	8 14.5	12.1	8.83	96	12.4	8.85	98	12.6	8.88	98	94	+2	−1	−1	93	+1
255	9 3.5	12.3	8.85	100	12.8	8.88	100	12.4	8.85	98	99	+1	+1	−1	100	−1
270	9 16.4	12.4	8.85	100	12.9	8.88	100	12.4	8.85	98	99	+1	+1	−1	101	2
285	10 5.3	12.2	8.84	98	12.8	8.88	100	12.3	8.85	98	99	1	+1	−1	97	+2
300	10 18.2	11.7	8.81	91	12.4	8.85	93	11.9	8.82	91	92	−1	+1	1	89	+3
315	11 7.1	10.9	8.77	83	11.4	8.80	83	10.8	8.77	81	82	+1	+1	−1	78	+4
330	11 20.0	8.4	8.68	68	9.0	8.70	66	8.0	8.67	65	66	+2	0	−1	68	2
345	12 8.9	4.0	8.58	54	4.7	8.59	[redacted]	4.2	8.58	53	58	+1	−2	0	60	−7
										Mean	. .	±1.9	±2.5	±1.5	..	±2.8

These residuals are much larger than in the case of *ζ Geminorum*; but this is to be expected, since the variations in light are greater. Evidently, if the changes were small, any two smooth curves would agree closely. Their average value amounts to about .04 of a magnitude, and their greatest value does not exceed the greatest difference of each of the observed curves from the others. The greatest errors of observation are those of the light of the comparison stars. The residuals near the principal minimum may be greatly reduced if the fainter comparison stars are assumed too faint, with a corresponding change in the value of the fainter grades. The rejection of *ε Lyræ* in Table V., while its effect on Table VI. cannot be eliminated, may account for this apparent error. An increase in the logarithm of the

light by about .01 for grades 11 to 12 would reduce the residuals corresponding to $v = 45°$, $60°$, $210°$, $225°$, $300°$, and $315°$. The residuals for $v = 120°$ and $135°$ would, however, be increased. These changes are not to be recommended unless indicated by future photometric measures of the comparison stars.

η Aquilæ. Argelander gave a light curve of this star in 1842, Astron. Nach., xix. 399, based upon 174 observations taken by himself and by Heis. In 1857, he gave a second curve dependent on 411 of his own observations, Astron. Nach., xlv. 97. In Table VII. the light of the comparison stars are given in grades and in logarithms, according to Wolff. The columns have the same meaning as in Table V. As *ν Aquilæ* was not observed by M. Wolff, it is unavoidably excluded from the comparison.

TABLE VII. — COMPARISON STARS FOR η AQUILÆ.

Name.	Wolff.	Grades.	Curve.	$W-C$	Grades.	Curve.	$W-C$
δ Aquilæ . .	8.85	13.3	8.85	.00	12.9	8.86	+.01
β Aquilæ . .	8.74	8.0	8.72	+.02	8.1	8.71	—.03
ε Aquilæ . .	8.63	6.0	8.66	—.03	6.1	8.64	+.01
ι Aquilæ . .	8.57	8.0	8.57	.00	8.0	8.55	+.02
μ Aquilæ . .	8.43	—1.4	8.43	.00	—0.6	8.44	—.01
ν Aquilæ . .	...	—2.4	...	..	—1.8	...	..

The first seven columns of Table VIII. have the same meaning as in Table III. and give the values of v, of the time, of the light in grades, in logarithms, and in percentages, a computed value, and the residuals from this, or the fifth column minus the sixth. The computation is effected by the formula $L = 73.6 + 20.0 \sin (v - 60°) + 6.0 \sin (2v - 120°)$. The last four columns give the light in grades, logarithms, and percentages, and the residuals according to the second curve of Argelander. The same theoretical formula is used in this case, adding 1 so that the positive and negative residuals shall be nearly equal. The light in this case, $L' = L + 1 = 74.6 + 20.0 \sin (v - 60°) + 6.0 \sin (2v - 120°)$.

δ Cephei. Argelander has given a light curve of this star in the Astron. Nach., xix. 395. Curves are also given by Oudemans in the paper cited above, and by Schönfeld in Astron. Nach., lxxv. 14. The relation of grades to logarithms is given in Table IX., for Argelander and Schönfeld. Oudemans has already reduced his scale to that of Argelander. Unfortunately, Wolff only measured those of the five

TABLE VIII. — VARIATION IN LIGHT OF η AQUILÆ.

v.	Time.		Gr.	Log.	Obs.	Comp.	$O-C$	Gr.	Log.	Obs.	$O-C$
°	d.	h.									
0	0	0.0	1.2	8.51	50	51	—1	2.1	8.52	54	+2
15	0	7.2	1.7	8.53	52	54	—2	2.4	8.53	55	0
30	0	14.4	3.2	8.57	58	58	0	3.5	8.56	59	0
45	0	21.5	4.8	8.63	66	65	+1	4.8	8.60	65	—1
60	1	4.7	6.3	8.67	72	74	—2	6.2	8.65	72	—3
75	1	11.9	8.1	8.72	81	82	—1	7.8	8.70	81	—2
90	1	19.1	9.6	8.76	89	89	0	9.5	8.75	91	+1
105	2	2.3	10.9	8.79	96	94	+2	10.6	8.78	98	+3
120	2	9.4	11.4	8.81	100	96	+4	10.9	8.79	100	+3
135	2	16.6	10.9	8.79	96	96	0	10.5	8.78	98	+1
150	2	23.8	10.1	8.77	91	94	—3	9.8	8.76	93	—2
165	3	7.0	9.2	8.75	87	90	—3	9.0	8.74	89	—2
180	3	14.2	8.5	8.74	85	86	—1	8.2	8.71	83	—4
195	3	21.3	8.0	8.72	81	82	—1	8.0	8.70	81	—2
210	4	4.5	7.6	8.71	79	78	+1	7.8	8.70	81	+2
225	4	11.7	7.2	8.70	78	76	+2	7.2	8.68	78	+1
240	4	18.9	6.8	8.69	76	74	+2	6.6	8.66	74	—1
255	5	2.0	6.4	8.67	72	71	+1	6.0	8.64	71	—1
270	5	9.2	5.8	8.65	69	69	0	5.4	8.62	68	—2
285	5	16.4	4.7	8.62	65	66	—1	4.9	8.61	66	—1
300	5	23.8	3.7	8.59	60	62	—2	4.2	8.58	62	—1
315	6	6.7	2.9	8.56	56	57	—1	3.6	8.57	60	+2
330	6	13.9	2.1	8.54	54	54	0	3.0	8.55	58	+3
345	6	21.1	1.5	8.52	51	51	0	2.4	8.53	55	+3
					Mean . .		±1.3				±1.8

comparison stars of δ *Cephei*, and the logarithms of two of these he found differed by only one hundredth of a unit. Accordingly, we can do no better than to draw a straight line nearly through the points designated by these stars, or assume that the value of a grade is constant. The columns of Table IX. have the same meaning as the corresponding columns of the previous similar tables.

TABLE IX. — COMPARISON STARS FOR δ CEPHEI.

Name.	Wolff.	Grades.	Curve.	$W-C$	Grades.	Curve.	$W-C$
ζ Cephei . .	8.84	11.4	8.84	.00	12.4	8.86	—.02
ι Cephei . .	8.83	10.8	8.82	+.01	10.9	8.82	+.01
7 Lacertæ .	...	7.1	...	..	6.6	.	..
ξ Cephei . .	...	3.0	...	..	...	..	..
ϵ Cephei . .	8.53	2.0	8.53	.00	1.9	8.53	00

Table X. compares the various light curves with theory. The columns have the same meaning as those of Table V. The theoretical values are computed by the formula, $L = 72.1 + 20.0 \sin (v - 45°) + 7.0 \sin (2v - 120°)$

TABLE X. — VARIATION IN LIGHT OF δ CEPHEI.

v.	Time.	Argelander.			Oudemans.			Schönfeld.			Mean.	$A-M$	$O-M$	$S-M$	Comp.	$M-C$
		Gr.	Log.	Obs.	Gr.	Log.	Obs.	Gr.	Log.	Obs.						
°	d. h.															
0	0 0.0	2.8	8.56	55	3.4	8.58	55	3.0	8.56	58	56	−1	−1	+2	52	+4
[illegible]	0 5.4	3.0	8.56	55	3.2	8.57	54	3.1	8.57	59	56	−1	2	+3	55	+1
30	0 10.7	3.5	8.57	56	3.3	8.57	54	3.5	8.58	60	57	−1	−3	+3	61	4
45	0 16.1	4.7	8.62	63	5.3	8.64	68	4.6	8.61	65	64	1	−1	+1	69	−5
60	0 21.5	6.5	8.68	72	7.5	8.71	74	6.6	8.68	76	74	−2	0	+2	77	−3
75	1 2.8	8.4	8.74	83	10.3	8.81	93	8.7	8.75	89	88	−5	+5	+1	86	+2
90	1 8.2	9.9	8.79	93	11.1	8.83	96	10.0	8.79	98	96	−3	+2	+2	92	+4
105	1 13.6	10.7	8.82	100	11.2	8.84	100	10.4	8.80	100	100	0	0	0	96	+4
120	1 19.0	10.1	8.80	96	10.9	8.83	98	10.0	8.79	98	97	1	+1	+1	98	−1
135	2 0.3	9.0	8.76	87	10.4	8.81	93	9.3	8.76	91	90	3	+3	+1	96	−6
150	2 5.7	8.5	8.75	85	9.6	8.78	87	8.6	8.74	87	86	−1	+1	+1	91	−5
165	2 11.1	8.4	8.74	83	8.6	8.75	81	8.0	8.72	83	82	+1	−1	+1	86	−4
180	2 16.4	8.3	8.74	83	7.7	8.72	76	7.8	8.72	83	81	+2	−5	+2	80	+1
195	2 21.8	7.8	8.72	79	6.8	8.69	71	7.6	8.71	81	77	+2	−6	+4	75	+2
210	3 3.2	7.1	8.70	76	6.2	8.67	68	6.8	8.69	78	74	+2	6	+4	71	+3
225	3 8.5	6.3	8.67	71	5.6	8.65	65	6.2	8.67	74	70	+1	−5	+4	69	+1
240	3 13.9	5.6	8.65	68	5.0	8.63	62	5.6	8.65	71	67	+1	−5	+4	67	0
255	3 19.3	5.2	8.64	66	4.4	8.61	59	5.1	8.63	68	64	+2	5	+4	66	−2
270	4 0.6	4.7	8.62	63	4.0	8.60	58	4.7	8.62	66	62	+1	−4	+4	64	−2
285	4 6.0	4.3	8.61	62	3.7	8.59	56	4.3	8.60	63	60	+2	−4	+3	62	−2
300	4 11.4	3.9	8.59	59	3.6	8.58	55	3.9	8.59	62	59	0	−4	+3	59	0
315	4 16.7	3.4	8.58	58	3.6	8.58	55	3.6	8.58	60	58	0	−3	+2	56	+2
330	4 22.1	3.2	8.57	56	3.6	8.58	55	3.3	8.57	59	57	1	−2	+2	53	+4
345	5 3.5	2.9	8.56	55	3.5	8.56	55	3.1	8.57	59	57	−1	−1	+3	51	+6
									Mean	. .		±1.4	±3.2	±2.3	..	±2.9

These residuals are not large, considering the differences between the different observed values. There is, however, a curious alternation of the positive and negative signs. As a similar alternation appears in some of the other residuals, it is important to compare them to see if they can be shown to follow any law. There appear to be three maxima and three minima, or the variation repeats itself at intervals of about 120°. We should then exaggerate this effect by adding each set of the three residuals differing by 120°; that is, the residuals corresponding to 0°, 120°, and 240°, to 15°, 135°, and 255°, etc. This is done in Table XI., in which the first value of v, in each set, is given in the first column, and the sums of the three residuals for the four stars are given in the second, third, seventh, and eleventh columns. The residuals of ζ *Geminorum* are so small that we should expect no evidence of systematic error. In the other three cases marked variations are shown. In each case there are only two changes of sign, while there should be on the average four if the variations were accidental. The residuals of β *Lyræ* are well represented by subtracting from the computed value 3 cos 3 v. The residuals which then remain are given in columns four, five, and six. Their average value is 1.6 instead of 2.8, or they have been reduced nearly one half. The residuals of η *Aquilæ*, in like manner, leave 1.1 instead

of 1.8, by subtracting the term 3 sin $(3v - 45°)$. Those of δ *Cephei* become 1.4 instead of 2.8, if we subtract 4 sin $3v$.

TABLE XI. — TERMS INVOLVING $3v$.

v.	ζ Gemino.	β Lyræ.				η Aquilæ.				δ Cephei.			
		Sum.	Residuals.			Sum.	Residuals.			Sum.	Residuals.		
0°	0	−9	−4	0	+4	+5	0	+1	−8	+ 3	+4	−1	0
15	0	−9	−3	−1	+1	0	0	+1	−1	− 7	+4	−3	+1
30	−2	−2	0	0	−2	−4	+2	0	0	−11	0	−1	+2
45	+1	+7	+2	−1	0	−4	+2	+1	+2	−11	−2	−1	+1
60	+1	+9	0	0	0	−8	−1	−2	+1	− 2	−3	+1	0
75	−1	+9	0	+1	+2	−2	−2	−2	+2	+ 6	−1	−1	−1
90	0	0	−1	+8	−2	+6	−1	0	+1	+11	0	−1	0
105	−1	−6	−1	+6	−5	+7	0	+2	0	+11	+1	−2	+3
Mean. .		±1.6				±1.1				±1.4			

No natural explanation can be offered for such terms, and the reduction might be thought accidental did it not occur in so many different curves. A careful distinction must be made between these terms and those which might be assumed empirically, since their form is clearly pointed out by the residuals. If we tried to represent the residuals by a function of $4v$, we should soon see that the effect was wholly different, nor would any values of the arbitrary constants in this case materially reduce the residuals.

Neglecting these last terms, as their reality may be questioned, we may write the equations of the four stars under each other thus: —

ζ Geminorum, $L = 89.6 + 10.2 \sin (v - 11.3°)$
β Lyræ, $L = 81.1 + 4.1 \sin (v - 90°) + 20.0 \sin (2v - 90°)$
η Aquilæ, $L = 74.6 + 20.0 \sin (v - 60°) + 6.0 \sin (2v - 120°)$
δ Cephei, $L = 72.1 + 20.0 \sin (v - 45°) + 7.0 \sin (2v - 120°)$

To compare them, it will be convenient to make the mean brightness equal to unity in all cases, or to divide by a the equation $L = a + m \sin (v + \alpha) + n \sin (2v + \beta)$. Instead of making $v = 0$, when the light is a minimum, it will also be better to take as the starting-point the position in which the shorter axis of the star is turned towards the observer. If $v' = v + \gamma$, we may write $L' = 1 + m' \sin (v' + \alpha') + n' \cos 2v'$. The various values of these constants are given in Table XII., which contains in successive columns the name of the star, the value of γ, of α', of m', and of n'. Independently of the form of the star, its light would vary, owing to the unequal bright-

ness of the two sides from $1 + m'$ to $1 - m'$. The brightness of the darker side would therefore equal $\frac{1 - m'}{1 + m'}$ times that of the brighter. In like manner, if the surface was uniformly bright, the variation in area of the disk, or the length of the shorter axis in terms of the longer, would be $\frac{1 - n'}{1 + n'}$. These quantities are given in the sixth and seventh columns. The last two columns give the average residuals in percentages before and after applying the terms which are functions of $3\,v$.

TABLE XII. — COMPARISON OF LIGHT CURVES.

Name.	γ	a'	m'	n'	$\frac{1-m'}{1+m'}$	$\frac{1-n'}{1+n'}$	Av. Resid.	Av. Resid.
ζ Geminorum	-11.8°	..	+.114	..	0.80	..	0.5	..
β Lyræ . . .	-90.0	0°	+.051	+.247	0.90	0.60	2.8	1.6
η Aquilæ . .	-105.0	+45	+.268	+.080	0.58	0.85	1.8	1.1
δ Cephei . .	-105.0	+60	+.277	+.097	0.57	0.82	2.8	1.4

From the column $\frac{1 - m'}{1 + m'}$ we see that in every case the darker side is more than half as bright as the other, and that the difference in the case of β *Lyræ* amounts only to ten per cent. In other words, if this effect is due to spots, we must conclude that they cover only one-tenth of the hemisphere in the case of β *Lyræ*, and about two-fifths in the cases of η *Aquilæ* and δ *Cephei*. The next column also shows that β *Lyræ* is much elongated, the ratio of its axes being as five to three, while the two stars following have this ratio about as six to five.

The dark portion of β *Lyræ* is at one of the ends, since $\alpha' = 0^{\circ}$ for this star; it appears also to be symmetrically situated as regards the longer axis. The dark portions, both of η *Aquilæ* and of δ *Cephei*, are placed somewhat preceding an end, that is, they are turned towards the observer before the end has been directed to him. For this reason the time from minimum to maximum is greater than that from maximum to minimum. This is probably a general law of stars of this class, as it has been noticed in several other instances.

One source of systematic error has been disregarded in the above comparison of observation with theory. In the value of L' the term $m' \sin (v' + \alpha')$ may be regarded as the measure of the effect of the difference in brightness of the two sides, and $n' \cos 2\,v'$ as due to the form of the body. Their combined effect, however, would not strictly equal their sum, but would be found by adding each to unity and

taking the products of these sums. The actual light would equal $(1 + m' \sin (v' + a')) (1 + n' \cos 2 v') = 1 + m' \sin (v' + a) + n' \cos 2 v' + m'n' \sin (v' + a) \cos 2 v'$. The value of L' given above is then subject to the systematic error of $m'n' \sin (v' + a) \cos 2 v'$. The maximum value of this would equal $m'n'$, and it would generally be much less. The maximum value for *β Lyræ* would be about 1 per cent; for *η Aquilæ*, 2; and for *δ Cephei*, 2.6 per cent. If the star underwent much greater change of light, it might be necessary to take this term into account; but in the present case it does not seem to sensibly affect the average value of the residuals.

Various attempts have been made to determine the light curve of *β Lyræ* photometrically. The observations of Zöllner and Wolff are reduced according to the same method in the photometric work of the latter, p. 110. The accuracy of the results does not make this a promising method of determining the light curve, unless the number of observations is greatly increased. The maxima and minima were also determined at the Harvard College Observatory.* Calling the light at either maximum 100, that at the two minima would be 55.8 and 64.7, which agrees very closely with that given by computation, if we neglect the term $3 \sin 3 v$.

One great advantage of the study of the stars by physical instruments, such as the spectroscope and photometer, is that some clew is given to certain laws, for our knowledge of which we must otherwise depend on theoretical considerations alone. While the conclusions to be drawn from micrometric measurements are in general much more precise, and the effect of the errors can be more certainly computed, they fail entirely to aid us in studying such laws as those here considered. For example, the present investigation serves to study the following important problem in cosmogony, to which micrometric measures contribute nothing, and which can otherwise only be examined from the standpoint of theory. If we admit a common origin to the stars of the Milky Way, a general coincidence in their axes of rotation seems not improbable, especially as such an approximate coincidence occurs in the members of the solar system. If the coincidence was exact, the direction must be that of the poles of the Sun, or, approximately, that of the pole of the ecliptic. On the other hand, since the stars of the Milky Way are supposed to be arranged in the general form of a flattened disk, we should more naturally expect that the axes of rotation would be symmetrically situated with regard

* Annals, xi. 185.

to it, or would coincide with its shortest dimension. According to this theory, then, the axes of rotation would be directed towards the poles of the Milky Way. If now we suppose that a great number of variable stars, of the form described above and rotating around parallel axes, were distributed over the heavens, it is evident that those seen in the direction of their axes would not appear to vary, since as they turned they would always present the same portion of their surfaces to the observer. Those at right angles to this direction would show the greatest variation, and, other things being equal, would appear to be more numerous since they would be more likely to be detected. If then the axes are coincident, we should expect that most of these variable stars would lie along the arc of a great circle whose pole would coincide with their axes of rotation. An inspection of a plot of the stars of Class IV. showed that they agreed closely with a great circle whose pole is in R. A. 13^h and Dec. $+20°$. To compare these stars in this and in other respects, they are arranged in the order of their periods in Table XIII. They are divided into three sections; first, those known to be of the fifth class; secondly, those of the fourth class, including all of a shorter period than *β Lyræ;* thirdly, the remaining variables of longer period, whose position in Class IV. may be open to question. The first column gives the name of the star, and the second its period in days. The distance from the great circle whose pole is in R.A. 13^h and Dec. $+20°$ is given in the third column. It was found by measurement on a globe, instead of by calculation, and is not therefore exact to the nearest degree.

In measuring the stars of the fifth class at the Harvard College Observatory, much difficulty was experienced from the absence of adjacent comparison stars. Stars of the fourth class, on the other hand, have, in almost all cases, stars near them. An unprejudiced comparison is made in the next two columns, by giving the magnitude and distance, in minutes, of the nearest star of the Durchmusterung. The lines for the southern stars are therefore left blank. If the stars of the fourth class lie near the Milky Way, we should expect an increased number of companions due to this cause. Accordingly, a count has been made of the Durchmusterung stars in a square degree, in which each star is contained. This area is defined as the portion of the Durchmusterung zone in which the star is situated, having an average length of one degree, one half preceding, the other half following, the variable. The results are given in the sixth column. If these stars were connected with the variables, we might expect that they would lie, approximately, in a plane at right angles to the axes of rota-

TABLE XIII. — COMPARISON OF VARIABLE STARS.

Name.	Period.	Dist.	Mag.	Dist.	No. Stars.	Ang.	Birm.
Class V.							
δ Libræ . . .	2.32	+51°	..	..	..	..	..
— Cephei . . .	2.49	+11	9.5	5'.1	20	..	..
β Persei . . .	2.87	—24	8.8	7.5	15	..	55
U Coronæ . . .	3.45	+58	9.4	11.5	7	..	..
λ Tauri . . .	3.95	—41	9.5	17.5	5	..	..
S Cancri . . .	9.48	+25	9.1	11.6	16	..	..
Mean		±35°	..	10'.6	12.6	..	..
Class IV. — Short Periods.							
R Muscæ . . .	0.89	0	..	..	..	..	..
T Triang. Austr.	1.00	+ 3	..	..	..	..	..
— Sagittarii . .	2.42	— 1	..	..	..	..	..
S Monocerotis .	3.40	— 4	9.4	1.0	29	—31	..
R Triang. Austr.	3.40	+ 1	..	..	..	..	..
N Velorum . .	4.25	+ 1	..	..	..	..	..
δ Cephei	5.37	— 5	7.5	1.5	31	—33	..
S Coron. Austr. .	6.20	— 9	..	..	..	..	..
U Sagittarii . .	6.75	+ 2	..	..	..	..	445
X Sagittarii . .	7.01	+ 8	..	..	..	..	..
η Aquilæ . . .	7.18	— 9	9.2	3.2	17	+20	..
W Sagittarii . .	7.59	+ 3	..	..	..	..	..
κ Pavonis . . .	9.10	—16	..	..	..	..	..
ζ Geminorum .	10.16	+ 5	8.5	1.8	33	+ 1	..
β Lyræ . . .	12.91	+14	8.5	1.5	34	—20	..
Mean		±5°	..	1'.7	28.8	±21°	..
Class IV. — Long Periods.							
W Virginis . .	17.27	+66	..	..	..	..	284
T Monocerotis .	27.00	— 8	9.5	6.1	23	0	..
l Carinæ . . .	31.25	— 1	..	..	..	..	..
u Herculis . .	38.50	+34	9.4	5.0	15	+79	405
U Monocerotis .	46.00	+ 2	..	..	..	..	..
R Lyræ	46.00	+16	7.1	9.7	17	—78	474
R Coron. Austr. .	54.00	— 9	..	..	..	..	..
S Vulpeculæ . .	67.50	0	9.5	5.2	20	+53	517
R Sagittæ . . .	70.42	— 9	9.3	5.0	26	+76	540
R Scuti	71.10	+ 3	..	..	..	..	462
Mean		±13°	..	6'.2	20.2	±57°	..

tion, since the planes of revolution of the planets do not differ greatly from the solar equator. Moreover, if the elongation of the variable was caused by one or more disturbing bodies, we should expect that they would lie in this plane. Of course, the present distance of these

companions is far too great to sensibly affect the variables, but other nearer objects may lie in the same plane. The approximate position angle of the companion was computed from its Durchmusterung place. The position angle of the pole of the variable stars was measured by a protractor, laid upon the globe over the position of the variable star, and stretching a thread to the pole. Each of these determinations is liable to an error of some degrees, but the results which are given in column seven are sufficiently exact for our present purposes. Some of these stars are red, and when they are contained in the Catalogue of Birmingham * their numbers are given in the last column.

The numbers of the third column show that the stars of the fifth class are not concentrated in the assumed plane. If uniformly distributed all over the heavens, their average distance should be about 30°, since one half of each hemisphere is contained in a zone of this width. In the short-period stars of the fourth class, however, the agreement is most remarkable. None have yet been found more distant than 16° from the circle, and with two exceptions none are more distant than 10°. There is only one chance in four that a given star should lie within 15° of a given great circle, and about one in six that it should lie within 9° of it. Evidently the chances would be many millions to one against the observed arrangement being accidental. As an argument in favor of the parallelisms of the axes, this distribution of the stars fails by proving too much. We should expect, if the axes were parallel, to find nearly as many stars between 10° and 20°, as between 0° and 10°, since the variation would be a function of the cosines of these angles. If the axes were not exactly coincident, we should find the stars still more widely distributed.

Of course it is possible that the distribution of these stars may be partly due to the parallelism of their axes of rotation. But we have shown that the latter cause is insufficient. Since then we must assume an arrangement of the stars approximately in a plane, we cannot be sure that their apparent distribution is not wholly due to it, and the evidence in favor of parallelism of their axes is much weakened.

It is a little singular that this plane appears to pass through the Sun. We should expect that while the more distant stars might lie upon a great circle, the nearer, and therefore presumably the brighter, stars, would lie on the opposite side of it from the Sun. As, however, the positive and negative signs are nearly equally distributed, we must

* Trans. Roy. Inst. Acad., xxvi. 249.

infer that the distance of the Sun from the plane of these stars is small compared with its distance from them. If the stars lay exactly in one plane we might infer their distances from the Sun from these residuals. As the residuals of the brighter stars show no systematic arrangement, it seems probable that the variables of the fourth class lie nearly, but not exactly, in a plane. This plane approaches that of the Milky Way, but does not coincide with it. The pole of the latter is nearly in R. A 12^h 40^m and Dec. $+28°$. Evidently the residuals in column three would be greatly increased if we moved the pole from its assumed position of R. A. 13^h and Dec. $+20°$, by more than 10° to the pole of the Milky Way. The position of the Milky Way, as given in the "Atlas Coelestis Novus" of Heis, agrees, however, more closely with the plane of the variable stars.

It is not certain whether the stars of longer period given in the third section of Table XII. should be included with those of the fourth class of variables. With two exceptions, *W Virginis* and *u Herculis*, they lie near the plane of the others.

The total number of stars in the Durchmusterung north of the equator is 315,048. Since the area of the hemisphere is 20,626 square degrees, this corresponds to 15.3 stars per degree, or an area of 236 square minutes to each star. A circle having a radius of 8′.7 would have an area equal to this. If, then, a circle having this radius is described around any star as a centre, it will be an even chance that another star will be contained within it, provided that the presence of the second star is no way affected by that of the first. For circles of other radii the chances will vary as the squares of the radii, or as the areas. We know from the existence of clusters and multiple stars that one star is not without influence on the presence of another, and that this effect may extend to some distance, as is shown in the Pleiades and in Præsepe. This principle may still be used in comparing different classes of stars, although the distance 8′.7 should be diminished. It is, therefore, surprising that the average distance of the companions of stars of the fifth class is as great as 10′.6, especially as two of them, *S Cancri* and *λ Tauri*, lie near the large clusters Praesepe and the Hyades, where the average intervals between the stars is much less. A circle of radius 10′.6 has only two-thirds the area of that of 8′.7, hence these companions are only two-thirds as thickly placed as the stars in other parts of the heavens. This effect extends to the square degree, as is shown in the sixth column. It appears to be probable that there is no physical connection of these stars with the variables, and that their sparseness is due to their distance from the Milky Way.

Passing now to the second part of the table, we find a wholly different condition of things. Every star has a companion near it at an average distance of only 1'.7, or these stars are twenty-six times as thickly placed as in the rest of the heavens, since $8.7^2 : 1.7^2 = 26 : 1$. This effect is partly due to the surrounding square degree, which contains nearly double the average number of stars. Only a small part of this effect may, however, be thus explained. We may, therefore, infer that there is a physical connection between these variables and their companions, or that they are at nearly the same distance from the Sun, and not optically double. The singular character of these stars renders them interesting objects for the measurement of parallax. This is especially the case with those of very short period, since from the rapidity of the changes we might infer that they were really small, and therefore near. Now an observer would be very likely to select the companions as points to measure from, since their distances are much greater than that separating the components of most stars which are binary, or are supposed to be physically connected. A measurable parallax might thus escape detection.

The stars of longer period occupy an intermediate position as regards the distances of the components, and the number of stars in the square degree.

If the direction of the components depended wholly on chance, we should find that they would differ, on the average, from that given by any theory, by about 45°. It therefore seems scarcely probable that, in each of the five cases, a chance distribution would give the angle less than 45°, for the stars of short period. The uncertainties in the measurements would in general increase the discrepancies, so that it is to be expected that a more accurate determination would diminish the mean value, although it would doubtless alter the separate results by many degrees. The position of the components of the stars of the fifth class has not been determined, as it seems very improbable that they have any physical connection. The stars of long period, with one exception, give results which do not agree at all with theory. Some more precise test of the class to which these variables should be assigned, is therefore much needed. They are distinguished from many of the stars of the second class only by the length of their period, no other known variables having a period less than that of *R Vulpeculæ*, or 137 days. Stars of Class II. have banded spectra, and are of a red color. This suggested a test dependent upon observations already made. The last column shows what stars have been regarded as red, and may, therefore, in some cases belong to Class II. The only

star of Class V. given in Birmingham's Catalogue is β *Persei*, and many observers may be surprised that this should have been called a red star. It is remarkable that but one star of short period, *U Sagittarii*, is called red. On the other hand, six of the variables of long period are given in the catalogue, including all of those which have shown marked discrepancies. Excluding these disposes of the large deviations, 66° and 34°, in column three; and we find no star more distant than 16° from the assumed plane in which the variables lie. Again, the large discrepancies of the last column but one are removed, and *T Monocerotis* probably placed with the variables of the fourth class. This view is confirmed by the light curve given by Schönfeld, page 32 of the catalogue cited above, which shows that in the form of its variations this star closely resembles η *Aquilæ* and δ *Cephei*. Another reason for excluding *W Virginis* and the last four stars of the list is, that their light is variable at their maxima, and in four of the five cases at their minima. This frequently happens with stars of Class II., but would not be readily explained in stars of Class IV.

The Uranometria Argentina adds *U Monocerotis* to the list of red stars. All stars whose period lies between 32 and 72 days have, therefore, been called red, except *R Coronæ Australis*. This star is so faint that its color might well have been overlooked.

A further discussion would have been made of *T Monocerotis*, but no means exist for converting into light ratios the scale of magnitudes of its light curve. As the brightness of the comparison stars are not given, we have no means of knowing whether a tenth of a magnitude corresponds to the same light ratio when this star is faint, as when it is bright. A preliminary trial showed that the maximum appeared to occur more suddenly, and the minimum more slowly, than theory would indicate. The large range of variation of this star renders it well suited for study, and the same may be said of some others of the list, as a slight increase in the difference between the maximum and minimum greatly increases the severity of the test the light curve offers to theory.

The system which appears to govern the position of the companions to these stars suggests an investigation which might lead to important results. The planes of the orbits of the binary stars are defined by their inclination and the position angle of the node. Since we cannot determine micrometrically the direction in which the orbit is inclined, we can only say that the pole of this orbit lies in one of two places. Should any law be discovered, we might then decide for any particular star what sign should be given to the inclination, and also whether the

motion was direct or retrograde. It might also help to determine the amount of the inclination when the latter is not large enough to be determined precisely by micrometric measurements. Such a law would render an important aid to the study of the orbits of the dark companions of stars of the fifth class. They would afford a check on the observed inclination, and would define the position angle of the major axis of the orbit, which is now wholly indeterminate. An inspection of the orbits of the binaries fails to show any law, but it is possible that this might be brought out by a more careful examination, as has been done with the proper motion of the stars. The conclusion regarding the motion of the Sun in space is liable to large error, in case systematic errors exist in the catalogues on which the positions of the stars depend. Such an error in Bradley might greatly change the conclusion generally accepted. The orbits of the binaries, on the other hand, are wholly independent of each other, and there is little danger of a systematic error affecting all.

The elegant method of Argelander for determining the light curve of the variable stars leaves little to be desired as a means of determining their periods and the times of their minima. Its simplicity, and the need of no instrument but a telescope powerful enough to show the variable, are strong arguments in its favor when comparing it with the best photometric methods. If, however, we wish to determine the true light curve, the following sources of error become perceptible. As the comparison stars are selected from the immediate neighborhood of the variable, they are few in number; and if any one of them proves to be itself variable, the errors introduced are large. It is difficult to obtain independent estimates, since there is but little range of choice in the star to be selected at any given time. Much skill is required on the part of the observer to make a grade the same when the variable is bright as when faint, to make it the same on different nights, and to make the interval of two grades double that of one. In reducing the light to logarithms, it appears to be impossible to render the errors of the measures of the comparison stars as small as those of the light curves. The comparisons given above show that the errors of the measurements of the comparison stars probably exceed those from all other sources combined.

Three methods may be used for determining the brightness of the stars without a photometer. First, the observer may keep a certain scale in mind, and by it estimate the light of the stars in tenths of a magnitude. He should first estimate several known stars, and compare his result with their true brightness, so as to apply mentally to his

scale proper corrections for the effect of haze, moonlight, etc. He may also observe a large number of known stars, and afterwards reduce his scale for the evening from a discussion of their light.

In the second method, which is that of Argelander, the observer selects a comparison star of very nearly the same light as the star to be measured, and estimates the difference in grades, a grade being a small interval nearly equivalent to a tenth of a magnitude. A discussion of all the observations serves to determine the intervals in grades between the comparison stars. The value of one grade is then determined from photometric measures of the comparison stars. According to the third method, the observer selects two comparison stars, one a little brighter, the other a little fainter, than the star to be observed, and estimates its difference in magnitudes from the brighter component, with the difference between the two comparison stars. The first of these methods is the most rapid, and is well adapted to zone observations, or to any work with a meridian instrument. More skill is, however, required on the part of the observer than by either of the other methods. Besides being able to judge of small intervals of brightness, as in the other methods, he must be able to prevent any changes from taking place in his scale, at least during a single evening. The second method requires less skill, since the observer must merely keep the values of his grades constant; but in the third method even this is not needed. It is, therefore, probably the most exact, when the results are to be reduced by photometric measures of the comparison stars. The three methods are directly comparable with those which may be used in estimating linear measures. We may estimate the length of a bar directly in inches, or its excess in inches over a similar bar of known length; or, thirdly, we may compare it with two bars, one a little longer, the other a little shorter, and estimate its relative length compared with them. It can hardly be doubted that the last of these methods would give the most accurate results. When applied to the stars, the third method has also an advantage in reducing the accidental errors of the photometric measures, since the comparison is made with two stars instead of one.

The light curve of a variable may then be determined as follows: — Select as comparison stars all those of nearly the brightness of the variable, and not too far distant, excepting any which may be thought to be variable, to differ from the variable in color, or which are near other stars. Photometric measures should be obtained, during the period over which the observations of the variable extend, of all of those stars which are used. Each star should be measured in turn under precisely the same conditions, by a Zöllner photometer or other

instrument, and this should be repeated on several evenings. The relative light will thus be obtained with great accuracy, as the same errors will be likely to affect them all. If this cannot be done, the Uranometria Argentina, with the measures now in progress at the Harvard College Observatory, will give the brightness of all the naked-eye stars, with an error probably less than a tenth of a magnitude.

The light of the variable would be found by selecting two comparison stars, one a little brighter, the other a little fainter than it, and comparing the interval between the variable and the brighter, with that between the two comparison stars, which may be assumed equal to 10. Thus, *a* 4 *b* will denote that the interval between the bright comparison star *a* and the variable is estimated at only four-tenths of that between the two comparison stars. Of course the time of each comparison must be recorded. This measure should be repeated with different pairs of comparison stars. Thus, if *a* and *b* are brighter and *c* and *d* fainter than the variable, we may compare the latter with *ac*, *ad*, *bc*, and *bd*. In like manner, with six comparison stars we may obtain nine independent measures. The reduction is very simple, since it is useless to carry the estimates beyond tenths of a magnitude.

The above paper has suggested several researches of importance, and which are accordingly placed together below: —

1. Determination of the light curves of any of the variables of short period, except β *Persei*, ζ *Geminorum*, β *Lyræ*, η *Aquilæ*, and δ *Cephei*, for which satisfactory curves have already been obtained. The method of Argelander, or that proposed above, may be used with advantage.

It must be remembered that the observations will have little value, unless they are reduced and the light curve found. A vast number of excellent observations of these stars already exist, including the larger part of those of Argelander, which will have no value until they are reduced.

2. Determination of the light curve of the stars of the fourth class photometrically. This may be done with great accuracy by an instrument similar to that described in the Annals of the Harvard College Observatory, xi. 4, Figs. 1 and 2. The proximity of the companions render these objects especially suitable for photometric measurement.

3. Photometric measures of the comparison stars used in (1), of those used by previous observers, and the reduction of the observations by these measures to light intensities.

4. Search for variables of the fourth class, selecting from the Durchmusterung those fulfilling the conditions named above. They may be readily identified by their companions, and observed very rapidly by a

transit instrument, or small equatorial. The first of the three methods of estimating their light is to be recommended for this work. It is sufficiently precise, and the scale used each evening would be readily found from the Durchmusterung magnitudes of the great mass of the stars which would, probably, be invariable in light. Any interesting variable would be detected by observations on a few nights.

5. Measures of the position angles, distances, and magnitudes of the companions. The approximate places given from the Durchmusterung in Table XIII. could thus be corrected, and the blanks for southern stars filled. The magnitudes could best be measured by the photometer recommended in (2). Otherwise especial care should be taken that the light of the fainter star was not affected by the proximity of the brighter.

6. Observations of the color and spectrum of these stars, to decide which ones, if any, should be included in the second class.

7. Distribution of the light in the spectra of these stars, and also of those of the second class at their maxima and minima.

8. Computation by Jacobi's method of the true diameter of a liquid ellipsoid in equilibrium, having given the period of rotation and the ellipticity of the equator.

9. Computation of the Galactic latitude and longitude (or distance and direction from the pole of the Milky Way) of variables of Classes II. and IV., of the planetary and other gaseous nebulæ, and of stars whose spectrum is of the fourth type.

10. Computation of the position of the poles of the orbits of the binary stars.

The object of the present paper is not to advocate a certain theory which may seem improbable, and, possibly to some, inadequate. It is rather intended to bring together the most important facts bearing on the study of an interesting class of objects, and to exhibit them in a form in which they may be subjected to any desired test. The hypothesis advanced has a value as affording a simple geometrical conception of the nature of the variations under consideration, even if it proves not to be the true explanation of the cause. The ingenious hypothesis of Zöllner, and other explanations of these phenomena, have not been overlooked. It seemed best, however, to leave to another to decide the comparative merits of views in which the precision of the effects must be considered as well as the probability of the causes.

One theory, that the variation is due to the absorption of a rotating mass of gas, deserves a moment's consideration. This explanation does not appear probable for stars of the fourth class, since no evidence

of absorption is in general shown in their spectra beyond the appearance of lines such as are seen in our Sun. For the stars of the second class, however, this view seems more reasonable, since many of them exhibit spectra which are strongly banded. Moreover, the great variation in light is thus explained. An excellent test of this hypothesis is afforded by the variation in light of the different portions of the spectra. For light of any given wave-length the logarithm of the transmitted ray will always vary proportionally to the thickness and density of the absorbing medium, the amount of absorbent effect for any given thickness varying with the wave-length. Accordingly, a study of the variation of each ray should show the same law. They would give very different coefficients of absorptions, those of the dark bands being large, and those of the bright zones being small. The great variation in light will render this test a severe one with even a moderate degree of accuracy in the observations. For the lack of any data, this method of study is for the present unavoidably postponed.

The principal conclusions of the above paper may be summarized as follows: —

Thirty-one variable stars are known whose period is less than 72 days. Of those six belong to the fifth class, or that of *β Persei*, in which the variation is probably due to the interposition of an opaque eclipsing satellite. Of the remainder, seven may be excluded, since they are red, and may belong to the second class, or that of *o Ceti*. Nineteen remain, whose periods vary from less than a day to 54 days, and which may be placed in the fourth class. All lie within 16° of a great circle whose pole is in R. A. 13^h, Dec. + 20°. The distances of eleven are from 0° to 5°, of five at distances 8° and 9°, one at 14°, and one at 16°. The average distance is 5°.5, while if the stars were distributed at random it should be 30°.

If the stars of the Durchmusterung were uniformly distributed, their average distance apart would be about 8′.7. The five stars of the fifth class have Durchmusterung companions at an average distance of 10′.6. In the fourth class, excluding the red stars, six are in the Durchmusterung, and have companions at an average distance of 2′.5, four being less than 2′.0 distance, one at 3′.2, and one at 6′.1. In all six cases the direction of the companions is within less than 34° of the plane near which the variables lie, or at an average distance of 18°, while, if distributed by chance, this angle should be 45°. Hence a method of discovering variable stars of this class is offered by looking in a certain part of the sky for those having near companions in a given direction.

The light curves of four stars have been determined with sufficient precision to permit a comparison with theory. All of these may be represented by the formula $L' = 1 + m' \sin (v' + a) + n' \cos 2v'$, in which L' is the light when the star has turned through the angle v'. The difference between observation and theory amounts on the average to only about 0.03 of a magnitude. In other words, the light of these stars at any time may be computed with this degree of precision.

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XX.

LARGE TELESCOPES.

By Edward C. Pickering.

Presented April 13, 1881.

The small amount of work accomplished with large telescopes has often been the subject of unfavorable comment. This criticism applies with especial force to this country, where there are nearly a dozen telescopes having an aperture of a foot or over, besides two of the largest size now in course of construction, and two of 26 and 24 inches aperture which are unmounted and have been for several years perfectly useless. Among so many, it seems as if one might be spared for a trial of the following plan, which if successful would produce at a small expense far more work than could be obtained with a mounting of the usual form.

Suppose that the telescope is placed horizontally at right angles to the meridian, and that a plane reflector inclined to its axis by 45° is placed in front of it. This reflector may revolve around an axis coinciding with that of the telescope. Such a mounting has been used in transit instruments, and gives much satisfaction in the meridian photometer of the Harvard College Observatory. The principal difficulty with a large instrument would lie in the flexure of the reflector. This difficulty has, however, been overcome in a great measure in reflecting telescopes by various ingenious devices. In the present case, since the reflector rotates only around one axis instead of two, the problem is much simplified. A slight motion at right angles of perhaps 5° would be a great convenience, as will be shown below, and would probably be insufficient to materially affect the flexure. It may be said that it is more difficult to make a plane surface than one that is curved. But the principal effect of a slight curvature would be to change the focus of the telescope, the aberration being much less than the effect of the varying flexure. Let us admit, however, that the best definition cannot be obtained, in considering the purposes to which such an instrument could be applied without disadvantage.

Many advantages will be apparent on comparing such a mounting with an equatorial. Great steadiness would be secured, since the mirror would be the only portion moved, and this would be placed directly upon a low pier. Instead of a large and expensive dome which is moved with difficulty, the mirror would be protected by a small shed, of which the roof could be easily removed. It would, therefore, be opened and ready for use in a very short time, and would quickly take the temperature of the surrounding air. The object-glass would be mounted directly upon a second pier, and, as it would not be moved, would be in very little danger of accident. The tube could be made of tin or other inexpensive material, as its flexure is of no importance. It could easily be protected from the changes of the temperature so troublesome in the tube of a large equatorial. If preferred, it might even be exhausted of air, or filled with hydrogen, and the effect of the changes of temperature thus greatly reduced.

The eyepiece could be mounted on a third pier, and would be so far distant horizontally from the mirror and object-glass that there is no reason that it should not be enclosed in a room which may be warmed. The comfort in winter of working in a warm room will be appreciated by those who have used a large telescope in a cold climate. The result is sure to be an increased precision in the observations, and a possibility of prolonging them over longer intervals. A similar effect is produced by the constant direction of the line of sight. No especial observing chair is needed. There is no limit to the size of the attachments which may be made to the eyepiece, since they need not be moved. This is a great advantage in certain spectroscopic and photometric measurements. A strong wind interferes seriously with many observations, as it is impossible to make a telescope so stiff that it will not be shaken by sudden gusts. In the plan here proposed the mirror alone is exposed, and its surface is too small to give trouble.

By means of a long handle the position of the mirror may be regulated from the eye-end, and the declination of the object observed read by small telescopes. If the mirror can be moved at right angles to the meridian 5° from its central position, an object at the equator may be followed for forty minutes, and other objects for a longer period. Without this motion an object may be followed for three or four minutes by moving the eyepiece alone. Clockwork may be applied to the mirror, or less easily to the eyepiece. The focal length may be increased almost indefinitely if desired, and certain advantages will be thus attained in the diminution in the defects of the object-glass, although those of the reflector will not be affected. If the telescope is

to be erected at a great elevation the advantages of the present plan are at once apparent. Many nights of observation would be secured which otherwise would be lost owing to the wind and cold. The simplicity in the construction of the building would be a great advantage, as a large dome in so exposed a situation would be kept free from snow with much difficulty, and might be a source of danger in winter storms. If found impracticable to observe during the winter, it would be possible to have a duplicate mounting below, and remove the lens and mirror from one to the other.

It is evident that the saving of cost would be very great not only in the observatory building and dome, but in the tube, observing chair, clockwork, etc.

If a reflector could be constructed whose surface was the portion of a paraboloid whose abscissa equalled that of the focus, the instrument could be much simplified. No object-glass would then be required, the reflector taking the place both of mirror and lens. All the light intercepted by the objective would thus be saved, and but a single surface need be adjusted and corrected. With the advance in mechanical methods this does not seem wholly impracticable, especially with a mirror of long focus. Since the final correction must always be made by hand, it makes but little difference what is the exact form of the surface.

In any case it would be a great advantage that the mirror could be reground, repolished, or resilvered without moving it from its place. It would only be necessary to place it horizontally, and the grinding machinery could be kept permanently near it. If plane, the perfection of its form could also be tested at any time by setting it on edge, and viewing the image it reflected by a collimating eyepiece attached to the large telescope. Another method would be to place a heliotrope a few hundred yards to the north or south of it, and the light from this would form an excellent artificial star, available whenever the sun shone.

The greatest advantage is the rapidity with which observations could be made. No more time would be lost in identification than with a transit instrument, so that a large number of objects could be examined in the course of a single hour. Any one who has worked with a large telescope knows how much time is lost in opening and closing the dome and in finding and identifying minute objects.

Let us now consider to what purposes a large telescope mounted as suggested might be applied.

I. Sweeping. For the discovery of new objects this mounting presents especial advantages. It might be used for the detection of new

double stars, of nebulæ, of red stars, or of objects having singular spectra, as planetary nebulæ, banded stars, and variables of long period. Suppose that the field of view had a diameter of somewhat over one minute of time, and that a small motor was attached to the mirror which would move it uniformly over 5° in declination in this time, and then bring it quickly back to its first position. The observer would then have presented to him a series of zones 5° long and one minute wide. The sweeps should overlap by a small amount, so that the entire region could be covered in a single evening. The observer could have a few seconds rest between each zone, while the motion of the mirror was reversed. If an object of interest was suspected, it could be located by merely noting the time at which it was seen. The right ascension would be given directly, and the declination would be found by interpolation from the time of beginning and ending the sweep. An examination of the object and a determination of its exact location should be made on another evening.

II. Measures of position. For many purposes positions could be determined with this instrument as in a transit circle. It would generally be better, however, to make the measures differential, leaving the mirror at rest and observing the transits of the object to be determined and of two or more companion stars. The method of the ring micrometer might be employed, or some modification of that with inclined lines. In the latter case the zero of position could be found by the transit of preceding stars, by setting the reticule by a divided position circle, or perhaps better by keeping it in a fixed position, determining the direction of the lines once for all, and applying a correction for the declination of the object observed. Stars could be compared differing nearly a degree in declination, as the eyepiece could be moved without danger of disturbing the reticule. For the same reason the star could be followed for three or four minutes, and its transit over a great number of wires observed. It is here assumed that the distortion produced by the mirror is not very great. A slight distortion would do little harm, as it would be the same for all stars of equal brightness. If the stars differ greatly in brightness, the observer should determine his personal equation between them in any case, and the same operation would eliminate the effect of the distortion. The large aperture of the instrument would permit the observation of stars quite beyond the reach of any meridian circle. The faintest asteroids could thus be readily measured, and could probably be followed in many cases on successive evenings to their stationary points. Zones of stars could be observed very conveniently for the formation of charts or catalogues, for the discovery of asteroids, stars with large proper motion, etc.

Probably the definition could not be sufficiently good for the measurement of the closer double stars, but if clockwork was attached, faint companions could be measured or approximate positions of the coarser pairs determined very rapidly. The positions of nebulæ could also be observed with a view to detecting their proper motion. Stars having a large proper motion might be observed, and the observations so arranged that any very large parallax would be detected. A similar search for a large parallax of variable stars, short-period binaries, minute planetary nebulæ, or stars having singular spectra, might lead to interesting results. The argument that no ordinary star is very near does not apply to such objects.

III. Spectroscopy. The increased dimensions which could be given to the spectroscope, and its steadiness, would compensate in a great measure for a defect in definition. By Zöllner's reversion spectroscope the slit might be dispensed with, and also the necessity of clockwork. So many stars could be observed in a single evening that systematic errors could be in a great measure eliminated, and as the spectroscope would not be moved, we should have a great assurance that the deviations were real. Of the six thousand nebulæ hitherto discovered we know nothing of the spectrum of more than three or four hundred, while the observation of all the others with a large horizontal telescope would not be a very formidable undertaking. It would also be interesting to observe the spectra of all the clusters. It is possible that some may consist of stars having singular spectra, or even of disconnected nebulous masses, in fact forming clusters of planetary nebulæ. The interesting discovery by Dr. Copeland that Burnham's double nebula in Cygnus is gaseous, shows the same tendency to aggregation in these bodies as in stars. Observations of the spectra of all the red stars and variables would also probably lead to interesting results.

IV. Photometry. Should the instrument be devoted to photometry, numerous problems suggest themselves. Variable stars could be observed near their minimum when too faint to be identified with an equatorial without great loss of time. Faint stars in zones or faint companions to bright stars could be measured very rapidly. The relative light of all the asteroids would be an interesting problem. Many coarse clusters appear to consist of stars of nearly equal brightness. Their light compared with their distances apart might aid our study of their formation. Another useful investigation would be to measure the brightness of all the nebulæ.

In the application of physics to astronomy doubtless many other problems will suggest themselves. Thus no satisfactory results have

been obtained in the attempt to measure the heat of the stars with the tasimeter. The use of this instrument would be vastly simplified if it was placed on a solid pier near the ground, was not moved during the observation, and could be perfectly protected from other changes of temperature than those which it was intended to measure.

As either of the problems proposed above would occupy the time of a telescope for at least one year, it is obvious that there could be no difficulty in keeping such an instrument occupied indefinitely.

The horizontal mounting is especially adapted to an elevated position, and would permit the use of a telescope where an equatorial mounting would be quite impracticable. On the other hand, to an amateur, or for purposes of instruction, an instrument which could be set quickly from one object to another, and where the observers need not be exposed to the cold, would offer many advantages. The impossibility of observing far from the meridian would be less important with a large instrument, where the number of objects to select from is very great.

There are certain purposes to which this mounting could not be advantageously applied. The study of close double stars and other objects requiring long examination and very perfect definition could be better left to other instruments. The sun, moon, and planets can also generally be better observed off the meridian. If, however, the entire time of an instrument can be employed to advantage, and it can collect several times as much material as an instrument of the usual form, it is no evidence against its trial that there are certain problems to which it cannot be advantageously applied.

The working force required for such an instrument should consist of at least one observer, an assistant to record, and a number of copyists and computers to prepare the working lists, reduce the observations, prepare them for the press, and read and check the proof-sheets. A large volume of valuable observations could thus be produced every year, which would require at least double the time and money to produce by the same telescope mounted equatorially. The difference in the amount of work will be evident when we compare the number of objects observed with a transit instrument per night, with those observed with an equatorial. A hundred objects in various declinations might be examined in a single evening, while it is seldom that the same number could be identified and measured by an equatorial in a week.

PHOTOMETRIC MEASUREMENTS

OF THE

VARIABLE STARS

β PERSEI AND DM. 81°25,

MADE AT THE

HARVARD COLLEGE OBSERVATORY.

BY

EDWARD C. PICKERING, Director,

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XXI.

PHOTOMETRIC MEASUREMENTS OF THE VARIABLE STARS β PERSEI AND DM. 81°25, MADE AT THE HARVARD COLLEGE OBSERVATORY.

BY EDWARD C. PICKERING, DIRECTOR, ARTHUR SEARLE AND O. C. WENDELL, ASSISTANTS.

Presented April 13, 1881.

OUR knowledge of the cause of variation in the light of certain of the fixed stars must be derived largely from the curves showing the intensity of their light at any given time. Two methods may be employed for determining the form of these light-curves, as they are called. First, that proposed by Argelander, in which the variable is compared by the eye with some adjacent stars of nearly equal brightness. The difference, if any, is estimated in terms of a small unit called a grade, which nearly equals a tenth of a magnitude. A discussion of the entire series of measures serves to determine the light of the comparison stars, and to reduce all the measures to a scale of grades. This method is so simple, and gives results of such precision, that it has heretofore been almost exclusively used. For determining the form of the curve qualitatively, and the times of maximum and minimum light, this method leaves little to be desired. For a quantitative study of these curves, however, we must reduce the scale of grades to light ratios by photometric measures of the comparison stars. If, meanwhile, any of the comparison stars vary in light, errors are introduced which cannot be eliminated, and these, with the errors in the photometric measures, are likely to greatly exceed the errors in determining the form of the light-curve. The second method consists in a photometric measurement of the light of the variable at different times, and thus determining directly the form of its light-curve. Although the errors in the final results in the second method may be no larger than in the first, yet they are rendered much more conspicuous, so that hitherto no very satisfactory light-curves have been obtained in this way. On the

other hand, a photometric measure has a great advantage on the score of independence, as it may be repeated many times in succession in a single evening. An observer cannot repeat a naked-eye comparison many times without being prejudiced in the later measures by those which have preceded them.

The photometer used in the following measurements is essentially the same as that described in volume xi. of the Annals of the Harvard College Observatory, p. 4, figs. 1 and 2. A double-image prism is placed between the object-glass and eyepiece of a telescope, and in front of the eyepiece a Nicol prism is inserted. A circle divided into degrees serves to show the angle through which the Nicol is turned. Two adjacent stars may be compared by this instrument with great accuracy. Two images of each will be formed by the double-image prism, and their relative brightness varied at will by turning the Nicol. Each image in turn will disappear when the Nicol is revolved 90°. There will therefore always be four positions in which the brighter image of the fainter star will be precisely equal to the fainter image of the brighter star.

β PERSEI.

The measurements of *β Persei* were made by comparing it with *ω Persei*, a fifth magnitude star 90′ distant. The two images were formed by two Rochon prisms, which produced a separation of about 100′. It was therefore necessary that they should be placed very near the object-glass of the telescope, in order that the images of the two stars should be near together. The focal length of the telescope is about seventeen inches, and its available aperture is limited by the size of the prisms to about an inch. A larger aperture would be preferable, but is not very important on account of the brightness of the stars. The telescope is placed horizontally with a right-angled reflecting prism in front of the object-glass. The line of sight is therefore horizontal, even when objects in the zenith are observed, and the stars are followed by rotating the telescope around its axis, and also by turning the stand around a vertical axis. The four images are placed in line, and the two central ones, which are compared, are reversed by moving the prisms to and from the object-glass by a handle attached to the tube carrying them. This reversal was essential, and was always made after the second setting in each set. Errors due to the position of the images are thus completely eliminated. As the two images are seen on the same background under precisely the same conditions, many sources of error are eliminated. The conven-

ient position of the observer, the line of sight being always horizontal, also conduces to the accuracy of the measures. Owing to the low power used (about nine diameters), clockwork was not needed, the stars being occasionally brought back to the centre of the field as they moved away. The readings were wholly independent, as it was quite impossible to distinguish the images of the two stars when brought nearly to equality.

The four positions of the Nicol, in which the images appeared to be equal, were read by the graduated circle to tenths of a degree. This was repeated three times, so that twelve settings constituted a single set. Successive sets were commonly taken by different observers, but when one observer only was present he generally took care to leave the instrument for a minute or so between the sets, so that the same sources of error should not recur. Three observers took part in the work, Mr. Arthur Searle, Mr. O. C. Wendell, and myself. They will be designated by the initials S., W., and P.

Observations were made on thirteen nights, from Sept. 29, 1880, to Jan. 1, 1881, and will be detailed in a future volume of the Annals of the Observatory. The total number of sets was 230, containing 2,748 settings, two of the sets being incomplete. Twenty-eight sets were taken when the nearest minimum was five hours or more distant. They serve to determine the full brightness of the variable. Twelve sets by Mr. Searle give the excess in brightness of β over ω of 2.45 magnitudes; five by Mr. Wendell give 2.68; and twelve by myself give 2.67. As these results are confirmed by the other observations, we may conclude that ω appeared brighter or β fainter to Mr. Searle than to the other observers by about 0.22 magnitudes. All of his measures have been corrected by this amount to reduce all to the same system. Fifty sets, or six hundred readings, were obtained on Oct. 7, extending almost continuously from quarter of seven in the evening to half-past three of the following morning. On Oct. 10, nineteen sets were obtained from half-past six until nearly eleven, when clouds prevented further measures. On Nov. 2, forty-two sets were taken between six o'clock and midnight. On Nov. 19, fifteen sets were obtained; on Nov. 22, thirty-three; on Dec. 9, fourteen; on Dec. 22, nine; and on Jan. 1, twenty-eight. On Nov. 19, the observations of Mr. Searle appeared to differ from the results of the other observers by about three tenths of a magnitude, even after applying the correction of 0.22 magnitudes noted above, or without this correction they differed about half a magnitude. The reduction was first made retaining these, but they introduced so large a discord-

ance that the entire reduction has been repeated, rejecting them. No explanation can be offered for this difference, which occurs in nine sufficiently accordant sets. As the alternate observations of Mr. Wendell on the same evening agreed with the results of the other evenings, the effect seems to be due to the observer, and not to a variation of the star. The remaining observations have been arranged in groups according to the time preceding or following the minimum. Each group extends over half an hour, the computed minimum being the middle of one group. The first and last group extend from 255 to 345, the observations being more scattering. The results derived from these groups are given in Table I. The first column gives the mean of the times before or after the computed minimum. The latter was taken from the Annuaire of the Bureau des Longitudes, 1880, p. 78, which depends on the formula given by Schönfeld,* Ep. E = 1867 Jan. 0^d 11^h 1.2^m M Z Paris + 2^d 20^h 48.9^m (E — 8534).

The second column gives the number of sets in each group, and the third the mean of the observed magnitudes. The points defined by these times and magnitudes were then plotted on rectangular paper, and a smooth curve drawn nearly through them. Various precautions were taken to avoid small irregularities in this curve. The ordinates were read off, and the residuals computed from straight lines nearly tangent to the curve. These were plotted in turn, and the smooth curve drawn through them served to correct the original curve. The discussion of the rate of change in the light and of the true time of minimum, given below, also furnished small corrections, so that the curve should not only pass nearly through the observed points, but should undergo no sudden change in its direction or curvature. The ordinates of the final curve are given in the fourth column, and the deviations of the observations from them in the fifth column.

An inspection of this table shows that the observed minimum precedes that given by computation by more than half an hour. To determine the exact time of minimum, we must find the mean of the times when the light is equal. If the light curve was symmetrical, each of these means would equal the true minimum. Suppose that points are constructed with abscissas equal to the mean times, and ordinates to the corresponding light. Suppose that a smooth curve is drawn through these points, and extended to the point whose ordinate equals the light at the minimum. The abscissa of this point will give

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* Sechsunddreissigster Jahresbericht des Mannheimer Vereins für Naturkunde, p. 94.

TABLE I.—LIGHT CURVE OF β PERSEI.

Time.	No.	Obs.	Curve.	O — C.
—	28	2.67	2.67	.00
— 278	8	2.68	2.67	+ .01
239	10	2.56	2.58	— .02
211	10	2.50	2.48	+ .02
181	9	2.29	2.31	— .02
151	11	2.15	2.14	+ .01
117	9	1.94	1.92	— .02
90	11	1.75	1.76	— .01
59	11	1.68	1.66	+ .02
— 29	11	1.64	1.64	.00
+ 1	17	1.70	1.70	.00
29	12	1.85	1.83	+ .02
63	9	1.98	2.00	— .02
91	12	2.19	2.17	+ .02
122	10	2.29	2.30	— .01
149	11	2.39	2.40	— .01
179	10	2.52	2.50	+ .02
212	10	2.59	2.58	+ .01
239	6	2.60	2.63	— .03
+ 275	6	2.66	2.66	.00
	221			± .013

the true minimum. Table II. gives for the various magnitudes contained in the first column the corresponding times at which the star attains this brightness during decrease and increase in the second and third column. The mean of these times is given in the fourth column.

TABLE II.—TIME OF MINIMUM.

Magn.	Dec.	Inc.	Mean.
2.6	— 246	+ 221	— 12.5
2.5	— 218	+ 180	— 19.0
2.4	— 196	+ 148	— 24.0
2.3	— 177	+ 122	— 27.5
2.2	— 161	+ 99	— 31.0
2.1	— 145	+ 80	— 32.5
2.0	— 131	+ 63	— 34.0
1.9	— 114	+ 45	— 34.5
1.8	— 97	+ 24	— 36.5
1.7	— 76	+ 1	— 37.5
1.68	— 70	— 5	— 37.5
1.66	— 60	— 13	— 36.5

From the above table we see that the true time of minimum preceded that given in the ephemeris by about thirty-seven minutes. An

ephemeris given by Dr. Schönfeld * for the present year differs by thirty-five minutes from his formula, or agrees within two minutes with the result of the present observations. The writer has shown in another place † that observations show a deviation from Schönfeld's formula of twenty-nine minutes at the end of 1878, and that this deviation is increasing at the rate of about three minutes a year, which would also give a correction of thirty-five minutes.

Any portion of the observations, as those of a single observer, or of one evening, would in general be better satisfied by moving the curve horizontally, or by assuming a different time of minimum. We wish, therefore, to know what correction t to the minimum is indicated by such observations. Let R equal the residual found by subtracting the value given by the assumed curve from that found by observation, and let r equal the residual when the minimum is altered by t. Also, let a equal the differential coefficient of the light in terms of the time, or the change of the light in magnitudes per minute. Then $R = r + at$, in which r and t are unknown. Solving with regard to t, we obtain, $t = \frac{R}{a} - \frac{r}{a}$. The weight to be assigned to such a determination of t will be proportional to a, since the errors are almost entirely due to erroneous determinations of the light, the error in the time being wholly insensible. Accordingly the effect on t of an error of a hundredth of a magnitude will be inversely as the rate of change of the light, or the weight should be proportional to a. Whatever the sign of a, the weight must always be positive. Multiplying the above value of t by a, we have $at = \pm R \pm r$, in which a is positive, and the signs of R and r will always be those of $\frac{R}{a}$ and $-\frac{r}{a}$. Taking the sum of all these equations, we obtain $\Sigma at = \Delta R - \Delta r$, in which Σ denotes the arithmetical sum of all the separate values, Δ their algebraic sum, taking into account the signs assigned them above. But $\Sigma at = t\Sigma a$, since, although t is unknown, it is the same for all the observations. Again, $\Delta r = 0$, since the separate values of r are arranged according to accident. Therefore, $t\Sigma a = \Delta R$, or $t = \frac{\Delta R}{\Sigma a}$. The computation is made by taking the algebraic sum of all values of R after changing the signs of those, in which a is negative, and dividing the result by the arithmetical sum of all the values of a.

* Vierteljahrsschrift, xv. 14. † Proceedings American Academy, xvi. 36.

The probable error, e, of the resultant value of t may be found from n, the number of residuals, their magnitudes r, and their weights a. The value of each expressed in minutes will be $\frac{r}{a}$, but since a weight of a should be assigned to it, we must write $a \times \frac{r}{a} = r$. The sum of all these terms will be Σr, and the sum of their weights Σa. The probable error will therefore be, $e = \frac{0.845\ \Sigma r}{\sqrt{n-1}\ \Sigma a}$. We cannot determine Σr directly, since r has not been computed. If t is not very large, ΣR will not greatly exceed Σr; we shall not therefore cause a large error if we write $e = \frac{0.845\ \Sigma R}{\sqrt{n-1}\ \Sigma a}$. The probable error thus found will be somewhat too large, so that the substitution from which it results cannot exaggerate the accuracy of the observations. $\frac{\Sigma R}{n}$ equals the average deviation D, and if n is large we may write $e = \frac{0.845\ \sqrt{n}\ D}{\Sigma a}$.

To apply this method we must determine the values of R and a corresponding to each set. The light corresponding to the time of each observation was read off from the light curve, and subtracted from the observed brightness. The value of a was determined as follows: A silk thread was kept stretched perfectly straight by making it the string to a bow of whalebone. It was then laid upon the curve so as to be tangent in turn to the points whose abscissas differ by twenty-five minutes. The ordinates of the points where the thread intersected two vertical lines, whose abscissas differed one hundred minutes, were next read. The difference in these ordinates, divided by one hundred, gave the change in magnitude per minute or a. Table III. gives, in the first and second columns, the corresponding times and values of a derived in this way, from the portion of the light curve preceding the minimum. Points were next plotted with the times as abscissas, and the values of a as ordinates, and a smooth curve drawn through them. The ordinates of this curve are given in the third column, and the residuals found from the observed values of a in the fourth column. The close agreement testifies to the smoothness of the curve and the precision of the measures. From the curve thus found, the values of a were read for each set. The last three columns correspond to the portion of the curve following the minimum.

TABLE III. — RATE OF CHANGE IN LIGHT.

Time.	Decreasing.			Increasing.		
	Obs.	Curve.	$O-C.$	Obs.	Curve.	$O-C.$
300	—	.0000	—	—	.0000	—
275	—	—.0014	—	—	+.0007	—
250	—.0030	—.0027	—.0003	—	+.0013	—
225	—.0037	—.0040	+.0003	+.0020	+.0019	+.0001
200	—.0054	—.0051	—.0003	+.0025	+.0026	—.0001
175	—.0058	—.0059	+.0001	+.0031	+.0031	.0000
150	—.0064	—.0063	—.0001	+.0035	+.0038	—.0003
125	—.0063	—.0063	.0000	+.0041	+.0045	—.0004
100	—.0056	—.0056	.0000	+.0048	+.0050	—.0002
75	—.0036	—.0036	.0000	+.0059	+.0054	+.0005
50	—.0011	—.0013	+.0002	+.0054	+.0054	.0000
25	+.0008	+.0010	—.0002	+.0046	+.0046	.0000
0	+.0036	+.0034	+.0002	—	—	—

This table also affords a method of determining the point of minimum light. At this point the rate of change should be zero, or should change from positive to negative. This evidently occurs between the times 50 and 25 minutes. Interpolating with the values given in either the second or third column gives for the exact time 36 minutes. This value agrees closely with 37 minutes, the value derived above from the points of equal light. The best method of determining the time of beginning and ending of the variation in light is from this same table. It will necessarily be subject to considerable error, since the observed curve must be extended according to the judgment of the observer. The times —300 and +300 have been found in this way. In other words, the star begins to diminish about 263 minutes before the minimum, and does not recover its original brilliancy until 337 minutes after. The most rapid diminution would occur at —140 or 100 minutes before the minimum. The variation would be then 0.0064 per minute.

The most rapid increase would occur at 100 minutes after the minimum, and would amount to 0.0055 magnitudes per minute.

In Table IV. the values of R and a are arranged in groups. A current number in the first column is followed in the second by the condition determining the groups. The next columns give the number of sets of twelve readings each, the arithmetical sum of the values of a, the arithmetical sum of the residuals, and their algebraic sum, giving to each the sign of R divided by a. The seventh column gives the correction to the assumed minimum found by dividing the sixth column by the fourth. The eighth column gives the probable error of the

resulting time, or $\frac{0.845 \, \Sigma R}{\sqrt{n-1} \, \Sigma a}$. The last column but one gives the average residuals, or the fifth column divided by the third. The last column gives the average deviation of the three sets of four readings each, of which the sets of twelve readings are composed. It serves to show the accordance of the successive readings.

The first seven lines give the results for the seven minima which were observed. The next three lines group together all the observations of each observer. Lines 11 and 12 place together all those in which the light is decreasing, and those in which it is increasing. The results of all these sets is given in line 13. The rejected sets obtained on Nov. 19 are given in line 14. Line 15 groups those in which the star has its full brilliancy, or when the nearest minimum was more than five hours distant. The last line gives the results of lines 13 and 15, or the entire series, excepting those of Nov. 19. A set taken Oct. 7 is also included, which was taken so near the minimum that a was sensibly equal to zero. For this reason line 16 is not exactly equal to the sum of lines 13 and 15.

TABLE IV. — COMPARISON OF RESULTS.

No.	Group.	No. Sets.	Σa	ΣR	ΔR	$\frac{\Delta R}{\Sigma a}$	Prob. Err.	Av. Resid.	Av. Dev.
1	Oct. 7.....	49	.1937	4.00	−0.62	− 8.2	2.5	.081	.060
2	" 10.....	19	.0837	2.30	+0.28	+ 3.5	5.5	.121	.047
3	Nov. 2.....	43	.1284	2.64	−0.60	− 5.0	2.7	.061	.066
4	" 19.....	6	.0335	0.27	−0.11	− 3.6	8.0	.045	.056
5	" 22.....	33	.1362	3.32	+2.28	+17.5	2.6	.001	.083
6	Dec. 9.....	13	.0353	1.01	+0.11	+ 3.6	7.0	.078	.078
7	Jan. 1.....	28	.1214	2.47	−0.03	− 0.2	3.3	.088	.067
8	Obs. of P.	86.	.3103	6.33	−0.35	− 1.2	1.9	.078	.063
9	" S.	45	.1864	5.30	+1.94	+10.4	2.9	.117	.085
10	" W.	60	.2355	4.38	−0.28	− 1.2	2.0	.073	.057
11	Decrease ..	81	.3342	7.27	+0.17	+ 0.5	2.1	.089	.066
12	Increase ..	110	.3980	8 74	+1.14	+ 2.9	1.6	.079	.066
13	Total......	191	.7322	16.01	+1.31	+ 1.8	1.3	.083	.066
14	Nov. 19. S.	9	.0385	3.55	−3.55	[− 92.2]	[27.8]	[.394]	.092
15	Full Light..	28	.0000	3.04	+0.06	—	—	.108	.068
16	Total......	220	.7322	19.14	+1.28	—	—	.086	.066

The observations of Nov. 22 show a large correction to the minimum. This is not easily explained unless the deviation is real. The measures before the minimum give a correction of + 15 minutes; those after, of + 18; those of P. alone, + 26; of S., + 14; of W., + 12. As the probable error of the mean result is only 2.6 minutes, and a nearly equal number of measures were made on each side of

the minimum, it is difficult to understand what instrumental errors could have caused the deviation. Including this minimum, the mean deviation for the seven nights is 5.2 minutes, or excluding the observations of Nov. 22, 3.2 minutes, the corresponding probable error would equal 4.7 and 3.0 minutes. The mean of the probable errors given in the next column is 3.8 minutes. This compares favorably with the results of naked-eye observations. Schmidt * gives the probable error of a single minimum observed by Argelander to be 6.0 minutes; of those of Schönfeld, 4.6 minutes; and of those by himself, 8.0 minutes. Probably still better results could have been obtained photometrically had the observations been designed for determining the time of minima. The mean of the whole series of measures would imply a correction to the adopted curve of + 1.8 minutes, with a probable error of 1.3 minutes. But if the observations of Nov. 22 are rejected, the correction becomes — 1.6 minutes. It therefore seems better to retain the correction to the ephemeris of 37 minutes, already adopted.

We have now a means of determining more precisely the constant difference between the different observers. The differences so far assumed are, P = 0.00, S = — 0.22, and W = 0.00 magnitudes. If either observer had taken an equal number of observations before and after the minimum, — or more strictly, if the weight of his observations before and after the minimum were equal, — an error in this correction would not affect the result. It would, however, very slightly exaggerate the residuals, and consequently the probable errors. If these personal differences were zero, the algebraic sum of the residuals of each observer should be zero. In fact, their values for the three observers are, for P = — 2.43, for S = + 1.66, and for W = + 1.84. As the total number of sets in the three cases are 98, 57, and 64, we obtain by division the deviations — 0.02, + 0.03, and + 0.03. Combining with these the correction of 0.22 already derived from Mr. Searle's observations of the full light of the variable, we find that the correction required to reduce his measures to mine is + 0.17, and to reduce Mr. Wendell's — 0.05, magnitudes. The effect of these changes on the final result would probably be wholly insensible.

Line 14 of the above table shows clearly that the observations of Nov. 19 should be rejected. They would indicate an error of an hour and a half in the minimum, if the deviations were not so large that the present method could not be applied.

* Astron. Nach., lxxxvii. 204.

A comparison may now be made with the light-curve given by Schönfeld in the paper cited above. As has been already stated, the great difficulty lies in deciding what brightness shall be assumed for the comparison stars. In a previous article,* the light of these stars in grades assumed by Schönfeld have been reduced by means of the formula $L = 8.446 + 0.025\,g$, in which L gives the light and g the number of grades. This formula is derived from a comparison with the measurement of the comparison stars by Seidel and Wolff. These stars have since been measured with the meridian photometer of the Harvard College Observatory. Each star has been observed on the average on ten nights.

Three methods of reducing the grades of Schönfeld by these stars may be used. We may adopt the formula given above, which was found by a least square solution of the measures of Seidel and Wolff. Secondly, we may apply the method of least squares to the Harvard College Observatory measures, and deduce the formula most nearly satisfying them. This gives the value of one grade in logarithms equal to 0.029. In both these cases we have assumed that the value of a grade is the same for bright and for faint stars, and that the deviations are due to accidental errors, or to variations which have taken place in the light of the stars. As a third method, we may draw a curve through the points whose co-ordinates equal the light in grades and the measured brightness, and reduce the grades by means of this curve. We now assume that the errors are unimportant, but that the grade varies in different parts of the scale.

Table V. gives, in successive columns, the name of the star, its light in grades, the number of nights on which it has been observed at Cambridge, the resulting magnitude, the probable error, and the logarithm of the light, adopting the same unit as that of Seidel. Observations of *β Persei* have been included in this list, excluding those made within a few hours of its minimum. Three columns of residuals exhibit differences between the measures of Seidel, of Wolff, and of the Harvard College Observatory, and the values computed by the formula $L = 8.446 + 0.025\,g$. The next column gives the H. C. measures reduced to logarithms, minus those given by the formula $8.391 + 0.029\,g$. The last column gives the difference between the measures of the stars and the values of their light derived from the smooth curve.

* Proc. Am. Acad., xvi. 21.

The last two lines give the mean results in logarithms and in magnitudes.

TABLE V. — Comparison Stars of β Persei.

Name.	Gr.	No. Nights.	Mag.	P. E.	Log.	S-C.	W-C.	HC-C	HC-C′	HC-Curve.
γ Andromedæ.	23.4	11	1.89	.05	9.085	+.007	—.010	+.054	+.015	.000
β Persei.....	20.8	13	2.05	.03	9.021	—	—	+.045	+.017	.000
ι Aurigæ.....	17.3	8	2.40	.12	8.881	—.181	—.075	+.003	—.011	.000
β Arietis.....	16.7	12	2.48	.05	8.849	+.038	—.002	—.015	—.027	—.006
ϵ Persei......	12.8	10	2.75	.06	8.741	+.034	—.020	—.025	—.021	—.002
γ Persei......	10.9	10	2.85	.03	8.701	—.019	—.027	—.017	—.006	—.007
β Trianguli...	9.1	14	2.86	.04	8.697	+.042	+.042	+.023	+.042	+.003
δ Persei......	7.8	11	2.90	.05	8.681	+.100	+.053	+.040	+.064	+.011
α Trianguli...	3.5	12	3.26	.05	8.537	—.003	+.054	+.003	—.016	—.001
ν Persei......	0.9	10	3.71	.06	8.357	—	—.038	—.111	—.060	.000
Mean in logarithms				±.020		±.052	±.045	±.034	±.028	±.003
Mean in magnitudes				±.05		±.18	±.11	±.08	±.07	±.01

The eighth and ninth columns show that the agreement of our measures with the estimates of Schönfeld is better than that of either Seidel or Wolff. This is the case even when the value of g is derived from the measures of these observers. The last column shows that a curve could be made to follow the observations almost exactly, the small deviations being allowed rather than that too sharp a change of curvature should be given to the curve.

The form of light-curve deduced from the above measures is shown in Table VI. The first column gives the time, and the second the corresponding magnitude, found by reading the ordinates of the curve drawn through the observed points as described above. A correction to the ephemeris of thirty-seven minutes in the time of minimum is assumed, and the points correspond to intervals of thirty minutes from this time. The logarithm of the light is given in the third column, and is found by multiplying the magnitudes by 0.4 and subtracting 1.068. The relative intensity of the light compared with the full brightness is given in the next column. The observations of Schönfeld are next reduced by assuming the value of g to be successively 0.025 and 0.029; and, thirdly, by means of the curve described on page 380. The residuals in the last three columns are found by subtracting the logarithms given in the third column from these three sets of values.

TABLE VI. — LIGHT CURVE OF β PERSEI.

Time.		Mag.	Log.	Light.	Schönfeld.		
					$g = .025$	$g = .029$	Curve.
−4	30	2.67	0.000	1.000	—	—	—
4	0	2.67	0.000	1.000	−.014	−.016	−.023
3	30	2.60	.972	.938	−.002	−.007	−.018
3	0	2.50	.932	.855	+.015	+.007	−.013
2	30	2.35	.872	.745	+.042	+.028	−.012
2	0	2.18	.804	.637	+.058	+.036	−.018
1	30	1.98	.724	.530	+.057	+.022	−.014
1	0	1.80	.652	.449	+.040	−.009	+.008
−0	30	1.68	.604	.402	+.032	−.025	+.010
0	0	1.64	.588	.387	+.031	−.030	+.005
+0	30	1.68	.604	.402	+.031	−.027	+.008
1	0	1.79	.648	.445	+.022	−.031	−.004
1	30	1.94	.708	.510	+.017	−.027	−.025
2	0	2.12	.780	.603	+.029	−.002	−.051
2	30	2.26	.836	.686	+.038	+.019	−.037
3	0	2.39	.888	.773	+.035	+.023	−.012
3	30	2.48	.924	.840	+.036	+.030	+.012
4	0	2.56	.956	.904	+.030	+.028	+.022
4	30	2.68	.984	.964	+.016	+.016	+.016
5	0	2.66	.996	.991	+.004	+.004	+.004
+5	30	2.67	0.000	1.000	—	—	—
Mean in logarithms					±.029	±.019	±.016
Mean in magnitudes					±.07	±.05	±.04

It does not seem to be practicable to obtain at present more accurate values from the observations of Dr. Schönfeld, on account of the uncertainty in the value of a grade. The observations themselves are much more precise, and determine the time of minimum, as has been shown above, with an accuracy nearly equal to that of the photometric measures. Even if more accurate measures of the comparison stars should be made, we should still be in doubt whether to assume that g is constant, or that the reduction should be made, as in the last column, by a curve. From the residuals it appears that the various deduced values differ from each other more than they differ from the photometric measures. It accordingly appears scarcely safe to correct the latter by the former. The three values of the minimum corresponding to the last three columns are 1.72, 1.56, and 1.65 magnitudes, their mean agreeing exactly with the photometric measure of 1.64.

It is to be noticed that the value of $g = 0.029$ is confirmed by the photometric measure of β *Persei*, since the residuals are less than when g is taken equal to 0.025. A wholly independent test of the accuracy of the meridian photometer measures is thus afforded. Since

the residuals are smallest in the last column, it seems probable that the value of a grade is not always the same.

The results of the two methods agree as closely as would be expected, even if no systematic errors increased their discordance. The residuals of the photometric observations indicate a probable error of 0.024 magnitudes for each group. Assuming an equal accordance in the observations of Schönfeld, the two methods should differ by 0.04, or by the amount found in Table VI.

DM. 81°25.

The variability of this star was detected by M. Ceraski, of Moscow, during the summer of 1880. It was soon shown that it belonged to the Algol class, or that every few days it lost a large portion of its light for several hours; the interval in the case of this star is somewhat less than two days and a half. Measurements of its light were made according to the method described above in the case of *β Persei*. The photometer was attached to the 15-inch telescope of the Harvard College Observatory, since as much light-gathering power as possible was desired, owing to the faintness of the star. The same observers took part in the work, and the observations were made in the same way as with *β Persei*, except that the images were reversed by turning the photometer instead of by moving the prism. This could be done very conveniently by a pinion, which served to rotate the entire tail-piece of the telescope. The prism was therefore set once for all, and the images reversed and separated by any desired amount with great nicety by turning a milled head. The star DM. 81°26, which is estimated in the *Durchmusterung* to be of the 9.5 magnitude, and is nearly north at a distance of 5′, was used for comparison. DM. 81°30 would have been better on account of its greater brightness, but its distance of 8′ is so great that both images could not be easily brought together. The large angle of the prisms and their distance from the object-glass rendered the light-pencils divergent. At first this gave much trouble, but it was remedied by placing the images always in the same part of the field. Two cardboard points visible against the background of the sky secured this condition. The great northern declination reduced the errors of the driving clock to about one sixth of what they would be for an equatorial star.

The first measures to determine the full brightness of the variable in terms of that of the comparison star were made on February 6, 1881. On the following evening the variable attained its minimum at about half-past eleven. Forty sets or four hundred and eighty settings were obtained between a quarter past six and half past ten. The

later observations were made through clouds which finally stopped the measurements. On February 17 seventy-five sets or nine hundred settings were obtained; the observations extended from seven o'clock in the evening until the variable had regained its full light, at about half-past two on the following morning. During this time no interval of more than five minutes elapsed during which an observer was not comparing the two images. During most of the time the observers took sets alternately, so that there was only an interval of a few seconds between the sets. On February 22 observations began at half past six and continued until ten o'clock, when they were stopped by clouds. Twenty-six sets were obtained in this time. A long period of cloudy weather intervened, and the next measures were made on March 24. Thirty-six sets were taken through clouds, from quarter-past nine to quarter-past twelve. Owing to the small distance between the stars, no perceptible error seems to be introduced by these clouds, as long as they are not dense enough to render the stars invisible. Some measures were obtained on March 14, but apparently the wrong star was observed. They were stopped by the deposition of dew on the object-glass, which may have caused an error, since the two pencils include different portions of the objective. No use has been made of these observations. On April 3 another minimum was observed. Fifty-two sets of six hundred and twenty-four settings were made between seven o'clock and midnight, when the star had recovered its full brightness. Forty-four sets of five hundred and twenty-eight settings were also made on other evenings to determine the undiminished light of the star. Fifteen of these sets by Mr. Searle give its light as 3.64 magnitudes brighter than DM. 81°26. Sixteen sets by Mr. Wendell gave 3.59, and thirteen sets by myself gave 3.71. As the evidence of systematic difference is not conclusive, the mean of all, or 3.64, has been adopted.

The entire number of measures, not including those of March 14, is 273 sets or 3276 settings.

Table VII., like Table I., gives the results of these measures arranged in groups in the order of times from the computed minimum. The columns give the mean of the times, the number of sets of twelve settings, the mean magnitude, the corresponding magnitude derived from a curve drawn nearly through them, and the difference of the last two columns. Each group extends over thirty minutes, except the first, which extends from —311 to —258, and the last three, which include all the measures made when the nearest minimum was more than five hours distant. Their limits are +852 to +991, +1350 to +1495, and +1887 to +2250 minutes.

TABLE VII.—LIGHT-CURVE OF DM. 81°25.

Time.	No.	Obs.	Curve.	O — C.
— 286	6	3.39	3.45	—.06
237	8	3.30	3.27	+.03
208	14	3.07	3.09	—.02
178	11	2.91	2.85	+.06
148	15	2.44	2.44	.00
118	14	1.81	1.80	+.01
89	14	1.34	1.33	+.01
62	14	1.27	1.25	+.02
31	12	1.16	1.24	—.08
— 8	9	1.29	1.24	+.05
+ 31	12	1.28	1.24	+.04
60	14	1.32	1.32	.00
90	18	1.93	1.93	.00
120	17	2.48	2.48	.00
150	14	2.94	2.93	+.01
179	14	3.22	3.23	—.01
207	15	3.42	3.42	.00
237	8	3.54	3.54	.00
922	12	3.66	3.64	+.02
1411	19	3.65	3.64	+.01
+2127	18	3.62	3.64	—.02
	273			±.019

From the last three groups we may infer that the light of this star, like that of the others of the same class, is constant except during the few hours immediately preceding or following the minimum.

The same precautions were taken as with *β Persei* in drawing the light-curve that it should be free from sudden changes in curvature. From the small residuals in the last column we may therefore infer that the accidental errors are very small. About an hour before the minimum the light ceases to vary, and remains nearly constant for an hour and a half, when it begins to rapidly increase. The exact time of these changes may be found more precisely by subdividing the groups whose means are —89 and +60. Making the period of the groups ten minutes instead of thirty, we replace the first group by three containing 4, 5, and 5 sets, having mean times 99, 90, and 80, and magnitudes 1.54, 1.30, and 1.21. The other group similarly subdivided gives for the mean times 52, 60, and 69, the magnitudes, 1.25, 1.26, and 1.44. From these we might infer a somewhat longer period of uniform light than would be indicated by the curve already drawn. The number of observations is, however, too small to determine this point with certainty.

The correction to the ephemeris of the minima is best found by Table VIII., which gives the time at which the light is equal while

decreasing and while increasing. The successive columns give the light in magnitudes, the corresponding times before and after the minimum, and the mean of these times.

TABLE VIII. — TIME OF MINIMUM.

Magn.	Dec.	Inc.	Mean.
3.6	—362	+256	—53.0
3.5	—305	+225	—40.0
3.4	—270	+204	—33.0
3.3	—246	+190	—28.0
3.2	—226	+175	—25.5
3.1	—209	+164	—22.5
3.0	—194	+155	—19.5
2.9	—182	+147	—17.5
2.8	—173	+140	—16.5
2.7	—164	+133	—15.5
2.6	—157	+127	—15.0
2.5	—151	+121	—15.0
2.4	—146	+115	—15.5
2.3	—141	+109	—16.0
2.2	—137	+104	—16.5
2.1	—132	+ 99	—16.5
2.0	—128	+ 94	—17.0
1.9	—123	+ 88	—17.5
1.8	—117	+ 83	—17.0
1.7	—112	+ 78	—17.0
1.6	—107	+ 73	—17.0
1.5	—101	+ 68	—16.5
1.4	— 94	+ 64	—15.0
1.3	— 87	+ 59	—14.0

From the numbers in the last columns we may infer a correction of 13 minutes when the light equals 1.24, or at the minimum. In other words, thirteen minutes should be subtracted from the adopted ephemeris of the minima. The minimum can evidently be determined with much precision from any observations of the times at which the light is equal when diminishing and increasing. If the light is less than 2.9, or the interval between the times less than five hours and a half, it is only necessary to take the mean of the two times and subtract from two to four minutes. The exact correction is found from the last column of the table after subtracting thirteen minutes. The observation is easily made with a small telescope, as there are so many comparison stars of suitable brightness near the variable. Doubtless a very precise determination of the minimum could thus be easily obtained.

To reduce the separate observations we must determine the rate of change in light. The method employed for *β Persei* has again been used; the results are given in Table IX. The columns have the same meaning as in Table III.

TABLE IX.—RATE OF CHANGE IN LIGHT.

Time.	Decreasing.			Increasing.		
	Obs.	Curve.	O — C.	Obs.	Curve.	O — C.
300.	—	—.0003	—	—	+.0004	—
275	—	—.0017	—	—	+.0012	—
250	—.0040	—.0032	—.0008	+.0026	+.0024	+.0002
225	—.0057	—.0050	—.0007	+.0040	+.0040	.0000
200	—.0067	—.0073	+.0006	+.0063	+.0054	+.0009
175	—.0106	—.0108	+.0002	+.0086	+.0088	—.0002
150	—.0180	—.0171	—.0009	+.0125	+.0125	.0000
125	—.0198	—.0201	+.0003	+.0166	+.0162	+.0004
100	—.0157	—.0152	+.0005	+.0188	+.0196	—.0008
75	—.0025	—.0024	—.0001	+.0205	+.0198	+.0007
50	.0000	—.0006	+.0006	+.0026	+.0030	—.0004
25	.0000	.0000	.0000	.0000	.0000	.0000
0	.0000	.0000	.0000	.0000	.0000	.0000

The greatest change in light amounts to two hundredths of a magnitude a minute, or at the rate of a magnitude and two tenths an hour. This is much greater than the change of any other known variable, being over three times that of *β Persei*. Accordingly, we should expect a corresponding increase in the accuracy with which the time of minima could be determined.

The observations of DM. 81°25 are grouped in Table X. The successive columns, like those of Table IV., give a current number, the condition limiting the group, the number of sets, the arithmetical sum of the residuals, their algebraic sum giving to each the sign of R divided by a, and the correction to be inferred, or ΔR divided by Σa. The remaining columns give the probable error, the average of the residuals, and the average difference of the three sets of four contained in each set of twelve settings.

TABLE X.—COMPARISON OF RESULTS.

No	Group.	No. Sets.	Σa	ΣR	ΔR	$\frac{\Delta R}{\Sigma a}$	Prob. Err.	Av. Dev.	Av. Resid.
1	Feb. 7....	88	.3463	8.31	— .35	—1.0	1.3	.087	.065
2	" 17....	62	.7071	7.55	+ .01	0.0	1.1	.121	.063
3	" 22....	23	.2408	2.49	— .01	0.0	1.9	.108	.082
4	March 24..	36	.4312	3.03	+ .35	+0.8	1.0	.084	.059
5	April 3....	38	.4000	3.22	+ .74	+1.8	1.1	.084	.051
6	Obs. of P..	76	.8618	6.41	— .01	0.0	0.7	.084	.055
7	" S..	69	.7248	9.11	+1.69	+2.3	1.3	.132	.072
8	" W..	52	.5388	4.08	— .94	—1.7	0.8	.078	.062
9	Decrease ..	91	.9089	9.23	+ .39	+0.4	0.9	.101	.071
10	Increase ..	106	1.2165	10.37	+ .35	+0.3	0.6	.097	.055
11	Total......	197	2.1254	19.60	+ .74	+0.3	0.6	.099	.062

The average probable error of the five minima observed is 1.3 minutes, or about one third of that of β *Persei.* This ratio, as has been already stated, was to be expected, since the rate of variation of the stars is about as three to one. The average deviations from the ephemeris, after applying the constant correction of thirteen minutes, is only 0.7 minutes. It becomes still less if we adopt another ephemeris, as will be shown below. Clouds or twilight prevented observations on both sides of the minimum on every night except on February 17. Accordingly, from a complete observation of a minimum under favorable circumstances we may expect an error of but a few tenths of a minute.

The systematic difference between the observers is found by dividing the algebraic sum of the residuals of each by their number: the algebraic sum of 86 residuals by Mr. Searle is —5.30; of 85 by Mr. Wendell, —0.87; and of 102 by myself, +5.62. The corresponding corrections are, —0.06, —0.01, and +0.06. As each of these represent over a thousand settings, the differences are not probably due to accident. The excess of the computed probable error in the eighth column of Table X. over that to be inferred from the residuals in the seventh column is partly due to the neglect of these differences. If applied to the observations, they would make them appear more accordant. They would not probably sensibly affect the form of light-curve or the times of minima, owing to the distribution of the measures of each observer.

The variation in light is given in Table XI., which is derived from the light-curve described above, after applying a correction of thirteen minutes to the assumed minimum. The ratios of light are given for every half-hour, expressed in differences of magnitude, in logarithms, and in numbers, the full brightness being assumed as the unit in the last two columns.

Some interesting theoretical deductions may be drawn from this light-curve. For about an hour and a half the light remains sensibly constant at 0.110, or about one ninth of its full intensity. This interval is over one third of that during which the light is increasing or diminishing. If the variation in light is admitted to be due to a dark, eclipsing satellite, the diameter of the latter must be $\sqrt{1 - 0.110} = 0.943$ of that of the star, in order to sufficiently reduce the light. A somewhat less diameter is possible if we admit that the star, like our sun, is darker near the edges than in the centre. The effect of this is probably slight, or it would show itself in other ways. The longest period of uniform minimum light would occur if the satellite produced a central

TABLE XI. — LIGHT-CURVE OF DM. 81°25.

Time. h. m.	Mag.	Log.	Light.
—6 30	8.64	0.000	1.000
6 00	8.62	9.992	0.982
5 30	3.58	9.976	0.946
5 00	8.52	9.952	0.895
4 30	8.44	9.920	0.832
4 00	8.34	9.880	0.759
3 30	8.19	9.820	0.661
3 00	2.99	9.740	0.550
2 30	2.68	9.616	0.413
2 00	2.12	9.302	0.247
1 30	1.71	9.228	0.169
1 00	1.27	9.052	0.113
—0 30	1.24	9.040	0.110
0 00	1.24	9.040	0.110
+0 30	1.24	9.040	0.110
1 00	1.26	9.048	0.112
1 30	1.67	9.212	0.163
2 00	2.26	9.448	0.281
2 30	2.76	9.648	0.445
3 00	3.13	9.796	0.625
3 30	3.35	9.884	0.766
4 00	3.51	9.948	0.887
4 30	3.59	9.980	0.955
+5 00	3.64	0.000	1.000

annular eclipse. In this case, if the motion was uniform, the duration of the minimum light would equal only one ninth of that of increase or decrease. The effect of the curvature, or ellipticity, of the path would not greatly affect this conclusion. A very great ellipticity is not admissible, or at the periastron the satellite would strike the star. We are therefore obliged to admit that the eclipse is total (that is, that the star is entirely covered by the satellite), and that the light during the minima is due to one of the two following causes: first, that the satellite is self-luminous, and that the light at the time of the minimum is that received from the satellite, the star itself being completely obscured. In this case we should expect to find a corresponding diminution midway between the minima when the star was in front of the satellite, and accordingly cut off a portion of its light. The loss of light would, however, be small, and might easily escape detection. The greatest effect would occur when the transit was central. In this case, to produce the observed duration of the minimum, assuming the motion to be uniform, the diameter of the satellite should be about 1.3, that of the star being taken as unity. Since the light of the satellite is supposed to be 0.110, that of the satellite and star together being taken as unity, it follows that if the star passes in front of the satel-

lite, it will cut off $\frac{1}{(1.3)^2}$ of its light, or produce a diminution in the total light of $\frac{1}{(1.3)^2} \times 0.110 = 0.065$. The secondary minimum would therefore reduce the light from 1.000 to 1.000 — 0.065 = 0.935, or about 0.07 magnitudes. This will be the greatest effect, and would be less if the transit was not central. An eccentricity in the orbit of the satellite might even reduce it to zero by carrying the satellite at superior conjunction entirely to one side of the star.

The light reflected by the satellite from the star does not account for this phenomenon, since during its transit the dark side of the satellite would be turned toward the observer. In no case would the light reflected be sufficient, because the satellite does not receive one ninth of the light of the primary; so that, even if all were reflected, it could not emit a sufficient amount of light.

A second hypothesis would explain the prolonged diminution of light by admitting that the satellite consisted of a cloud of meteors so scattered that about 0.110 of the light could pass through the central portions. We should then expect that somewhat more light would pass through the edges, and accordingly that the light would vary slightly during the whole obscuration, attaining a true minimum when the centres of the star and satellite appeared to coincide.

In a recent note Dr. Vogel informs me that he has found no perceptible approach or recession of Algol by means of the spectroscope.* If this observation is confirmed with the other similar variables, we should infer that the masses of the eclipsing satellites were small, or that the second hypothesis is the more probable of the two. In any case, an excellent example is afforded of the value of indirect observations, like those with the photometer or spectroscope, in solving certain problems where direct measurements are valueless.

The next step is to compare the results of other observers, and to derive the correction to the ephemeris. Following the example of Argelander by reducing to Paris mean time, the ephemeris for the time at which any minimum will occur may be expressed by the formula, —

$$\text{Time of minima} = 1880 \text{ June } 23^d\ 7^h\ 44.0^m + 2^d\ 11^h\ 50.0^m\ E.$$

The number of minima which have elapsed since the discovery of the variability is here designated by E, which accordingly equals zero

* See also Ber. der Königl. Säch. Gesell. xxv. 555, and Proc. Amer. Acad. xvi. 84.

on June 23, 1880. The first observations which can be reduced are those made by M. Glasenapp* on July 3, 1880. He adopted a series of comparison stars, which will probably be employed by other observers of this variable. Table XII. gives their Durchmusterung designations, and their right ascension, declination, and magnitudes taken from that catalogue. The next columns give the designation by Glasenapp, and the assumed light in grades. Measures of these stars were made on three evenings at the Harvard College Observatory with Photometer I.† attached to a telescope of four inches aperture. These measures must be regarded as provisional; a much more precise determination of their light will probably be obtained next year with a large meridian photometer. From these measures, which are given in the seventh column, the grades of M. Glasenapp are reduced to magnitudes by the formula, $m = 9.5 - 0.07\, g$, in which g denotes the number of grades and m the corresponding magnitude. The results are given in the eighth column. The last two columns give the residuals found by subtracting the H. C. measures from the magnitudes of M. Glasenapp and of the Durchmusterung.

TABLE XII. — Comparison Stars for DM. 81°25.

DM.	R.A.		Dec.		Mag.	Desig.	Gr.	*H.C.*	*G.*	*G-HC.*	*DM-HC.*
	m.	s.	°	′							
80°23	41	14	80	57.8	9.2	c	0.0	9.6	9.5	—0.1	—0.4
81°22	42	04	81	07.5	9.2	d	1.6	9.4	9.4	0.0	—0.2
80°22	40	28	80	53.3	9.2	b	2.0	9.2	9.4	+0.2	0.0
80°21	39	05	80	48.9	8.9	a	4.6	8.9	9.2	+0.3	0.0
81°27	50	56	81	19.3	8.6	(3)	12.1	8.5	8.6	+0.1	+0.1
81°29	51	35	81	28.1	8.6	(4)	12.6	8.8	8.6	—0.2	—0.2
81°18	38	28	81	10.5	7.6	(5)	17.6	7.5	[8.3]	[+0.8]	+0.1
81°30	52	29	81	10.9	8.3	(2)	18.3	8.1	8.2	+0.1	+0.2
81°25	49	39	81	05.6	7.5	—	—	6.9	—	—	+0.6

The star DM. 81°18 is either variable, or its light in grades is erroneously given by M. Glasenapp. An examination on different evenings showed that it was decidedly brighter than 81°30. This is confirmed by the measures and by the Durchmusterung magnitudes. If the light in grades was written 17.6 by mistake for 27.6, the magnitude becomes 7.6 instead of 8.3, and the residual + 0.1 instead of + 0.8. This cannot be a typographical error, since the stars were arranged by M. Glasenapp in the order of brightness, and 81°18 is

* Astron. Nach., xcviii. 61.
† Annals, xi. p. 7, figs. 5 and 6.

placed before 81°30. The other residuals show a good agreement between the estimates and measures. The Durchmusterung magnitudes also agree well, if we correct for the difference of scale, which makes the residuals for faint stars negative and for bright stars positive.

The individual comparisons by M. Glasenapp are detailed in Table XIII., which gives a current number, the Moscow mean time, and the corresponding light in grades. By the formula $1.00 + 0.07g$ these are reduced to the same scale of magnitudes as that used in measuring the light in Table XI. The results are given in the fourth column. The next column gives the time of minimum derived from each of these observations by means of the light-curve adopted in Table XI. The last column gives the error in the observation of M. Glasenapp, if we assume the minimum to have occurred at $9^h\ 47^m$ Moscow mean time.

TABLE XIII. — M. GLASENAPP'S COMPARISONS OF DM. 81°25 ON JULY 3.

No.	M.M.T.	Gr.	Log.	Time Min.	$O - C$.
				h. m.	
1	10 40	4.1	1.28	[9 34]	+.03
2	43	2.2	1.15	—	—.10
3	45	4.8	1.34	[9 31]	+.08
4	50	3.4	1.24	—	—.02
5	55	3.6	1.25	[9 59]	—.03
6	57	4.8	1.34	[9 43]	+.05
7	11 1	5.4	1.38	9 45	+.04
8	8	6.8	1.48	9 47	.00
9	12	8.1	1.57	9 47	.00
10	15	8.8	1.62	9 48	—.01
11	17	9.3	1.65	9 46	—.02
12	19	12.3	1.86	9 40	+.15
13	22	10.4	1.73	9 47	—.05
14	24	13.5	1.94	9 40	+.13
15	27	11.8	1.83	9 49	—.05
16	31	12.2	1.85	9 52	—.10
17	37	13.2	1.92	9 54	—.14
18	41	14.2	1.99	9 55	—.15
19	51	14.5	2.02	[10 04]	[—.30]
20	59	14.5	2.02	[10 12]	[—.44]
21	12 23	16.2	2.13	[10 29]	[—.72]
22	47	17.2	2.20	[10 50]	[—.93]
23	13 23	17.4	2.32	[11 19]	[—1.03]

The error in the estimated light of DM. 81°18 seems to have affected the last measures of the variable. It would appear that after increasing for over two hours (or three hours after the minimum), the variable had not attained the brightness of DM. 81°30, or 18.3 grades! The last five observations have accordingly been bracketed. The

second and fourth comparisons cannot be reduced, since the light is less than that adopted for the minimum. All of those preceding eleven hours have also been bracketed, since the variation in light is so small that an exceedingly small weight should be assigned to them. Retaining them would not sensibly affect the result. The mean of the remaining twelve gives for the time of minimum 9^h 47^m Moscow mean time. The proximity of the minimum does not affect the residuals of the last column. The last five are alone rejected, since, owing to the cause stated above, they indicate errors too large to be accidental. The mean of the eighteen residuals retained is 0.06 magnitudes.

The most complete series of naked-eye observations of this variable are those of Dr. Schmidt of Athens. Five minima were observed by him in August.* As all the comparisons were made after the period of least light, he was obliged to wait for their reduction until October 8, when he observed the star both before and after the minimum. The first reduction of these observations was made from a curve derived from the measures of October 8. Later he has given a discussion of thirteen minima,† from which he infers a rapid increase in the period. In this paper he omits the observations of August 22, although in his former paper he had assigned to it and to the minimum of August 17 weights double those of any of the others. No reason is given for this omission. There also seems to be a misprint in line 12, p. 89, of this same article. December 7 should apparently be December 2, as this date is employed below. Otherwise, an error of nineteen minutes would be indicated in the observed minimum. A second reduction is given of the August observations, by which the time of minimum is increased more than half an hour. As the original comparisons have not been published, it is impossible to rediscuss them. As the star varies only a few hundredths of a magnitude during nearly two hours, it is obvious that large differences may arise in the assumed time when the light is least. Dr. Schmidt has also determined the period by a method free from this criticism. He has compared the intervals between the times at which the variable equals one of the comparison stars in brightness. Unfortunately, he has not stated the times at which this occurs, so that a comparison with other observers is not practicable.

Mr. George Knott has also observed seven of the minima by the method of Argelander.‡ On September 23 and 28 the variable was

* Astron. Nach., xcviii. 283. † Astron. Nach., xcix. 87.

‡ Astron. Nach., xcix. 109. Nature, xxiii. 542.

observed at the Harvard College Observatory by the same instrument which was used in determining the light of the comparison stars. The image of *α Ursae Minoris* was rendered equal to the variable and to DM. 81°30 alternately. Seven settings were made in each set, beginning and ending with the variable. Systematic errors were thus greatly reduced, since those only would enter which affected one star and not the other. Although a large number of readings were taken, the results were not satisfactory and probably have but little value. The result for September 23 was $8^h\ 57^m$, Cambridge mean time, and for September 28, $8^h\ 7^m$, the difference between the two being five days less fifty minutes instead of five days less twenty minutes. On neither evening were observations obtained before the minimum, owing to twilight.

TABLE XIV. — Comparison of Observed Minima.

No.	E.	Comp. Time.				Obs.		Meridian.	O — C.	Authority.
			d.	h.	m.	h.	m.		m.	
1	0	June	23	7	44	—	—	—	—	Ceraski.
2	4	July	8	7	04	9	47	Moscow.	+22	Glasenapp.
3	16	Aug.	2	5	04	7	10	Athens.	+40	Schmidt I.
4	"	"		"		7	58	"	+83	Schmidt II.
5	18	Aug.	7	4	44	6	54	"	+44	Schmidt I.
6	"	"		"		7	30	"	+50	Schmidt II.
7	20	Aug.	12	4	24	6	15	"	+25	Schmidt I.
8	"	"		"		7	03	"	+73	Schmidt II.
9	22	Aug.	17	4	04	5	41	"	+11	Schmidt I.
10	"	"		"		6	28	"	+58	Schmidt II
11	24	Aug.	22	3	44	5	21	"	+11	Schmidt I.
12	37	Sept.	23	13	34	8	57	Cambridge.	+17	H. C. O.
13	39	"	28	13	14	8	07	"	—13	"
14	43	Oct.	8	12	34	14	16.5	Athens.	+16.9	Schmidt.
15	47	"	18	11	54	13	33.6	"	+14	"
16	49	"	23	11	34	11	27	Greenwich.	+02	Knott.
17	51	"	28	11	14	12	44 0	Athens.	+ 4.4	Schmidt.
18	53	Nov.	2	10	54	12	31.0	"	+11.4	"
19	"	"		"		11	0	Greenwich.	+15	Knott.
20	55	Nov.	7	10	34	11	55.3	Athens.	— 4.8	Schmidt.
21	59	"	17	9	54	11	29.3	"	+ 9.7	"
22	61	Nov.	22	9	34	11	5.7	"	+ 6.1	"
23	"	"		"		9	30±	Greenwich.	+ 5	Knott.
24	65	Dec.	2	8	54	9	0	"	+15	"
25	"	"		"		10	25.8	Athens.	+ 6.2	Schmidt.
26	69	Dec.	12	8	14	9	47 7	"	+ 8.1	"
27	79	Jan.	6	6	34	6	36±	Greenwich.	+11	Knott.
28	92	Feb.	7	16	24	10	18.2	Cambridge.	—12.0	H. C. O.
29	96	"	17	15	44	9	37 2	"	—13.0	"
30	98	"	22	15	24	9	17.2	"	—13.0	"
31	110	March	24	13	24	7	16.4	"	—13.8	"
32	112	"	29	13	04	12	45	Greenwich	—10	Knott.
33	114	April	8	12	44	12	24	"	—11	"
34	"	"		"		6	35.8	Cambridge.	—14.8	H. C. O.

All of the observations are compared in Table XIV. The columns give a current number, E or the number of minima which have elapsed since the discovery of the variability, the date, hour, and minute according to the approximate ephemeris used, and the observed minimum in mean time of the meridian of the observatory named in the fifth column. The last two columns give the correction to the ephemeris and the name of the observer. Observations made at the Harvard College Observatory are designated by H. C. O. Schmidt I. and Schmidt II. denote the two reductions referred to above.

For comparison with different ephemerides it will be convenient to group the observations of each observer, as has been done in Table XV. The observations of Dr. Schmidt in August according to his first and second reduction have been placed together, and also his later observations. The successive columns give a current number, the authority, the number of minima observed, and the mean value of E. The last four columns give the corrections in minutes to be applied to four ephemerides; that is, they equal the mean of the observed minus the computed value of each group according to the following four formulas: —

(A) Ep. E. = 1880 June 23^d 7^h $44.0^m + 2^d$ 11^h 50.0^m E.
(B) Ep. E. = 1880 June 23^d 10^h $13.1^m + 2^d$ 11^h 44.94^m E. + 0.04376^m E^2.
(C) Ep. E. = 1880 June 23^d 8^h $12.0^m + 2^d$ 11^h 49.6^m E.
(D) Ep. E. = 1880 June 23^d 7^h $41.0^m + 2^d$ 11^h 49.9^m E.

The first of these formulas (A) is extremely convenient, since the minutes repeat themselves every six minima. As the period differs from two days and a half by exactly ten minutes, the times of the successive minima may be written down directly. Every two hundred and forty minima, or every five hundred and ninety-nine days, the hours and minutes repeat themselves, so that the ephemeris can be easily extended over long periods. Whatever ephemeris is adopted, it may be more convenient to compute the minima by this formula first and apply the difference of the ephemerides as a correction.

Formula (B) is derived from the law proposed by Dr. Schmidt on page 90 of his article, reducing to Paris mean time and adopting June 23 as the starting-point, as in the other ephemerides. A minimum is assumed to have occurred on December 7 at 10^h 6.7^m, Athens mean time. The period at this time is taken as 2^d 11^h 50.812^m, with an increase of 0.08753 in each successive period. The ephemeris on page 91 is nearly, but not exactly, represented by this law. A part

of the discrepancy is due to an error by which the interval between the minima of October 3 and 6 is about five minutes too small. This will affect all the minima preceding or all of those following it. The number of decimal places employed by Dr. Schmidt has been retained, although the accuracy of the observations does not seem to justify it. Since we do not know the period within some tenths of a minute, it seems scarcely advisable to carry the result to thousandths of a minute in the formula used to represent it.

Formula (C) is proposed as that which best satisfies all the observations, if we admit that the period is invariable.

Formula (D) is that which best represents the later measures obtained at the Harvard College Observatory.

TABLE XV. — COMPARISON OF EPHEMERIDES.

No.	Authority.	No. Min.	E.	A.	B.	C.	D.
1	Glasenapp.	1	4	+22.0	—105.1	— 4 4	+25.4
2	Schmidt I.	5	20.0	+26.2	— 36.3	+ 6.2	+31.2
3	Schmidt II.	4	19.0	[+73.5]	+ 7.8	[+58.1]	+78.4
4	H. C. O.	2	38.0	+ 2.0	— 14.6	—10.4	+ 8.8
5	Schmidt.	9	55.9	+ 8.1	+ 1.3	+ 2 5	+16.7
6	Knott.	5	61.4	+ 9.6	+ 1.5	+ 6.2	+18.7
7	H. C. O.	1	92	—12.0	— 65.7	— 3.2	+ 0.2
8	"	1	96	—13.0	— 79.0	— 2.6	— 0 4
9	"	1	98	—18.0	— 85.7	— 1.8	— 0.2
10	"	1	110	—18.8	—133.9	+ 2.2	+ 0.2
11	Knott.	1	112	—10	—137.2	+ 6.8	+ 4.2
12	"	1	114	—11	—149.7	+ 6.2	+ 8.4
13	H. C. O.	1	114	—14.8	—158.7	+ 2.8	+ 0.4

An examination of the residuals from Dr. Schmidt's formula shows that this ephemeris alone satisfies all of his measures, if we admit his second reduction of his observations in August. It, however, entirely fails to represent the later determinations. The deviations exceed two hours, both in the Harvard College measures and in those of Mr. Knott When this ephemeris was published, these observations had not been made, and of course such a deviation could not have been foreseen. It is, however, remarkable that Dr. Schmidt should not have noticed the large discordance in the minimum observed by M. Glasenapp. This observation has especial value as a test of any ephemeris, since it is much earlier than any other measures. Dr. Schmidt's ephemeris would give a minimum at $11^h\ 32^m$, Moscow mean time, which is at once seen to be in error on inspecting Table XIII. or the original publication of M. Glasenapp. In fact, the reduction shows a correction to the time of minimum of over an hour and a half.

All the observations are fairly represented by formula (C), except the second reduction of those of Dr. Schmidt. His original reduction leaves a residual which might well be due to errors of observation. Although the residuals of the Harvard College measures are small, they are still much larger than their probable errors, and their values evidently indicate systematic error. The last formula satisfies these completely, giving average residuals of only 0.3 minutes, but does not agree with the other observations. If we admit a variation in the period, the value $2^d\ 11^h\ 49.9^m$ would seem to be that between $E = 90$ and $E = 114$, but not between $E = 0$ and $E = 90$.

It is scarcely worth while at present to discuss the relative probability of these various formulas, since further observations which will doubtless soon be made will serve to decide between them with certainty. If the original observations were published, so that all could be reduced according to the same method, doubtless much greater accordance would be found in the results. There seems to be no reason why the error in determining each minimum from observation during the decrease and increase should not be reduced to two or three minutes or much less than those of *β Persei*. Should the discrepancy of certain measures, as those in August of Dr. Schmidt, be confirmed, they would indicate the existence of some disturbing body which might also account for such a deviation as that noted in the minimum of *β Persei* on Nov. 22. No correction has been applied for the aberration. The star is so near the pole of the ecliptic that the correction would never exceed two minutes, and would be masked by the other errors.

Harvard College Observatory,
Cambridge, Mass.

Report of the Committee on Standards of Stellar Magnitude.

In selecting a series of stars as standards of stellar magnitude, it would obviously be impossible to choose those which should represent any assigned brightness. Stars could not be found which should have magnitudes of exactly, 1.0, 2.0, 3.0, etc. If the scale was made to conform to the stars, subsequent measures would be sure to show that its divisions were irregular. Moreover, an observer might have difficulty in determining fractions of a magnitude, if the light of all his comparison stars were expressed as integer numbers. A much more precise method seems to be, first, to select suitable stars as standards; secondly, to measure their relative light; and, thirdly, to express these measures in terms of any convenient scale of magnitudes that may be finally adopted. Subsequent measures will then serve to increase the accuracy with which this scale is defined, by determining more precisely the brightness of the comparison stars.

International coöperation is to be desired in order that the system recommended may be adopted by astronomers in all parts of the world. Accordingly, the Royal Astronomical Society and the Astronomische Gesellschaft were invited to aid in this work. A committee consisting of Messrs. Hind, Knobel, Knott, Stone and Christie was appointed by the Royal Astronomical Society, and Dr. Schönfeld was named as its representative by the Astronomische Gesellschaft. Unfortunately, the somewhat voluminous correspondence of your Committee has been delayed by the great distances to be traversed, and although the following plans are under consideration by the Committees named above, final action has not yet been taken. Stars may be conveniently divided according to their brightness into three classes:—

I. Lucid stars, or those brighter than the sixth magnitude. These stars will form the standards of comparison of the brighter variable stars, and in general for all observations made with the unaided eye or with an opera or field glass. Most of the photometric measures hitherto made relate to these stars.

II. Bright telescopic stars, from the sixth to the tenth magnitude. This class includes most of the catalogue stars, and will furnish the standards for the fainter variables. Meridian observations and those with small telescopes are in general directed towards these objects.

III. Faint telescopic stars, fainter than the tenth magnitude. Large telescopes are required for the convenient study of these stars. They will form convenient standards for the asteroids, for very faint variables, and for the components of clusters, etc.

It is proposed that the first of these classes be assigned to the Royal Astronomical Society, the second to the Astronomische Gesellschaft and the third to the American Association. In accordance with this scheme the following plan is recommended for the fainter stars.

The standard stars to be so selected that they will form twenty-four groups near the equator and at approximately equal intervals in right ascension. Each group to consist of a series of stars decreasing in brightness by differences of about half a magnitude, from the tenth magnitude to the faintest object visible in the largest telescopes. The groups to be located by bringing a star visible to the naked eye into the field of the telescope, waiting for two minutes, and then forming a chart of the zone ten minutes wide passing through the centre of the field of the telescope during the next four minutes. This zone will therefore be defined as the region from five minutes north to five minutes south of the bright star, and from two to six minutes following it. The stars to be selected from this zone, which may in some cases have to be extended. Care to be taken that no star is near enough to another to be sensibly affected in apparent brightness by its proximity. The following stars are proposed as leading stars for these groups: —

γ *Pegasi*, θ' *Ceti*, α *Piscium*, α *Ceti*, γ *Eridani*, α *Tauri*, ϵ *Orionis*, γ *Geminorum*, α *Canis Minoris*, ϵ *Hydræ*, α *Leonis*, θ *Leonis*, η *Virginis*, α *Virginis*, α *Bootis*, β *Libræ*, δ *Ophiuchi*, η *Ophiuchi*, η *Serpentis*, δ *Aquilæ*, θ *Aquilæ*, β *Aquarii*, α *Aquarii* and α *Pegasi*.

Two other groups formed of stars near the poles to be added to these, with which all may be compared, to avoid large systematic errors in different right ascensions.

The advantages of this system are that an observer in any part of the earth and at any season will find comparison stars conveniently situated for observation. Moreover, he will often be able to bring some of the standard stars into the field without moving the dome or reading the finding circles of his instrument. This is a great advantage when working with a large telescope with which alone the smaller stars can be observed. The leading stars will also form convenient standards in observing the others photometrically. For this reason none fainter than the third or fourth magnitude have been selected.

If the above plans are adopted, Dr. C. H. F. Peters will undertake the preparation of charts of the small zones. By the help of these the standards will be selected and their positions determined. Measures of their light will then, if desired, be undertaken at the Harvard College Observatory. It is greatly to be hoped that similar measures may also be made at some other Observatories, and if possible by different methods. The owners of very large telescopes are also invited to examine these regions and locate very faint stars which may be beyond the reach of the other instruments employed in this work.

When the measurements are completed the light of all the standards selected will be expressed in such a scale as may seem best. Any observer may then compare the scale he is accustomed to use with this, by estimating the light of a number of comparison stars. Uniformity may thus be secured where discrepancies amounting to several magnitudes now occur.

Respectfully submitted,

EDWARD C. PICKERING, Chairman.
LEWIS BOSS,
S. W. BURNHAM,
ASAPH HALL,
WILLIAM HARKNESS,
EDWARD S. HOLDEN,
SIMON NEWCOMB,
C. H. F. PETERS,
ORMOND STONE,
C. A. YOUNG.

[From the PROCEEDINGS OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Vol. XXX, Cincinnati Meeting, August, 1881.]

ON A CONVENIENT METHOD OF EXPRESSING MICROMETRICALLY THE RELATION BETWEEN ENGLISH AND METRIC UNITS OF LENGTH ON THE SAME SCALE. By WM. A. ROGERS and GEO. F. BALLOU, of Cambridge, Mass.

IF we adopt the relation between the yard and the metre given by Kater we have:

$$.001^{\text{cm}} = .0003937079 \text{ inch.}$$

The aliquot part of the inch which is nearest this value is:

$$\tfrac{1}{2500}^{\text{inch}} = .0004000000 \text{ inch.}$$

The difference between these values is .000006291 inch. If therefore two scales are ruled side by side, one of which is 1000 to the centimetre and the other 2500 to the inch, we shall have a coincidence at the 63rd line of the inch scale, or more exactly at line 62.57 if there is an exact coincidence between the first lines of the two scales. If the assumed relation between the inch and the centimetre is maintained in the graduation, there will also be coincidences at lines 125.1 — 187.7 — 250.3 — 312.8, etc.

Since there is a coincidence for sixty-three spaces, each having

the constant value .0004 inch, an error of one line in the measured coincidence corresponds to an actual error of sixty-five ten millionths of an inch, counting from the first line.

The following is a description of the ruled bar to which the test here described has been applied.

This bar was ruled for the use of the American Watch Co., upon a machine built for me by Mr. Chas. V. Woerd, the mechanical superintendent of the watch factory. By his kindness I am permitted to exhibit it at this meeting.

On Tuesday, Aug. 9, commencing at 9h. 10m. A. M., Mr. Ballou subdivided four inches into forty equal parts. At 9h. 40m. A. M. the machine was started at 2500 spaces to the inch, and at 11h. 20m. P. M. the space of four inches had been subdivided into 10000 equal parts. The ruling carriage was then set back for coincidence with the first line and the machine was started at 1000 spaces to the centimetre. At 1h. 0m. P. M. of Aug. 10, the space of one decimetre had been subdivided into 10000 equal parts. The carriage was then set back for coincidence with the first line and the decimetre space was again subdivided into 100 equal parts.

It was hardly to be expected that in the continuous work of the machine for more than twenty-nine consecutive hours the theoretical values of the coincidences should be maintained under the wide variation of temperature which took place between the day and the night. Moreover the tremors occasioned by the heavy machinery of the factory, and especially the blows of a power hammer which is not very far distant, were sufficient to account for a portion of the errors introduced. Nevertheless the matching of the (triple) lines representing $\frac{1}{10}$ of an inch with the corresponding lines of the band is nearly perfect. In the metric band there is at one point a deviation amounting to about one-half of the width of the lines, but the coincidence is soon recovered.

In counting the coincidences between the two bands, of which there are 158, the following method was pursued. After Mr. Ballou had completed the graduation, he arranged a simple carriage for facilitating the count. He carefully counted the coincidences and communicated the results to me. I then compared them with the computed numbers.

It is to be noted especially that Mr. Ballou had no means of knowing whether his count would agree with the actual number of

lines forming a coincidence. The deviations from the computed values are given in the following table, for each of the 158 coincidences.

+ 0	+ 2	+ 5	− 1	− 1	+ 0	− 2	− 2
+ 0	+ 4	+ 5	− 2	− 3	+ 0	− 2	− 2
+ 0	+ 4	+ 5	− 5	− 1	+ 0	− 2	− 2
− 1	+ 4	+ 6	− 5	− 2	− 1	− 4	− 1
− 1	+ 4	+ 5	− 5	− 2	− 1	− 2	− 2
+ 0	+ 5	+ 4	− 5	− 3	+ 1	− 2	− 1
+ 0	+ 4	+ 4	− 6	− 1	− 1	− 2	− 1
− 1	+ 4	+ 3	− 1	− 1	− 2	− 1	+ 0
+ 0	+ 4	+ 3	− 1	+ 0	− 2	− 3	+ 0
+ 2	+ 5	+ 3	+ 1	− 3	− 1	− 2	+ 1
+ 2	+ 5	+ 3	− 2	− 2	− 1	− 2	+ 1
+ 2	+ 5	+ 0	+ 0	− 1	− 2	− 2	+ 1
+ 3	+ 4	+ 2	− 1	+ 1	− 2	− 1	+ 1
+ 3	+ 5	+ 2	− 3	− 2	− 2	− 1	+ 1
+ 3	+ 6	+ 2	− 3	− 2	− 3	− 1	+ 1
+ 3	+ 5	− 3	− 4	+ 0	− 3	+ 0	+ 0
+ 4	+ 4	− 3	− 2	− 3	− 1	− 2	− 2
+ 1	+ 4	− 1	− 3	+ 1	− 2	+ 0	− 2
+ 2	+ 5	− 1	− 3	+ 0	− 3	− 2	....
+ 1	+ 5	− 2	− 4	− 1	− 3	− 3	....

It will be seen that the greatest cumulative error is near line 2000, the maximum value of the *difference* between the two values of the two units at this point amounting to about one thirty-thousandth of an inch. It is to be noted that the apparent periodicity of the numbers representing the coincidences does not necessarily indicate a periodicity in the screw. In fact, this method of comparison gives us no information on this point, since the coinciding lines in the two systems were ruled at the same part of the screw. Finally, the four inches of this bar are nearly standard at 62°, and the decimetre is nearly standard at the same temperature.

[Salem Press, May, 1882.]

16.

THIRTY-SIXTH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE, NOVEMBER 10, 1881,
AND LAID BEFORE THE BOARD OF OVERSEERS,
JANUARY 11, 1882.

CAMBRIDGE:
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1882.

THIRTY-SIXTH

ANNUAL REPORT

OF THE

DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF HARVARD COLLEGE.

1880–81.

To the President of the University:

Sir, — The present condition of the Harvard College Observatory furnishes a test of an opinion expressed in some of its recent annual reports, namely, that any addition to the resources of a scientific institution already supplied with the funds needed to keep it in working order will prove an advantageous investment. Such a sum is directly applicable to the extension of scientific inquiry. A similar gift to establish a new institution, or to support a feeble one, will be largely consumed in the mere preparation for such inquiry, and the head of the institution must spend much of his time in routine work, even if he is not required to devote a large part of his energies to teaching, as well as to research.

The proper division of labor, and the continuous use of apparatus, are advantages as important in science as in the arts. These advantages are possessed by a liberally endowed observatory as they are by the factory of a rich proprietor, who can afford any immediate expense required to secure the most abundant returns. It seldom happens that the endowment permits the use of the instruments to their full capacity. Until this point is reached, great results may always be expected from a slight increase of income.

The subscription raised in 1878 for the support of the Observatory for five years has given it for that time a much improved position for conducting researches. Many of the contributors to

the fund paid their subscriptions in full at the outset, and the Observatory thus acquired a large balance to the credit of its account with the University, which will of course progressively diminish until the five years have expired.

One of the most important advantages resulting to the Observatory from this temporary enlargement of its resources consists in the increased number of its assistants, which is now sufficient to allow the work of each to be more advantageously directed than was previously possible.

The observations with the large equatorial and meridian photometer are made by Mr. Searle, Mr. Wendell, and myself. Messrs. Cutler and Eaton assist in recording these observations and in their reduction. Miss Farrar and Mrs. Fleming also take part in the reductions. The meridian circle remains in the charge of Professor Rogers, who is aided in the observations by Mr. Brown, and in their reduction also by Mrs. Rogers and by Misses Saunders, Bond, and Winlock. The time-signals are in charge of Mr. Edmands, assisted by Mr. Chandler. These gentlemen and Mr. Howard are making the naked-eye observations of northern stars. The meteorological observations are ordinarily made by Mr. Searle, who also has charge of the accounts, correspondence, &c., and has devoted much time to work connected with the revision and publication of the equatorial observations. The compilation of the bibliography of the variable stars, and the computation of cometary orbits as noted below, are conducted by Mr. Chandler.

Three instruments, the equatorial, meridian circle, and meridian photometer, have been kept actively at work during the past year. The last of these instruments has proved so satisfactory that a large one is being constructed to carry on observations of stars too faint for the present instrument. The new photometer will have object-glasses four inches in diameter, and will be capable of measuring any star brighter than the tenth magnitude. It is expected that such measurements will form a part of the permanent routine work of the Observatory. The relation of a meridian photometer to other photometers resembles that of a transit instrument to a sextant. There are the same advantages of rapidity, ease of reduction, and probably of precision. On the completion of the new photometer we shall have the same advantages for measuring the light of any star that an observer would of

finding its position if he possessed the only transit instrument in existence. It is hoped that the use of this instrument will become more general when its advantages are appreciated. I understand that a distinguished German astronomer is planning a photometer for similar work, and at his request a description of the instrument used here has been forwarded to him.

The work of these various instruments will now be considered. That of the large telescope may be classified as below, with reference to the principal objects observed.

Eclipses of Jupiter's Satellites. — One hundred and fifty-three of these eclipses have been observed photometrically, sixty-four during the past year. One improvement has been introduced by which the number of settings may be nearly doubled. A single observer can make but three settings a minute, or one in twenty seconds. With an assistant to record, the time is reduced to about nine seconds. This time has been reduced to five seconds by the employment of two assistants, one of whom reads the photometer circle, while the other records and observes the time by the chronometer. As the observer does not remove his eye from the eye-piece, it is probable that the accuracy of the observations will be increased, and the satellite followed nearer to the point of disappearance.

Objects having Singular Spectra. — The search for these objects has been much interrupted by the pressure of other work. Only perfectly clear nights when the moon is absent are available, and these rarely occur at certain seasons of the year. The general survey of the heavens with a telescope of four inches aperture has been completed from twenty to twelve hours of right ascension. The observations during the summer on the remaining eight hours were so much interrupted by the long twilight and other causes that it was deemed best to reserve this work for another season. The most important result of this search has been the discovery of the peculiar spectrum of the star Ll. 13412. Three bright lines were found having wave-lengths of 545, 486, and 466 millionths of a millimeter. The second of these lines probably coincides with the F line of hydrogen. The other two were unexplained until Dr. Konkoly, in "The Observatory," IV. 257, described the spectrum of Comet 1881 III. (the Great Comet), as containing the five lines 560, 545, 515, 472, and 468. Three of these, the first, third, and fourth, are the usual cometary lines;

the other two appear to be coincident with the lines of Ll. 13412. Accordingly, while other comets have a spectrum identical with that of the stars of Secchi's fourth type, and apparently largely composed of carbon, this comet contains a substance as yet unknown, which one star only is as yet known to contain. The bright-line stars in *Cygnus* and in *Sagittarius* also give the line 467, but differ in the position of the other lines.

Another interesting discovery is that the star *L^2Puppis* has a banded spectrum. This star is more than forty-four degrees south of the equator, and at the time of observation was less than two degrees above the horizon.

Two stars were found, one in *Puppis*, the other in *Ursa Minor*, which from their color and spectrum were at once thought to be variables of long period. These predictions were verified by observation, and a very rapid method of discovering variable stars is thus suggested. An announcement of the last of these discoveries was telegraphed to the Astronomische Gesellschaft as an example of the cipher code referred to below.

The spectra of all the stars north of —40°, marked as red or colored in the Uranometria Argentina, have been examined in the large telescope. In most of them no peculiarity of spectrum could be detected. From the remainder, and from some miscellaneous sweeping for such objects, a list of about eighty stars having banded spectra has been compiled and published.

Variable Stars.—The photometric study of the stars *β Persei* and DM. 81° 25, proposed in my last report, has been carried out. The number of comparisons made of *β* and *ω Persei* was 2,748, six hundred during a single night. On this occasion the observations were continued almost without intermission from a quarter of seven in the evening to half past three in the morning. In like manner, 3,276 comparisons of DM. 81° 25 and 26 were made on five evenings. Nine hundred settings were obtained during one night. The probable error of a single minimum determined in this way is only about 1.3 minutes.

Comets.—This Observatory has taken an important part in the early distribution of circulars relating to the comets recently discovered, by defraying the cost of telegrams, and by furnishing the necessary observations and computations. In co-operation with the Dun Echt Observatory and the Science Observer, circulars have been distributed in Europe and America, giving an

early announcement of the elements and ephemerides of the five new comets discovered in the first ten months of 1881. The average interval between the time of announcement of the comet and the issue of the circular has been less than a week. The interval between the last of the observations needed to determine the orbit and the time of telegraphing the elements to Europe has been less than two days. The computations have been generally made by Mr. Chandler, who has not hesitated, whenever necessary, to continue them through the entire night. No important error has occurred in their telegraphic transmission to Europe, by means of the ingenious cipher code devised by Messrs. Chandler and Ritchie. These preliminary orbits proved sufficiently accurate for the purpose for which they were intended. That of the Great Comet of the year (Comet 1881 III.), was published within four days of the first appearance of the comet in the northern heavens, and agreed more nearly with observation than any other published elsewhere for some time. This agreement was due to the use of an early observation made at Rio. Barnard's Comet was not found in Europe until the publication of the ephemeris computed here. Denning's Comet was early suspected by Mr. Chandler to be periodic. Elliptic elements were computed by him; but a later mail showed that he had been preceded by Dr. Schulhof of Paris in this work.

The use of the large telescope in the observation of comets has not, in general, been allowed to interfere with the other work of this instrument, except when early observations were needed for the determination of their obits. Almost all of the observations were made by Mr. Wendell, many of them late in the night, when the telescope was not otherwise occupied. Comet 1880 IV. was observed on twenty-nine nights; Comet 1881 II., on two; Comet 1881 III., on forty-three; Comet 1881 IV., on fifteen; Barnard's Comet 1881, on ten; and Denning's Comet 1881, on eight.

Some miscellaneous measurements have also been made of these comets, particularly of Comet 1881 III. The form of this comet's tail was measured, and the brightness of portions of it at various distances from the head.

Meridian Circle. — During the past year observations have been made on 250 days. This number is distributed by months as follows: —

	Month.	No. Days.		Month.	No. Days.
1880.	November . . .	26	1881.	May	20
1880.	December . . .	21	"	June	18
1881.	January. . . .	24	"	July	22
"	February . . .	18	"	August	12
"	March	17	"	September . . .	23
"	April.	24	"	October	25

The whole number of observations made during the past year is as follows: —

		No. Obs.	Total No. Obs.
Polaris.	Upper Culmination	122 . . .	
Polaris.	Lower Culmination	110 . . .	232
Sun.	Vernal Equinox	31 . . .	
	Autumnal Equinox	45 . . .	
	North Solstice	13 . . .	
	South Solstice	15 . . .	158
	Fundamental Stars		1999
Polar Stars.	Upper Culmination	293 . . .	
	Lower Culmination	170 . . .	463
Miscellaneous	. . . .		350

The observations for the determination of the horizontal flexure by means of the collimators have been continued throughout the entire year as often as the atmospheric conditions have been favorable. The whole number of observations is fifty-eight. It was stated in the last report that the horizontal flexure for January of any year had been found to be about 0″.6 greater than for July of the same year. Since that time the values of this function derived from the fundamental stars observed have been computed for the years 1876 and 1877. We have now, therefore, sufficient data to determine the reality of this apparent variation. The values given below represent the difference between the value of the correction described as horizontal flexure for the mean of the months of January and December of any year and for the month of July of the same year both from the fundamental stars and from the collimators.

From the Fundamental Stars.		From the Collimators.	
	$\frac{\text{Jan.}+\text{Dec.}}{2}$ — July Δ R_4.		Jan. — July Δ R_4.
	″		″
1872	+0.64	1879	+0.73
1873	+0.39	1880	+0.56
1874	+1.09	1881	+0.48
1875	+0.73		
1876	+0.14		
1877	+0.71		
1878	+0.62		
Means	+0.62		+0.59

The action of temperature upon the screws by which the two halves of the tube of the telescope are joined to the cube of the circle is suggested as the probable cause of this difference. Observations with the long collimator for the determination of the variation of the instrumental constants of the instrument have been continued throughout the year. For the most part there are two series during each day on which stars have been observed. During the autumn and winter, the image of the concentric rings, or the focus of the collimator, remains remarkably steady; but during the summer, it has been found necessary to limit the observations to the evening and to the early morning hours. As a further check upon the variability of the index-error of the circle, some experiments were made with the reserve level of the Russian transit; this method, however, proved unsatisfactory. A level of the form invented by Mr. John Clark was ordered of Fauth & Co., and was mounted upon the cube of the telescope in May. The use of this instrument has been limited to the determination of the variability of the level of the telescope and of the index-error of the circle. A definitive opinion of the value of this method of detecting changes in the position of the zero of the circle must be deferred till the value of the index-error has been derived from the observations of Polaris. According to present indications the level gives somewhat better results than the long collimator. At present the observations are arranged for the detection of changes occurring between the morning and evening hours.

A decided improvement in the performance of the Standard Clock, Frodsham No. 1327, has been maintained since its removal to the cellar of the building, as is shown below in the account of the Time Service. The extreme variation of the thermometer placed within the room and read through a glass window from the outside has been 8°. Comparisons of this clock with the mean-time clock at Waltham have been continued as before by means of the line of telegraph erected for the purpose.

The investigation of the errors of the fixed circle of the telescope for each degree was completed in August. A metal plate having an arc of 15° subdivided to 5′ was used for the investigation. It was attached to the frame which carries the microscopes opposite the circle to be examined. The frame proved to be capable of revolution with no sensible disturbance of the focus of

the microscopes. Each 15° of the fixed circle was compared with the constant arc upon the plate. Two series, one in May and the other in August, show a good agreement. The maximum error of any space was found to be about 1″.4, and the accumulated systematic error for 180° does not exceed 1″.3. The errors of the single degrees were found by comparing each degree with the first degree of the metal plate. From the provisional reductions it appears that the accidental errors of division rarely reach 1″ and that with two or three exceptions they never exceed 1″.3. The investigation is to be completed by an examination of each space of 5′.

Observations of certain stars were made at the request of Professor Davidson of the United States Coast Survey, of Professor Hall of the Naval Observatory, and of Captain Tupman of the Greenwich Observatory. Two observations were made of Comet 1881 III., and a special series of 280 observations was made for the purpose of ascertaining whether the telescope moves in a vertical plane during its revolution.

The chronograph sheets for 1880–81 have been read off to July of the current year. Since the last report, considerable progress has been made in the reduction of the zone observations. In the preliminary reductions of the fundamental stars, the observations of a few stars were found to indicate persistently a disagreement with the fundamental positions of Publication XIV.

By comparing the separate values of $(\Delta T+m)$ and of the equator-point correction for each date with the mean values of these functions for the same date, a list of stars showing substantially the same discordances from year to year was made out. From this list twelve stars in right ascension and forty stars in declination were selected for special examination. A list of the resulting corrections to the positions in Publication XIV. was submitted to Professor Auwers, with a request for instructions in regard to the proper course to be pursued in this case. He has subsequently authorized, on behalf of the Council of the Astronomische Gesellschaft, the use of the provisional corrections already found. The actual determination of the definitive values of $(\Delta T+m)$ and of the equator-point correction has been unavoidably delayed by this inquiry.

The right ascension constants have now been determined for the entire series from 1870 to 1879, and the declination constants

have also been found for the same time, except for the year 1871–72. The computation for this interval is nearly completed.

The entire number of dates, for which the constants have been determined, is 1,263. The tabular reductions to apparent place and the refractions have been completed for the entire series of zone observations. The reductions have been made for intervals of twenty minutes in right ascension and two degrees in declination. The whole number of days for which these computations have been made is 633, and the number of individual reductions is about 12,000.

Meridian Photometer. — The work originally proposed for this instrument of measuring on three nights the light of each of the naked-eye stars visible in this latitude was essentially completed August 25, 1881. The observations will be continued during another year, as the necessary delay for reduction and publication will not thereby be greatly increased. Of the four thousand stars in our catalogue, about fifteen hundred have been selected for re-observation on three nights each. We shall thus obtain at least six observations of these stars, which include all those brighter than the fourth magnitude, all those used in the principal nautical almanacs as standards, and all those of which the first measures proved discordant. About seventy-six thousand measures have been made in all; thirty-six thousand since November 1, 1880.

From a discussion of the upper and lower culminations of a hundred circumpolar stars, the law of atmospheric absorption has been deduced, and the corresponding correction for each of the stars observed determined. All the readings have been reduced to magnitudes, corrected for the difference in transparency of the prisms, the mean taken when the stars are completed, the residuals derived, and the probable error deduced. This work has also been in a great measure checked and entered in the sheets of copy ready for publication.

A comparison of the photometric observations with those made by the naked eye is much to be desired. The Uranometria Argentina furnishes the means of doing this with the southern stars, but extends only to 10° north. Accordingly, all the stars in the Atlas Coelestis Novus of Heis north of the equator, and brighter than the sixth magnitude inclusive, are being measured by the eye, aided, when necessary, by an opera-glass. Each star is to be measured by three observers, who are to compare it with two stars

in the vicinity of the pole, one being a little brighter, the other a little fainter. The interval between the two is supposed to be divided into ten parts, and the brightness of the star to be measured is estimated in terms of this interval. The standard stars, one hundred and fifty-seven in number, include all stars brighter than the second magnitude, and those between 60° and 75° north, which are used in determining the atmospheric absorption. The total number of comparisons required for this work is about nine thousand, of which nearly a quarter have already been made.

Time-Signals. — The distribution of time-signals continued in charge of Mr. F. Waldo until June 1, and since then Mr. Edmands has assumed their care. Special attention is now given to keeping the error always small, instead of reducing it to zero at 10 A. M. By graphical methods, the accuracy of the determination of the error during long periods of cloudy weather has been much increased. It is believed that the error rarely exceeds two tenths of a second in clear weather, and four tenths when cloudy, and that the error seldom changes more than two tenths of a second from one day to the next.

In July, the mean-time clock, Bond 394, stopped, after sending the signals continuously for four years. It was kept in use, however, till October, when it was dismounted and thoroughly repaired. Meanwhile, from October 7 to October 21, the time-signals were sent by the clock of Messrs. W. Bond & Son, in Boston. It is hoped that hereafter it will not be necessary to take this course, as the sufficiently exact regulation of a clock so far from the Observatory proved to be a troublesome and laborious undertaking. In August, the standard sidereal clock, Frodsham 1327, was cleaned and removed to the new brick vault under the west Equatorial. For a period of twenty-five days, beginning August 30, during which it was left wholly undisturbed, its performance was remarkable both for the uniformity and minuteness of its rate. Its error did not vary two tenths of a second on the thirty-two occasions on which it was compared with the stars.

Various ingenious devices have been adopted by Mr. Edmands for use in the Time Service. Among others may be named those for making a single clock run two independent observing circuits, without the use of relays, and without interference of the observations. Spark arresters have been introduced, that on the

standard clock being a sounder which gives the breaks of the clock without those of the observer. Another sounder gives the time-signals uninterrupted by the five-minute break. Experiments are in progress with a view of improving the methods of distributing standard time. One of the most important of the proposed improvements is a contrivance by which the signals may be delayed by any desired fraction of a second without affecting the clock. An error in the signals may thus be corrected without interference with the pendulum. By this means many variations in the rate of the clock may be prevented, and its performance proportionally improved.

The time-ball has been dropped correctly on 322 days at twelve o'clock by telegraph. Owing to repairs to the roof of the building, it was not dropped during an interval of thirty-five days. On five days it was dropped at twelve o'clock by hand. On two days it failed to drop at twelve, and was dropped five minutes later, according to the rule; and on one day ice on the mast prevented it from being raised.

Publication. — Volume XIII. of the Annals is now in process of publication, and will be issued as soon as it can be got through the press. It will contain the results of the work with the large telescope, under the direction of the late Professor Winlock, and also the micrometric measurements made up to the present time. Among other subjects treated will be a discussion of the focal length of the telescope and the pitch of the micrometer-screw; measurements of double stars; observations of nebulae and of their spectra; satellites of Saturn, Uranus, and Neptune; satellites of Mars during the opposition of 1877 and 1879; positions of asteroids and comets. An appendix will be added, giving an important list of errata in the zone observations of Professor Bond, kindly furnished by Dr. C. H. F. Peters. As soon as this volume is completed the printing of Volume XIV. will probably be commenced. It will contain the measures made with the meridian photometer.

The papers mentioned below have appeared during the year as communications from officers of the Observatory: —

Variable Stars of Short Period. By Edward C. Pickering, Proc. Am. Acad., xvi. 257. Reprinted in The Observatory, iv. 238, 264, 284.

Large Telescopes. By Edward C. Pickering. Proc. Am. Acad., xvi. 364. Reprinted in Nature, xxiv. 389.

Photometric Measurements of the Variable Stars *β Persei* and DM. 81° 25, made at the Harvard College Observatory. By Edward C. Pickering, Director, Arthur Searle, and O. C. Wendell, Assistants. Proc. Am. Acad., xvi. 370.

On the Probable Period of Swift's Comet. By S. C. Chandler, Jr. Astr. Nachrichten, xcix. 45.

Comet III. 1869. By Edward C. Pickering. Astr. Nachrichten, xcix. 95.

Observations of Last Contact of Solar Eclipse. By Edward C. Pickering. Astr. Nachrichten, xcix. 107.

Elements of Pechüle's Comet (*f*) 1880. By S. C. Chandler, Jr. Astr. Nachrichten, xcix. 109.

The Comparison of Sirius. By Edward C. Pickering. Astr. Nachrichten, xcix. 219.

On Certain Zodiacal Phenomena. By Arthur Searle. Astr. Nachrichten, xcix. 91, 369.

Objects remarkable for their Colors or Spectra. By Edward C. Pickering. Astr. Nachrichten, xcix. 375.

The Coefficient of Safety in Navigation. By W. A. Rogers. Proc. U. S. Naval Institute, vii. No. 2.

A review of Vol. XI. Part II. of the Annals of the Observatory, by Dr. E. Lindemann, appeared in the Vierteljahrsschrift der Astr. Gesellschaft, xvi. 108. A review of the articles on Variable Stars appeared in the Crónica Científica (Barcelona), iv. 401. Other notices of the Observatory and its work will be found in Nature, xxii. 483, 611; xxiii. 321, 338, 604; in Science, ii. 33, 107, 141, 164, 329, 447, 482; in The Observatory, iv. 81, 113, 116, 154, 245; in Copernicus, i. 139, 140.

With reference to the article on large telescopes, it may be noted that should any addition be made to the equipment of the Observatory, a telescope, mounted according to the method there described, is recommended. It is believed that in no other way could the same amount of work be obtained with a given expenditure. With our present resources, however, it is much more desirable to keep the instruments we now have actively at work than to make any addition to their number.

An important piece of bibliographical work has been undertaken, and placed in charge of Mr. Chandler. The references to obser-

vations of stars of known or suspected variability have been collected, and those of each star brought together. This portion of the work is more than half completed. The comparison stars are then to be measured photometrically, and, finally, a reduction to a uniform system to be made of all the observations of variables of long period.

Various improvements have been made in the condition of the buildings and grounds. Two wooden balconies, having an area of seven feet by seventeen, have been constructed, to replace the small iron balconies to the east and west of the large dome. They are exceedingly convenient for observations made with small instruments or with the naked eye.

In order to connect the previous determinations of longitudes by the U. S. Lake Survey with an established meridian, it was proposed by the Superintendent of the Survey to determine the difference of longitude between Detroit and Cambridge. Facilities for the work were accordingly furnished by this Observatory, and the observations required were made by officers of the Survey. Later in the season, a course of experiments on the force of gravity, by means of pendulum observations, was conducted at the Observatory by officers of the U. S. Coast Survey, who were provided with similar facilities for their work. The West Equatorial was lent for the summer months to Professor S. P. Langley, who used it in observations made on the summit of Mount Whitney, in California.

There are some advantages in an early announcement of proposed investigations, even when the preliminary experiments are not sufficiently advanced to insure their execution. The aid and advice of other astronomers may thus sometimes be secured, and the danger of duplication of work is greatly reduced. No change in the character of the work is proposed, except that an attempt will be made to restrict the work of the large telescope more closely to large pieces of routine work. Such investigations are less likely to be made at observatories where the corps of assistants is smaller, and have a value not attained by detached observations. With the ultimate object of determining the distribution of the light in the spectra of the banded stars, a stellar spectroscope has been ordered of Mr. Hilger, of London. With this it is proposed to measure the position of the principal lines in the spectra of all the banded stars as yet known. A catalogue of these objects has

already been compiled. With the approval and assistance of the Selenographical Society, the photometric determination of the brightness of various points on the surface of the moon has been undertaken. The Society has furnished a list of regions suitable for standards, and the observations may be begun after the decision of some further preliminary questions. The most important of these relates to the age of the moon at which the comparisons should preferably be made. In co-operation with other astronomers, a system of standards of brightness for faint stars has been devised, and measurements of their light will be shortly undertaken.

The new meridian photometer will be used for measuring variable stars and their comparison stars, and also for determining the light of the brighter asteroids. An important piece of work is now under consideration, which would occupy this instrument for several years. During the revision of the Durchmusterung, undertaken by a number of co-operating observatories, and now approaching completion, each of the zones into which the entire region to be surveyed was divided, was somewhat extended northward and southward, so as to overlap the adjacent zones. The object of this extension was to connect the work of the different observers. Hence, the portions of the sky occupied by the overlapping borders of the zones offer a promising field for photometric research, as each determination may be compared with the estimates of magnitude made independently by two observers, with the Durchmusterung itself, with the zones of Argelander and Bessel, and with various other authorities. The number of stars to be measured is estimated at about six thousand, and the time required for the observations with the new meridian photometer would be at least three years.

EDWARD C. PICKERING, *Director*.

17

GEODETIC FORMULÆ.

SIMPLIFIED METHODS FOR

COMPUTING LATITUDE, LONGITUDE, AZIMUTH, AND DISTANCE.

WITH TABLES AND EXAMPLES.

BY

J. RAYNER EDMANDS.

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GEODETIC FORMULÆ.

I.

Read June 9, 1880.

THE most convenient way of stating the relative positions of a large number of points, either for record or for mapping, is to give their latitudes and longitudes. But other methods have been suggested, mainly to avoid the complicated calculations involved by the fact that the horizontal surface, upon which everything is supposed to be projected, is not plane but spheroidal. Believing that it is desirable to extend the use of latitudes and longitudes, rather than to provide substitutes, the writer has been led to inquire to what extent we may simplify the formulæ, for application to points, where the distances involved are small, and where the accuracy sought is less than that required in primary or secondary triangulation.

The problem is to find the latitude and longitude of a point, the distance and direction of which are given from a point of known latitude and longitude. Various methods have been used, adequate to the needs of the most careful survey; and it is customary for secondary triangulation to omit some of the refinements necessary for long primary lines; but the writer has seen no further simplification, proposed for application to tertiary stations.

Let $K =$ the given geodetic distance between two points; let L and $M =$ the given latitude and the given longitude of the first point; and let $Z =$ the given azimuth of the line passing from the first point to the second, counted from the south around by the west continuously up to 360°. Also let L_1, M_1, and $Z_1 =$ respectively the required latitude, longitude, and azimuth for the second point; let L_0 and $M_0 =$ respectively the latitude and longitude of a point midway between the two; and let $Z_0 =$ the azimuth of the line at that middle point toward the second point. Finally let

$\Delta Z = Z_1 - Z \pm 180° =$ the variation of the azimuth of the line in moving from the first point to the second. Distances are to be expressed in meters, and differences of latitude, longitude, or azimuth in seconds of arc. North latitude and west longitude are to be taken positive, and remarks as to sign apply only to the northern hemisphere.

According to the equation recommended by Prof. J. E. Hilgard for use in the secondary triangulation of the U. S. Coast Survey,[1] the difference in latitude between the middle point and the second point would be,

$$L_0 - L_1 = B_0 \frac{K}{2} \cos Z_0 + C_0 \frac{K^2}{4} \sin^2 Z_0 + D_0 (B_0 \frac{K}{2} \cos Z_0)^2$$

where B_0, C_0, and D_0 are values corresponding to the latitude of the middle point, and given in tables published by the Coast Survey. Similarly, for the difference in latitude between the middle point and the first point, we should have

$$L_0 - L = -B_0 \frac{K}{2} \cos Z_0 + C_0 \frac{K^2}{4} \sin^2 Z_0 + D_0 (B_0 \frac{K}{2} \cos Z_0)^2$$

The difference of these two equations gives, for the difference in latitude between the first and the second point,

$$L - L_1 = B_0 K \cos Z_0, \tag{1}$$

the device of resorting to the middle point having secured the elimination of the other terms. As B_0 and K are always positive, the sign of $(L - L_1)$ will be that cos Z_0.

For the difference in longitude between the first and the second point, we may use the formula recommended for the secondary triangulation of the Coast Survey,

$$M_1 - M = \frac{A_1 K \sin Z}{\cos L_1}, \tag{2}$$

where A_1 is a value corresponding to the latitude of the second point, and given in the tables. As A_1, K, and $\cos L_1$ are always positive, the sign of $(M_1 - M)$ will be that of sin Z.

Treating this similarly to the difference of latitude, we should have

$$M_0 - M_1 = \frac{A_0 K \sin Z_1}{2 \cos L_0}, \text{ and } M_0 - M = \frac{A_0 K \sin Z}{2 \cos L_0},$$

where A_0 is the tabular value corresponding to the latitude of the middle point. Subtracting, and taking $\frac{1}{2} (\sin Z - \sin Z_1) = \sin Z_0$, very nearly, we then have,

[1] Report for 1860, Appendix No. 36; again in Report for 1875, Appendix No. 19.

$$M_1 - M = \frac{A_0 K \sin Z_0}{\cos L_0}. \tag{3}$$

For the quantity ΔZ, the formula recommended for the secondary triangulation of the Coast Survey is

$$-\Delta Z = (M_1 - M) \sin \tfrac{1}{2} (L + L_1). \tag{4}$$

But we may write without sensible error, $\sin \frac{1}{2}(L+L_1) = \sin L_0$, and substituting also the value of $(M_1 - M)$ given by (3), we have very nearly

$$-\Delta Z = K \sin Z_0 \cdot A_0 \tan L_0. \tag{5}$$

The quantity given by (4) or by (5) is called *the convergence of the meridians.* According to (4) it equals the difference of longitude, multiplied by a factor corresponding to the latitude. The following table gives the values of this factor for six latitudes.

Latitude.	Factor.	Latitude.	Factor.
23° 35′	0.4	44° 26′	0.7
30° 00′	0.5	53° 08′	0.8
36° 52′	0.6	64° 09′	0.9

The sign of ΔZ is opposite to that of $(M_1 - M)$, so that half the result of (4) or (5) will be numerically subtracted from Z when the second point is farther west than the first, and added when the second point is the farther east.

The calculation may be made in the following manner. Estimate the mean latitude L_0 and also the difference in longitude $(M_1 - M)$. With these, estimate the convergence of the meridians by aid of the remarks following (5). Half of this convergence added to Z (or subtracted from it, as the case may be) gives an estimate of Z_0. With these estimates solve (5), using $\log A_0 = 8.509$, or taking its value from the tables which accompany this paper, when four places are desired. Half this result added to or subtracted from Z (as the case may be) gives a sensibly accurate value of Z_0. With this solve (1), consulting the tables for the value of B_0 corresponding to the estimate of L_0. Finally solve (2), taking A_1 from the tables. A map will aid in the estimates, even when one locates the point upon it by the eye.

When the distance is short, and the point is already mapped with a fair degree of accuracy, first solve (4), using L_1 and

M_1 as derived from the map, and then solve (1) and (3). The use of (3) instead of (2) will enable the constants and the trigonometrical functions of Z to be taken from tables, for (1) and for (3) simultaneously; but a new value of L_0 should be used in (3), derived from that of L_1 just calculated.

When the distance is long, and no good estimate of M_1 is to be had, first solve (2) or (3) approximately. With this as an estimate of M_1, proceed as directed at first. A case might arise calling for a similar preliminary solution of (1).

Should ΔZ be wanted for its own sake with greater accuracy, make a final solution of the equation,[1]

$$-\Delta Z = (M_1 - M) \frac{\sin \frac{1}{2}(L + L_1)}{\cos \frac{1}{2}(L_1 - L)}. \qquad (4_2)$$

The tables give the logarithms of A, the number of seconds per meter in length of arc of the prime vertical (or section of the earth perpendicular to the meridian), and of B, the number of seconds per meter in length of arc of the meridian. They are derived from the voluminous tables published by the U. S. Coast Survey,[2] applying the corrections there tabulated for converting from the Bessel spheroid to that derived by Col. A. R. Clarke, of the Royal Engineers, embodying the results of additional measurements. Bessel's terrestrial elements were considered to be subject to an uncertainty of nearly 500 meters in the quadrant, or 1 in 20,000, which would correspond to an uncertainty of about two units in the fifth decimal place of logarithms. This agrees with the correction made in the length of arc of meridian in low latitudes; but for latitude 60° the correction amounts to about six units in the fifth decimal place of the logarithms, which is also roughly that to which the length of arc of the prime vertical has been subjected throughout. From this it would appear that, except for special geodetic purposes, the use of the sixth place in the logarithms would be assuming a better knowledge of the dimensions of the earth than can be asserted to exist.

The arrangement of the tables is, perhaps, original. The values of the logarithm progressing regularly by five in the fifth place, the tables give not only the latitude corresponding

[1] Recommended for the primary triangulation of the Coast Survey.

[2] To eight decimal places in the Report for 1860, and to seven in that for 1875.

to each four-place logarithm, but also the limits of latitude within which the latter is the nearest four-place value. The third column gives the number of units difference in the fifth place of the logarithm per minute of latitude, to aid in interpolating. Thus, while especially adapted to four places, the tables are convenient for five, and available to six.

In regard to the accuracy of the formulæ, notice that (2) and (4) are identical with those recommended for the secondary triangulation of the Coast Survey, that (1) is at least as accurate as the one there recommended, and that the approximations by which (3) and (5) were obtained, and by which Z is taken equal to $Z + \frac{1}{2} \varDelta Z$, introduce errors which are inappreciable on account of the small magnitudes of $\varDelta Z$ and of the distances for which (3) is recommended. Moreover (4), used by the Survey, is itself an approximation.

It remains to consider the accuracy with which the method can be carried out. An error as large as 0°.2 in the estimate of L_1 (0°.1 in L_0) will not seriously affect the logarithms of A_1 and B_0 in the fifth place. An error of 2′ in the estimate of L_1 or of $\varDelta Z$ (1′ in L_0 or in $\frac{1}{2} \varDelta Z$) will hardly cause an error of 1″ in $\varDelta Z$ as obtained from (5), where the difference of longitude does not exceed a degree; and only half this error enters (1) or (3). Equation (2) involves no estimates. An error of 20″ in the value of M_1 derived from a map, will give $\varDelta Z$ by (4) within less than 20″, which gives Z_0 within less than 10″ for use in (1) and (3). This last is all that is required for short distances, besides being as accurate as is called for by the character of many of the angular observations. Equations (1) and (2) should be solved with five-place logarithms, when it is desired to be accurate to one or two hundredths of a second of latitude and longitude with values of $(L-L_1)$ and (M_1-M) ranging from 1′ to 15′.

The equations are readily adaptable to calculating the azimuth when the positions of the two points are given. An interesting case calling for such a solution arose in running the Canadian boundary in 1845, when parties began at each end of a line to cut through the forest a distance of more than a hundred kilometers (about sixty-four miles), the directions having been calculated by Mr. Airy, the Astronomer Royal.

Dividing (3) by (1) and solving for $\tan Z_0$, we have

$$\tan Z_0 = \frac{B_0}{A_0} \cdot \frac{(M_1-M) \cos L_0}{L-L_1}. \qquad (6)$$

To calculate, solve (4) and (6) independently, and take

$$Z = Z_0 - \tfrac{1}{2}\Delta Z \text{ (very nearly).} \qquad (7)$$

B_0 and A_0 are to be taken from the tables which accompany this paper.

This method is sufficiently accurate for most cases, especially if $(L-L_1)$ or (M_1-M) be small. When neither of these is small, a method previously proposed[1] by the writer is more accurate. An established formula for azimuth[2] may be put in the form

$$\cot Z = \frac{A}{B} \cdot \frac{L-L_1}{(M_1-M)\cos L_1} - (M_1-M) \cos L_1 \tan L \frac{\sin 1''}{2} \qquad (8)$$

Writing $\cot(z+\delta) = \cot z - \frac{\sin 1''}{\sin^2 z}\,\delta$ (where δ is an increment of arc), equating the terms corresponding in order in the two equations, and solving for δ, we have very nearly

$$Z = z+\delta \text{ (9), where } \tan z = \frac{B}{A} \cdot \frac{(M_1-M)\cos L_1}{L-L_1}. \qquad (10)$$

$$\text{and } \delta = \tfrac{1}{2}(M_1-M) \cos L_1 \tan L \sin^2 z. \qquad (11)$$

The nature of the last term of (9) may be exhibited by overlooking the distinctions between L and L_1 in (11), and substituting (4) therein. Thus we have approximately

$$\delta = -\tfrac{1}{2}\Delta Z \sin^2 z. \qquad (12)$$

To calculate, solve (10) and look out $\sin z$ at the same time with z; then solve (11); and finally (9). As $\sin^2 z$ is always positive, the sign of δ is the same as that of (M_1-M).

Some care is necessary in solving (7) or (9) to place Z_0 or z in the right quadrant *before* applying the second term. The accompanying table will aid.

Quadrant.	Z.	$L-L_1$.	M_1-M.
I.	0° to 90°	+	+
II.	90° " 180°	−	+
III.	180° " 270°	−	−
IV.	270° " 360°	+	−

[1] "Appalachia," Vol. II. p. 88. Cos L is there erroneously printed cot L.

[2] See "Tables and Formulas," Professional Papers of U. S. Corps of Engineers, No. 12, p. 96.

The second terms of (7) and of (9) are to be numerically added when the second point is farther west than the first, and numerically subtracted when it is the farther east. A mistake in the sign of the last term will have the same effect in (8) as in (9), so that the two cannot be used to check each other. But (7) and (9) may be so used, unless the line run nearly east and west, when $\sin^2 z$ will be too near unity to make any marked difference between the respective last terms.

The distance is also readily obtainable in terms of the latitude and longitude. Dividing (1) by B_0, multiplying (3) by $(\cos L_0 \div A_0)$, squaring both equations, adding them, and remembering that $\sin^2 z_0 + \cos^2 z_0 = 1$, we have nearly

$$K^2 = \left[\frac{L-L_1}{B_0}\right]^2 + \left[\frac{(M_1-M)\cos L_0}{A_0}\right]^2. \qquad (13)$$

The error of (13) will not exceed that due to making the calculation with five-place logarithms, except in the case of desiring the distance between very remote points.

The direct calculation of azimuth and distance, by means so simple, will sometimes save recourse to the solution of triangles, in which two sides and the included angle are the given data, — a form of solution which is relatively slow, besides employing quantities not always published. The failure of the method to apply in many instances will be due, not to inaccuracies in the simplified equations, but to the fact that the differences of latitude and longitude often are not known with an accuracy comparable to the distances and angles of the triangulation.

In conclusion we would justify the use of simplified formulas for the tertiary lines of a survey by quoting a remark of Capt. John Herschel's. In the course of a review[1] of Col. Clarke's recent work on "Geodesy," he says: "Let it be remembered that the accuracy insisted on in trigonometrical surveying operations and reductions is far greater than is required for fiscal, commercial, or what are commonly called practical purposes. The object of this exceeding accuracy is geodetical. Thus, for instance, no one would dream of sur-

1 "Nature," Vol. XXI. p. 608.

veying a small isolated island with such accuracy. A great part of the cost of a continental survey, therefore, has to be reckoned as sunk for the sake of ultimately learning more about the exact shape of the earth than we could at present see any direct utility in."

Ex. 1. *Latitude, Longitude, and Convergence of Meridians.*[1]

(*Given*) Agamenticus, $L = 43° 13' 23''.18$ $M = 70° 41' 31''.88$

(*Req'd*) Isles of Shoals, $L_1 = 42° 59' 13''.32$ $M_1 = 70° 36' 49''.23$

$[L-L_1] = +0° 14'\ 9''.86$ $[M_1-M] = -0°\ 4' 42''.65$

(*Given*) Agam. to I. of Shoals, $K = 26990^m.7$ $Z = 346° 16' 38''.5$

(*Req'd*) " " " $\Delta Z = +3' 13''.1$

Above values of L_1, M_1, and ΔZ, being unknown as yet, assume $L_0 = 43° 00'$ and $(M_1-M) = -100''$ roughly, whence $\Delta Z = -\frac{2}{3}(M_1-M) = +67''$ roughly, $\therefore \frac{1}{2}\Delta Z = +33''$

Whence for use in (5), $Z_0 =$ (about) $346° 17'$

Convergence by Equation (5).

Without regarding signs of each of these quantities, take ΔZ positive, since (M_1-M) is negative

log K	4.431
log sin Z_0	9.375
log A_0	8.509
log tan L_0	9.970
[log ΔZ]	2.285

Whence by (5), $\Delta Z = +193'' = +3' 13''$

$Z = 346° 16' 38''.5$

$\frac{1}{2}\Delta Z = \frac{1}{2}(3' 13'') = +1' 36''.5$

Whence for use in (1) $Z_0 = 346° 18' 15''$

L_1 being as yet unknown, use also in (1) the estimate $L_0 = 43° 00'$.

Latitude by Equation (1).

log B_0	8.51062
log K	4.43121
log cos Z_0	9.98747
[log $(L-L_1)$]	2.92930

$(L-L_1)$ positive since cos Z_0 is positive.

L	$43° 13' 23''.18$
$-(L-L_1=849''.77)=$	$-\ 14' 09''.77$
	$L_1 = 42° 59' 13''.41$

Longitude by Equation (2).

log A_1	8.50904
log K	4.43121
log sin Z	9.37515
	2.31540
log cos L_1	9.86422
[log (M_1-M)]	2.45118

(M_1-M) negative since sin Z is negative.

$(M_1-M) = -282''.60 = -4' 42''.60$

[1] Data furnished by Capt. C. P. Patterson, Supt. U. S. Coast Survey. See Hitchcock's "Geology of New Hampshire," Vol. III. Part iii. p. 375.

The differences of latitude and longitude of the Coast Survey figures exceed those just obtained by 0″.09 and 0″.05 respectively. This is accounted for by the fact that the former are based upon the Bessel ellipsoid. According to the table[1] given by Prof. Hilgard for converting arcs of the Bessel into those of the Clarke ellipsoid, 0″.084 and 0″.042, in this case, should be numerically subtracted respectively from the former to give the latter. The calculation, therefore, checks within 0″.01 of latitude and longitude, and within 1″ of back azimuth (Z_1), although the difference of latitude is greater than that for which five-place logarithms can generally be relied on to give this accuracy.

Ex. 2.[2] *Latitude and Longitude for Short Distances.*

(*Given*) Powderhorn,	$L = 42° 24' 02''.71$	$M = 71° 01' 30''.95$
(*Req.*) Boston State Ho.,	$L_1 = 42° 21' 27''.61$	$M_1 = 71° 03' 30''.00$
	$[L—L_1] = +0° 02' 35''.10$	$[M_1—M] = +0° 01' 59''.05$
(*Given*) Powderhorn to B. St. Ho.	$K = 5505^m.9$	$Z = 29° 39' 10''$

Above values of L_1 and M_1 being unknown as yet, estimate from a map[3] $L_0 = 42° 22'.8$ and $M_1 = 71° 04'.0$, whence $(M_1—M) = +149''$ and by (4) $\Delta Z = —.674 \times 149 = —100''$ $\therefore \frac{1}{2}\Delta Z = \quad —50''$

Whence for use in (1) and (3) $Z_0 = 29° 38'.3$

Latitude by Equation (1).

$\log B_0$	8.51067
$\log K$	3.74083
$\log \cos Z_0$	9.93910
$[\log (L—L_1)]$	2.19060

$(L—L_1)$ positive since $\cos Z_0$ is positive.

$(L—L_1) = +155''.10 = +2' 35''.10$

$\therefore L_0 = \quad 42° 22' 45''$

Longitude by Equation (3).

$\log A_0$	8.50906
$\log K$	3.74083
$\log \sin Z_0$	9.69419
	1.94408
$\log \cos L_0$	9.86847
$[\log (M_1—M)]$	2.07561

$(M_1—M)$ positive like $\sin Z_0$.

$M_1—M = +119''.02 = +1' 59''.02$

[1] U. S. Coast Survey Report for 1875, p. 367.

[2] Data from Report of U. S. Coast Survey, 1864, p. 168.

[3] Simeon Borden's Topographical Map of Massachusetts. 1844.

According to the Coast Survey tables already referred to, 0″.015 and 0″.018 should be numerically subtracted respectively from the Coast Survey values of $(L-L_1)$ and (M_1-M), to reduce them to the Clarke ellipsoid, before comparing our results with them. This apparently places the errors of the calculation between 0″.01 and 0″.02, but it would require the correct results carried to thousandths in order to tell by comparison whether our error actually exceeds or falls within 0″.01. Notice that the result will not be changed by a second solution with revised constants.

Ex. 3. *Azimuth by Equations* (4), (6), *and* (7).[1]

(*Given*) Mt. Pleasant, Me.,	$L = 44°\ 01'\ 35''.17$	$M = 70°\ 49'\ 00''.88$
(*Given*) Ragged Mount,	$L_1 = 44°\ 12'\ 43''.97$	$M_1 = 69°\ 08'\ 43''.54$
	$-0°\ 11'\ 08''.80$	$-1°\ 40'\ 17''.34$
	$[L-L_1] = -668''.80$	$[M_1-M] = -6017''.34$
(*Required*)	$Z = 260°\ 38'\ 52''.3$	$Z_1 = 81°\ 48'\ 41''.6$

$$\tfrac{1}{2}(L+L_1) = 44°\ 07'\ 09''.57 = L_0 \text{ very nearly.}$$

$\log \sin \frac{1}{2}(L+L_1)$	9.8427	$\log (B_0 \div A_0)$	0.00152
$\log (M_1-M)$	3.7794	$\log (M_1-M)$	3.77940
		$\log \cos L_0$	9.85606
$[\log \Delta Z]$	3.6221		3.63698
ΔZ positive since (M_1-M) is negative.		$\log (L-L_1)$	2.82530
$\Delta Z = +4189'' = +1°\ 09'\ 49''$		$[\log \tan Z_0]$	0.81168

$$Z_0 = 261°\ 13'\ 46''$$
$$\mp \tfrac{1}{2}\Delta Z = \mp 0°\ 34'\ 54''$$
$$Z = 260°\ 38'\ 52'' \qquad Z_1 = 81°\ 48'\ 40''$$

The data are not sufficiently precise to test the agreement of the calculation within a single second of the azimuths given by the Coast Survey.

Ex. 4. *Distance by Equation* (13).[2]

$L =$	$43°\ 13'\ 23''.18$	$-\frac{1}{2}(L-L_1) =$	$-424''.88$
$L-L_1 =$	$+849''.77$	$\therefore L_0 =$	$43°\ 06'\ 18''$
$\log (L-L_1)$	$= 2.92930$	$\log (M_1-M)$	$= 2.45118$
$\log B_0$	8.51061	$\log \cos L_0$	9.86338
	4.41869		2.31456
	2	$\log A_0$	8.50904
$\left[\log \left(\frac{L-L_1}{B_0}\right)^2\right]$	8.83738		3.80552
[Gauss]	0.02505		2
$[\log K^2]$	8.86243	$\left[\log \left(\frac{(M_1-M)\cos L_0}{A_0}\right)^2\right]$	7.61104
$\log K$	$= 4.43121$		8.83738
		[Argument for Gauss.]	8.7737

[1] Data from Coast Survey Report for 1853, p. 20*.

[2] Data, the Coast Survey latitude of Agamenticus, and the logarithms of the differences of latitude and of longitude, calculated in Ex. 1.

The logarithm of K^2 is derived directly from the logarithms of the separate terms by Gauss's "Addition-Logarithms." When unprovided with such tables, the natural numbers must be taken out for each term, and added to get K^2. The calculated five-place value of $\log K$ agrees with that used in Ex. 1.

In the foregoing examples brackets have been used when there has been occasion to denote the meaning of *calculated* figures other than the results sought.

CURVATURE OF TERRESTRIAL ARCS IN SECONDS PER METER.

A = Curvature of Prime Vertical.

Latitude. ° ′	Log A.	Diff. per 1′.
23 07	8.50950	−.032
25 42	945	.034
28 07	940	.036
30 24	935	.038
32 35	930	.040
34 41	925	.041
36 44	920	.041
38 45	915	.042
40 44	910	.043
42 41	905	.043
44 37	900	.043
46 34	895	.043
48 31	890	.042
50 29	885	.042
52 29	880	.041
54 31	875	.040
56 36	870	.039
58 44	865	.037
60 58	860	.035
63 19	855	

Clarke's values for the Earth's Semi-axes.

Equatorial = 6 378 206.4 meters.
Polar = 6 356 583.8 "

B = Curvature of Meridian.

Latitude. ° ′	Log B.	Diff. per 1′.
23 03 −	8.51200	−.094
23 56	195	.096
24 48	190	.099
25 38 +	185	.101
26 28 −	180	.103
27 16	175	.106
28 03 +	170	.107
28 50	165	.110
29 35 +	160	.111
30 20 +	155	.113
31 05 −	150	.114
31 48 +	145	.115
32 32 −	140	.117
33 14 +	135	.118
33 57 −	130	.119
34 38 +	125	.121
35 20 −	120	.122
36 01 −	115	.122
36 42 −	110	.124
37 22	105	.125
38 02	100	.125
38 42	095	.126
39 22 −	090	.126
40 01 +	085	.127
40 41 −	080	.127
41 20	075	.128

The mean value of + or − after the minutes of latitude is ⅓ of 1′, e.g. 3′ − and 38′ + respectively equal 2′ 40″ and 38′ 20″ more nearly.

CURVATURE OF TERRESTRIAL ARCS IN SECONDS PER METER.

Latitude.	Log B.	Diff. per 1'.	Latitude.	Log B.	Diff. per 1'.
° ′		−.128	° ′		−.124
41 59	8.51070		53 06+	8.50985	
		.127			.123
42 38+	065		53 47	980	
		.129			.123
43 17	060		54 28−	975	
		.128			.121
43 56	055		55 09	970	
		.128			.121
44 35	050		55 50+	965	
		.129			.119
45 14−	045		56 32+	960	
		.128			.118
45 53−	040		57 15−	955	
		.128			.116
46 32−	035		57 58−	950	
		.129			.115
47 10+	030		58 41	945	
		.128			.114
47 49+	025		59 25	940	
		.128			.112
48 28+	020		60 10−	935	
		.127			.110
49 08−	015		60 55	930	
		.127			.109
49 47	010		61 41	925	
		.127			.107
50 26+	005		62 28−	920	
		.126			.104
51 06	.51000		63 16−	915	
		.126			.103
51 46−	.50995		64 04+	910	
		.124			.100
52 26	990		64 54+	905	
		.124			

II.

Read May 11, 1881.

In a preceding paper simplified methods were suggested for calculating the latitude and longitude of a subordinate point (a tertiary station, for example) when its distance and direction from a known point are given. Here a ready method is presented for obtaining the desired quantities without first knowing the distance, the data being the latitudes and longitudes of two or more occupied stations and the azimuths from them to the point sought.

The equations of the preceding paper may be so combined

as to give the latitude and longitude of the intersection of two observed lines. Again, they may be so combined, that for an assumed latitude we may calculate the longitude of a point lying on a given line of sight, or that for an assumed longitude the corresponding latitude may be calculated. When there are three or more lines we may calculate the position of the intersection of two of them, and the positions of neighboring points on the remaining lines, or (by obtaining at the start a close approximation for one co-ordinate of the station) we may calculate for each line the position of a point on the line and near the station. Whichever way we proceed, the object is to obtain the data for constructing a large-scale plot covering the vicinity of the station, with a portion of each line of sight represented upon it. On this plot the most probable position of the station is to be selected (by estimating it as nearly as may be), taking into account each plotted line with its due weight; and the co-ordinates of the point thus assumed are to be expressed in the form of latitude and longitude.

With three or more lines the method comprises an "adjustment" adequate to the purpose, although not possessing the precision of the method of least squares. When there are only two lines the advantage lies partly in the ease with which a revised position may be derived at a later date after other lines have been added, and partly in the fact that the length and direction of the base line are not required. Indeed the occupied stations may be tertiaries whose distances and directions from each other have never been calculated.

In a systematic survey one plot would be devoted to each station to be located, and all the plots, on sheets of uniform size, would be kept on file. The back of each would receive such facts, as the dates at which the respective lines were plotted, and at which the successive values for latitude and longitude were assumed as the lines accumulated, together with references to the places where the calculations could be consulted, and to the authorities for the data employed. Numerical values, already found and likely to prove convenient in future computations, would also be recorded on the backs of the plots to which they apply.

Using the notation of the previous paper with some omissions and additions, let L,M, L_1,M_1, and L_1',M_1' denote respectively the latitude and longitude of an occupied station, of the station whose position is sought, and of the origin of the plot, while L_0 denotes the latitude of a point midway between the stations. Let A and B denote the curvatures of the prime vertical, and of the meridian, for the latitude of the occupied station;[1] *i. e.* the number of seconds of arc per meter in length. Let A_1,B_1 and A_0,B_0 denote respectively corresponding quantities for the latitudes of the desired station and of the point midway on the line. (In practice we may use the mean latitude of the stations instead of the latter.) Let Z denote the observed azimuth of the line passing from the occupied station to the other, counted from zero at the south around by the west continuously up to 360°. Let ΔZ denote the *convergence of the meridians*, that is, the variation of the azimuth of the line in moving from the occupied station to that whose position is sought. Distances are in meters, and differences of latitude, longitude, or azimuth are in seconds of arc. North latitude and west longitude are positive, and remarks as to sign apply only to the northern hemisphere.

Before considering the calculations let us follow out the graphical work. The unit for plotting is to be some convenient length for the second of arc of meridian at the latitude of the origin of the plot. Five centimeters (about two inches) to the second of latitude is recommended. Simple rectangular co-ordinates may be used, by expressing the difference of longitude, not in seconds of the parallel, but as the number of seconds of the meridian measuring the same length; *i. e.* [ABSCISSA $= (M_1 - M_1')\ (B_1 \cos L_1' \div A_1)$]. Finely executed rectangular paper will prove very convenient for the plotting. The value of $[B_1 \cos L_1' \div A_1]$ is a constant for the particular sheet, and will be repeatedly used. Its logarithm, therefore, should be recorded on the back of the sheet when first found. The directions of the lines are to be constructed by tangents or cotangents of $[Z + \Delta Z]$, remembering that the meridian is the zero of azimuths.

When the collection of lines passing near the point is ex-

[1] See Tables.

hibited to the eye, we are concerned not so much with the intersections as with the perpendicular distances from the proposed position of the station to each of the lines. Thus, for three lines of equal weight, we should make these distances equal by selecting the centre of the circle inscribed in the triangle formed by the lines. In general the ideal solution would be to render a minimum the sum of products, formed by multiplying the square of each perpendicular distance by the weight of the line to which it is normal; but no calculations for this purpose are proposed.

If there was a signal on the station when observed, the weight may be diminished as the length of the line of sight increases. The same holds when an instrument of low precision was used without a signal. If an instrument of high precision has been used to observe a mountain summit without a signal, the weight is nearly independent of the distance, as such; but when under these circumstances the summit has been viewed with a considerable angular elevation or depression, the weight may be diminished as the vertical angle (positive or negative) increases in magnitude.

Having selected the position of the station on the plot, its co-ordinates with reference to the origin are to be measured. The abscissa is to be converted into seconds of longitude by dividing by the quantity $[B_1 \cos L_1' \div A_1]$, the value of whose logarithm will have been already found and recorded on the back of the sheet.

Combining equations (1) and (2) of the previous paper so as to eliminate K (the undetermined distance) and solving, we have for the difference of longitude

$$M_1 - M = (L - L_1) \frac{A_1 \sin Z}{B_0 \cos L_1 \cos (Z + \frac{1}{2} \Delta Z)} \qquad (14)$$

By analogy with (2) we may write (interchanging stations) $[M - M_1 = AK \sin (Z + \Delta Z \pm 180°) \div L]$. Combining this with (1) so as to eliminate K and solving, we also have for the difference of longitude

$$M_1 - M = (L - L_1) \frac{A \sin (Z + \Delta Z)}{B_0 \cos L \cos (Z + \frac{1}{2} \Delta Z)} \qquad (15_1)$$

For the abscissa to be plotted we have

$$(M_1-M_1')\,\frac{B_1\cos L_1}{A_1}=(M_1-M)\,\frac{B_1\cos L_1}{A}-(M_1'-M)\frac{B_1\cos L_1}{A},$$

whence taking $[M_1-M]$ according to (14)

$$\text{ABSCISSA}=(L-L_1)\frac{B_1\sin Z}{B_0\cos(Z+\frac{1}{2}\Delta Z)}-(M_1'-M)\,\frac{B_1\cos L_1}{A_1}\qquad(16)$$

Transposing (15_1) we have for the difference of latitude

$$L-L_1=(M_1-M)\,\frac{B_0\cos L\cos(Z+\frac{1}{2}\Delta Z)}{A\sin(Z+\Delta Z)}\qquad(15_2)$$

The basis for selection between (14), (15_1), and (16), when a longitude computation is to be made, will appear later. Neither would be applied, however, to a line running much nearer to the prime vertical than to the meridian, nor would (15_2) be applied for latitude to a line running much more nearly north and south than east and west.

Approximate latitudes may be used in entering the tables for the curvatures of terrestrial arcs, since their logarithms vary very slowly with the latitude. The ratio $[B_1\div B_0]$ occurring in (16) is nearly unity, and may be neglected in approximate calculations. Excepting for very long lines we may use for more accurate work the value

$$\log\frac{B_1}{B_0}=-\tfrac{1}{2}\,\varDelta\log B\,.\,\frac{L-L_1}{60}\qquad(17)$$

where $[-\frac{1}{2}\varDelta\log B]$ is half the magnitude of the "Diff. per 1′" tabulated[1] in the column following that of "Log B", where $[(L-L_1)\div 60]$ is the difference *in minutes* between the latitude of the occupied and the observed stations, and where the sign of $[\log(B_1\div B_0)]$ is that of $[L-L_1]$. Notice, however, that the unit of the tabulated value of $[\varDelta\log B]$ is in the fifth decimal place.

The value of the convergence of the meridians is to be taken according to the equation

$$-\varDelta Z=(M_1-M)\sin\tfrac{1}{2}(L+L_1)\qquad(4)$$

For approximate solutions a convenient and easily remembered value of $[\frac{1}{2}\sin\frac{1}{2}(L+L_1)]$ may be used for a section of country embracing several degrees of latitude. For exam-

ple, suppose that an error in $[-\frac{1}{2}\,\Delta Z]$ not greater than one second per minute of difference of longitude be allowable. Then between latitudes 34°.5 and 39°.3 we may use the relation $[-\frac{1}{2}\,\Delta Z = 0.3\,(M_1-M)]$, while the relation $[-\frac{1}{2}\,\Delta Z = 20''$ per minute of difference of longitude$]$ may be used between latitudes 39°.3 and 44°.6. *The sign of ΔZ is opposite to that of* $[M_1-M]$.

For the calculation of the longitude of the intersection of two lines let L', M', Z', etc., and L'', M'', Z'' etc, relate to the two lines, the first being that which runs the nearer to the meridian, while L_1, M_1, etc., relate to the intersection. Then we may write

$$M_1-M' = (L'-L_1)\,Q,$$

where $$L'-L_1 = (L'-L'') + (L''-L_1),$$ and where

$$Q = \frac{A' \sin(Z'+\Delta Z')}{B_0' \cos L' \cos(Z'+\frac{1}{2}\,\Delta Z')} = \frac{A_1 \sin Z'}{B_0' \cos L_1 \cos(Z'+\frac{1}{2}\,\Delta Z')}$$

according to (15_1) and (14) respectively. Whence

$$M_1-M' = (L'-L'')\,\frac{A' \sin(Z'+\Delta Z')}{B_0' \cos L' \cos(Z'+\frac{1}{2}\,\Delta Z')}$$
$$+ (L''-L_1)\,\frac{A_1 \sin Z'}{B_0' \cos L_1 \cos(Z'+\frac{1}{2}\,\Delta Z')}$$

In the last term we may put according to (14)

$$L''-L_1 = (M_1-M'')\,\frac{B_0'' \cos L_1 \cos(Z''+\frac{1}{2}\,\Delta Z'')}{A_1 \sin Z''}.$$

Whence finally we have

$$M_1-M' = (L'-L'')\,\frac{A' \sin(Z'+\Delta Z')}{B_0' \cos L' \cos(Z'+\frac{1}{2}\,\Delta Z')}$$
$$+ (M_1-M'')\,\frac{B_0''}{B_0'}\,\frac{\sin Z' \cos(Z''+\frac{1}{2}\,\Delta Z'')}{\sin Z'' \cos(Z'+\frac{1}{2}\,\Delta Z')} \qquad (18)$$

In this we may get $[\log(B_0''\div B_0')]$ by a process similar to that of (17); namely, multiply the difference *in minutes* between the latitudes L' and L'' by half the tabulated "Diff. per 1'" of $[\log B]$, and give to the product the sign of $[L'-L'']$. A value of M_1 is needed, sufficiently close to give $\Delta Z'$ and $\Delta Z''$ by (4) within the errors of observation. A provisional value of M_1 must be used in the last term of (18). Let m_1 denote this value of M_1, while m_2 is the value resulting from the solution. Also let c be the coëfficient of $[M_1-M'']$ in

the last term. Then the error of m_2 will be c times that of m_1. The value m_2 should therefore receive a correction $\left[(m_2-m_1)\div\left(\frac{1}{c}-1\right)\right]$ to obtain which is but the work of a moment, the logarithm of c being already found. Carefully regard signs. Carry the logarithms for the last term of (18) as far as would be done were not the value of M_1 provisional.

Substituting $[\cos(Z'+\frac{1}{2}\Delta Z')=\cos(Z'+\Delta Z')]$, in the first term of (18), and in the second term $[\cos(Z'+\frac{1}{2}\Delta Z')=\cos Z']$ and $[\sin Z''=\sin(Z''+\frac{1}{2}\Delta Z'')]$, which are the less inaccurate as the lines approach the meridian and the prime vertical respectively, and substituting also $[B_0''\div B_0'=1]$ we have approximately

$$M_1-M'=(L'-L'')\frac{A'}{B_0'\cos L'}\tan(Z'+\Delta Z')$$
$$+(M_1-M'')\tan Z'\cot(Z''+\tfrac{1}{2}\Delta Z'') \qquad (19)$$

Suppressing $\Delta Z'$ in the first term of (19), substituting $[M_1-M''=(M_1-M')-(M'-M'')]$ in the second and solving, we also have approximately

$$M_1-M'=\frac{[(L'-L'')A'\div B_0'\cos L']+(M'-M'')\cot(Z''+\frac{1}{2}\Delta Z'')}{\cot Z'-\cot(Z''+\frac{1}{2}\Delta Z'')} \qquad (20)$$

The preceding equations present a choice of means by which to carry out the calculations; but in any case the work will be easier when we can start with a moderately close estimate. To this end a chart is desirable on which are located all the stations within the area it covers. A scale of 1÷50,000 might serve the purpose. The occupied stations would be laid down by latitude and longitude. The others might be thus laid down after the calculations are made, or they might be constructed by azimuths beforehand. By providing that the observers in the field should construct the azimuths during foggy weather, any failure to intersect, due to gross error in the data, would be discovered, and additional observations would be made; while the existence of the map would induce, and assist in, the search for some points whose visibility might otherwise be overlooked. The laying down of the adopted latitude and longitude of a station whose position

had previously been constructed by azimuths would also serve as a check against gross error in the figures. But these advantages are less important than the fact that the chart would furnish a close first estimate of the desired position. If the azimuths have not been laid off, a fair estimate may still be made by taking on the map a point, so located that the azimuths from the occupied stations shall satisfy the eye.

The accuracy of the formulæ, the admissibility of the assumptions involved in their application, and the use of the tables were so fully discussed in the preceding paper, that it is unnecessary to treat them here. The numerical work, although differing slightly from what was exemplified there, is so similar in character as to render an example of it unnecessary here. But the synopsis of an example will be useful to illustrate the successive steps of the process, and will furnish an opportunity for the reader to test the formulæ numerically. The data are given below.[1]

Station.	Latitude.			Longitude.			Azimuth.		
	°	′	″	°	′	″	°	′	″
Spencer	41	40	41.07	71	29	19.79	298	02	52
McSparran	41	29	44.71	71	27	03.81	240	18	11
Copecut	41	43	15.08	71	03	15.56	46	32	23
Beaconpole	41	59	40.37	71	26	40.36	340	25	41
Pocasset	41	39	07.23	71	11	11.83	33	57	13
Great Meadow . . .	41	52	43.01	71	12	41.29	5	27	51
Quaker	41	34	55.17	71	14	57.32	. . .	. . .	. . .

The first column, headed *Station*, gives the names of six occupied stations, followed by that of the station whose position is desired. In the second and third columns, headed *Latitude* and *Longitude*, are placed the positions of the occupied stations, which are supposed in the example to be given, and also the position of Quaker, which is supposed in the example to be unknown. The last column, headed *Azimuth*, gives the azimuths of Quaker as observed at the six occupied stations. It is to be noticed that a strict accordance of our result with the tabulated position of Quaker is not to be ex-

[1] From U. S. Coast Survey Report, 1851, Appendix No. 12. Azimuths carried only to seconds in copying.

pected, because the above are not all the data bearing upon its position, and also because our computations will be based upon figures for the dimensions of the earth which had not been so accurately obtained as early as 1851. The azimuths, however, are more reliable than will ordinarily occur, since all the lines belong to the primary triangulation.

In order that the points determined by either of the equations (14), (15), or (16) shall lie within the boundaries of the plot, we desire if practicable to have beforehand a close approximation to one of the co-ordinates of Quaker, say within a second of latitude or one or two seconds of longitude. If Quaker had been laid down on a chart with a scale as large as 1 : 100 000, by constructing azimuths, this approximation would be given by direct measurement, although it is not best to make the construction for this purpose only. Possessing even a chart on a scale of 1 : 200 000, with the occupied stations located on it, and avoiding any graphical construction, a mere inspection of the observed azimuths will enable us to select the position within ten or twenty seconds. This will materially help, especially with a line running so near the meridian as does that from Great Meadow, and intersected so favorably as it is by the line from Spencer. Thus starting with $[M_1 = 71° \, 14' \, 40'']$, taking ΔZ equal (for the respective lines) to forty times the number of minutes of $[M_1 - M]$ with the sign reversed, and solving (19) with four decimal places in the logarithms, we obtain $+92''.0$ for the first term and $+45''.0$ for the second, or $[M_1 - M' = +137''.0]$ where M' refers to Great Meadow. Whence we get $[M_1 = 71° \, 14' \, 58''.3]$ within $1''$ of the tabulated value. We are now in a position either to apply (4) and (18) for the precise longitude of the intersection, or (better) abandoning the determination of this, to apply (15_2) to either of the lines from Spencer, McSparran, or Copecut.

Let us suppose, however, that we have neither the map estimate, nor so favorable a pair of lines as those from Great Meadow and Spencer. We then start with a rough estimate of longitude to get $[\frac{1}{2}\Delta Z'']$, and apply (20) to the lines from Beaconpole and McSparran. Assuming roughly $[M_1 = 71°.17']$ we thus calculate $[M_1 - M' = -705''.2]$ where M' refers to

Beaconpole. Whence [$M_1 = 71° 14' 55''.2$] or about $2''$ away from the tabulated figure. The application of (4) with this value of M_1 will give [$\frac{1}{2}\Delta Z'$] and [$\frac{1}{2}\Delta Z''$] within $1''$. The precise longitude of the intersection may be then obtained by solving (18). We thus obtain [$M_1 = 71° 14' 57''.83$] subject to the correction [$-0''.44$], giving [$57''.39$]. The latitude of the intersection would then be calculated by applying (15_2) to the McSparran line; and the position thus found would be taken as the origin when the other lines are to be calculated and plotted.

Let us return to the use of (14), (15), and (16). For longitude computation, after the origin has been fixed upon, (16) has the advantage of giving the distance to be taken on the scale in plotting, but (14) or (15_1) may be used to calculate a longitude which is to be assumed as that of the origin. Of these (14) is the less dependent upon accuracy in ΔZ; but (15_1) has the advantage when the errors of observation are liable to be larger than the uncertainty in ΔZ, since the result may be made to conform to an amended latitude by merely changing the logarithm of [$L - L_1$]. In this manner the use of (18), (19), or (20) may be avoided. Thus, suppose that a map has furnished the estimates [$L_1 = 41° 34' 58''$ and $M_1 = 71° 14' 55''$], differing several seconds from the fact, and suppose that the observations are hardly good to single seconds. The estimates being therefore good enough for obtaining [$\frac{1}{2}\Delta Z$] by (4), the value [$M_1 = 71° 14' 56''.99$], obtained by applying (15_1) to the Great Meadow line, will be sufficiently accurate for the longitude of the point (on that line) whose latitude is [$41° 34' 58''.00$]; but this latitude will be beyond the convenient limits of a plot. With this value of M_1 we should then solve (15_2) for the Spencer line, obtaining [$L_1 = 41° 34' 55''.02 = L_1'$]; *i. e.* it is taken as the latitude of the origin. We may now quickly amend the result of (15_1) for the Great Meadow line in accordance with the new value of L_1, obtaining [$M_1 = 71° 14' 57''.67 = M_1'$]; *i. e.* it is taken as the longitude of the origin. The point on the Great Meadow line will then lie upon the origin, while the point on the Spencer line will have an ordinate zero and an abscissa

[56.99—57.67 = —0.68] in seconds, or [—0.68 (B_1 cos$L_1' \div A_1$) = —0.51] in units of the plot. It will always be found convenient to arrange that every point shall lie upon one or other of the co-ordinate axes.

Enough has been written to show that the method can be planned to easily deal with any case which may arise. In conclusion it may be remarked that the length of the discussion has been caused by this adaptability, rather than by any complexity in dealing with a case in hand. The method is possibly less fitted in general for routine calculations, done according to specific directions issued from the headquarters of a large survey, than it is for the use of persons who have their own computations to make in extending the results of the larger survey. But where a routine is made of constructing the azimuths and measuring the resulting positions on a chart, the dependence upon the judgment of the computer reduces to a minimum, and the operation, except the final selection of the position on the plot, becomes a simple routine. In comparing the aggregate work with that by the old process, it must be remembered that the whole work of computing the triangle sides is saved.

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A PLAN

FOR

SECURING OBSERVATIONS

OF THE

VARIABLE STARS.

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A PLAN FOR SECURING OBSERVATIONS OF THE VARIABLE STARS.

FOR several reasons the investigations here proposed are especially suited to observers under very various conditions. The work is capable of indefinite sub-division. Small as well as large telescopes may be employed and many observations are needed which can best be made with an opera-glass or field-glass, or even with the naked eye. No attachment is needed to an ordinary telescope, so that no additional expense on this account is required. Useful observations may be made by an unskilled observer provided that he is capable of identifying a star with certainty. The work is quantitative, and the observer has, therefore, a continual test of the increased accuracy he has acquired by practice. As a portion of the investigation will probably lead to the discovery of interesting objects, the observations will possess an interest often wanting in quantitative research. The aid of the professional astronomer is earnestly requested for this scheme. Suggestions by which it may be modified and improved will be gratefully received. The professional astronomer, in consequence of his greater skill, instrumental appliances, and command of his own time, could fill gaps in the work, and thus greatly increase its value as a whole. Such observations could often be made in the intervals of other work or at times unsuitable for the observations to which he was especially devoting himself. It should be added that especial

care will be taken not to interfere with observations of variable stars now in progress. Observers of these objects are particularly requested to notify the writer what work they propose to carry out, so that a needless repetition of it may be avoided.

It is on the amateur and student of astronomy that we must depend largely for the success of the plan here proposed. Many such persons spend evening after evening at their telescopes without obtaining results of any permanent value. Either no publication is made and the results are therefore valueless, or time is spent on objects that can be much more usefully examined with a larger instrument. Most commonly the observer has no special plan and spends many hours without result, while the same time might have been employed with equal pleasure to himself and results of great value collected. Those who have not tried it do not realize the growing interest in a systematic research and the satisfaction in feeling that by one's own labors the sum of human knowledge has been increased.

Much valuable assistance might be rendered by a class whose aid in such work has usually been overlooked. Many ladies are interested in astronomy and own telescopes, but with two or three noteworthy exceptions their contributions to the science have been almost nothing. Many of them have the time and inclination for such work, and especially among the graduates of women's colleges are many who have had abundant training to make excellent observers. As the work may be done at home, even from an open window, provided the room has the temperature of the outer air, there seems to be no reason why they should not thus make an advantageous use of their skill. It is believed that it is only necessary to point the way to secure most valuable assistance. The criticism is often made by the opponents of the higher education of women that, while they are capable of following others as far as men can, they originate almost nothing, so that human knowledge is not advanced by their work. This reproach would be well answered could we point to a long series of such observations as are detailed below, made by women observers.

Variable stars may be defined as those which exhibit a varying degree of brightness at different times. The following classification of them is believed to be a natural one. (Proc. Amer. Acad. xvi, 1, 257.)

I. Temporary stars, or those which shine out suddenly, sometimes with great brilliancy, and gradually fade away. Examples, Tycho Brahe's star of 1572, new star in Corona, 1866.

II. Long period variables, or those undergoing great variations of light, the changes recurring in periods of several months. Examples *o Ceti* and *χ Ceti.*

III. Stars undergoing slight changes according to laws as yet unknown. Examples, *a Orionis* and *a Cassiopeiæ.*

IV. Short period variables, or stars whose light is continually varying, but the changes are repeated with great regularity in a period not exceeding a few days. Examples, *β Lyræ* and *δ Cephei.*

V. Algol stars, or stars which for the greater portion of the time undergo no change in light, but every few days suffer a remarkable diminution in light for a few hours. This phenomenon recurs with such regularity that the interval between successive minima may be determined in some cases within a fraction of a second. Examples *β Persei* (Algol) and S *Cancri.*

Stars belonging to the first of these classes are seen so rarely that the apparent discovery of one is to be received with the utmost caution. On the other hand, the importance of early observations of such an object is so great that no pains should be spared to secure an early announcement if one is really found. On the best star charts many stars are omitted of the brightness of the faintest objects given. But any star much brighter than these should be measured by the method given below, and a watch kept to see if any change takes place. If it proves to be a temporary star an immediate announcement should be made. If a telegram is sent to this Observatory the object will be at once examined, and, if verified, notification will be made in this country and in Europe with the name of the discoverer or sender of the telegram. A similar notification may be sent of any sus-

pected objects, which will be examined in the same way, and announced at once if they prove to be of interest. It is essential that the position of the object should be given with all the precision practicable, and that a letter should be sent by the next mail giving the observations in detail. This often proves of the greatest value in case the object is not readily found. It also serves to establish the claims of the first discoverer.

Nearly three quarters of the known variables belong to the second class. Most of them undergo very large changes of light, and may therefore be observed with comparative ease. Our knowledge of their variations is however very defective. Hitherto the attention of observers has been directed principally to determining the times at which they attain their maximum light, while their light at intermediate times has been neglected. It is now proposed to secure observations of these objects once or twice in every month, so that their light curves or variations throughout their entire periods may be determined. Again, many observers are accustomed to state their brightness in magnitudes without giving any clue to the scale which they employ. In most cases such observations have little value owing to the uncertainty of the scale of the fainter magnitudes.

According to Dr. Gould and some other observers most of the visible stars undergo slight changes of light and should therefore be assigned to the third class of variables. It is probable that our Sun also belongs to this class, as it is not likely that its light is the same during the maximum and minimum of the sun spot period. At present we are unable to tell in which case the light would be greatest. It by no means follows that when the spots are most abundant the Sun's total light is least, for the remaining portions of the Sun may then have an increased brightness more than compensating for their diminished area. As long as the suspected variations in light of the stars are small, not exceeding half a magnitude for instance, they seem in the present state of science to have comparatively little interest. They are so liable to be affected, or even caused, by errors of observation, that the observation of such objects does not seem

now to be advisable. Doubtless many such so-called variables are really due to errors caused by moonlight, the proximity of brighter stars, varying position of the images on the retina of the observer, and other similar causes. They will not therefore be considered further in this paper.

The stars of the fourth class as compared with the second are relatively few in number, and the changes in light small. While many of them need observation, especially to determine their light curves more precisely, it is advised that this work be left to those who have acquired a high degree of skill in these observations. That the work may be of value it is essential that the errors should be extremely small. As, however, nearly all are visible in an opera-glass, a skilful observer unprovided with a telescope may secure valuable results by their observation. This remark applies with especial force to many of those discovered in the southern heavens by Dr. Gould.

The phenomena of the Algol stars are in many respects the most striking of any. The rapidity of the changes, their surprising regularity, and the comparative rarity of these objects, combine to render the discovery of each new one a matter of unusual interest. As in the case of stars of the fourth class, however, the study of their light curves should be left to those who have acquired especial skill in this work. This is particularly desirable, when, as in this case, the unaided eye enters into competition with photometric apparatus, by which, as some think, it should properly be altogether replaced.

An elaborate bibliographical work on the variable stars has been undertaken at this Observatory by Mr. Chandler. It will include the collection of all available published observations of known or suspected variables. A catalogue of suspected variables has thus been prepared, doubtless containing many stars which are really important variables. But it is also likely that many objects have been introduced in the list by errors in the original observations. Such stars often appear in one catalogue after another of suspected variables, and it is difficult to prevent the continued circulation of such an error. Of course if an

experienced observer at any time estimates a star as above or below its normal brightness, it is impossible to prove that the observation was not correct, and the star really variable. No amount of subsequent observing could prove that it had not then, and then only, an abnormal brightness. We can, however, prove that in all probability it does not belong to one or more of the above classes, and thus make it more and more probable that the observation is due to an error. If the star varies in light by one magnitude, what will be the chances that we shall get a series of observations having a range of variation of one fifth of a magnitude? Evidently on the average, there will be only one chance out of five that any observation shall fall in the same fifth of a magnitude as another. The chances for three such observations will be only $\frac{1}{25}$, and for four $\frac{1}{125}$, etc. These ratios expressed decimally are .2, .04, .008, 0016, .0003, etc. Since the separate determinations of the light of a constant star by the method given below should not differ more than two or three tenths of a magnitude, it is obvious that if the variations of the star are large, a few observations would generally establish this fact. If the star belongs to class four, observations on half a dozen evenings would hardly fail to show the variation. Conversely, if no such variation is detected we may be almost certain that the star is not a variable of that class, or at least that the variation, if any, is not large. If the star belongs to class two, it will change so slowly when near its maximum or minimum that a variation might not be noted if the observations are near together. An interval of several months should therefore be allowed to take place, or perhaps it would be better to wait until the star is again visible the following year. The total variation in light is usually so great in these stars that the change will often be visible at the first glance.

To prove that a star does not belong to the fifth class is a matter of much greater difficulty. In fact it is almost impossible to prove that it may not be an Algol star with a long period between the minima. Since these stars may have their full brightness for nine tenths of the time, it is obvious that they

may be examined again and again without happening to be seen at the time of a minimum.

On the other hand, during a considerable portion of the time when it is varying, the light will be so much less than usual that a careful measurement is not needed to detect the change. Moreover, it will be useless to look for an increase of light, and the observation may be so planned as to detect a diminution only.

If we assume that only during one tenth of the time the change in light will be sufficient to be perceptible, the chance on any given evening will be 9 out of 10 or $\frac{9}{10}$ that the star will have its full brightness. For two evenings the chance will be $(\frac{9}{10})^2$ for three $(\frac{9}{10})^3$, etc. These quantities expressed decimally are .9, .81, .73, .66, .59, .53, .48, etc. Even after seven nights' observations, on which no change is noted, it will only be about an even chance that the star may not still be of the Algol type. A different method of observing is therefore recommended when the star is supposed to belong to this class. Select for comparison a star slightly fainter, so that a moment's glance will satisfy the observer that the suspected variable is the brighter. It is only necessary to repeat this observation night after night. If the star is bright enough to be visible with a field glass, a few seconds will be sufficient for this observation after the observer has become familiar with the vicinity. The fact that the light is normal, and the time to the nearest minute, should be recorded after each observation. When convenient, it is well to repeat the inspection two or more times during the night, as in determining the period all the observations will have a value, provided that they are separated by intervals of more than two or three hours. If the star is ever found below its normal brightness, comparisons should be made with the adjacent stars, and continued as long as possible, or until it has regained its usual brightness. The most complete proof that a star was not of the Algol type would be for observers in the polar regions to examine it at intervals of a few hours for several days, or for observers in different longitudes to make the

same observations. If it could thus be watched for a week or fortnight by enough observers to avoid interference by clouds, it would be nearly certain that it is not an Algol star unless its period is greater than that of any such object as yet discovered.

The problems to be undertaken may be defined as follows:—

1. To observe all the long period variables once or twice every month throughout their variations according to such a system that all the observations may be reduced to the same absolute scale of magnitudes.

2. To observe the stars whose variability is suspected and prove either that they are really variable, or that in all probability they do not belong to the first, second, or fourth class. If any are thought to belong to the fifth class, to watch them until such a variation is proved, or is shown to be improbable.

All of this work will depend on the possibility of readily determining the brightness of a star according to such a method that all the observations can ultimately be reduced to the same system. Herschel and Argelander have independently invented what appears to be the true method to be followed. If a star is seen to be very nearly equal to several others, from their light we can at any time define its brightness. It is essential that at least one of the stars selected should be a little brighter, another a little fainter, than the star to be observed. The range within which its light is known is thus also defined. Such observations will far exceed in value any direct estimate of magnitude. When stars are to be compared many times, it is convenient to designate them by letters for brevity. Let *v* represent a star which is suspected to be variable, and *a* an adjacent star of nearly equal brightness. Owing to fluctuations in the atmosphere, each star will appear to be constantly varying in brightness. If the stars appear equal after a careful examination, or if one appears brighter as often as it appears fainter than the other, we may denote this equality by *av* or *va*, these terms having precisely the same meaning. If one of the stars is suspected to be brighter, — that is, if it appears sometimes brighter and sometimes fainter, but more frequently brighter, the interval

may be designated as one grade. The observation may be written *a* 1 *v* or *v* 1 *a*, the brightest star being named first. If one star is certainly brighter than the other, the difference, however, being very small, so that they sometimes appear equal, the difference will be two grades, and may be written *a* 2 *v* or *v* 2 *a*. Greater intervals may be estimated as three or four grades, but such observations have much less value. It is found in practice that a grade thus estimated will slightly exceed a tenth of a magnitude. A useful exercise for an observer is to select two stars of known magnitude and several others of intermediate brightness. Arrange them in a series in the order of brightness, and estimate the intervals in grades. The difference in magnitude of the first stars divided by the total number of grades gives the value of one grade. By using different intermediate stars, the same standard stars may be employed repeatedly. The following well-known polar stars will be convenient, since they are always visible: — *a Ursæ Minoris*, 2.2 magn.; *γ Ursæ Minoris*, 3.0 magn.; *δ Ursæ Minoris*, 4.4 magn.; 51 *Cephei*, 5.4 magn.; *λ Ursæ Minoris*, 6.5 magn. The above method is essentially that of Argelander. Sir William Herschel had already employed a method which differed mainly in his notation, a . , and — being equivalent to one, two, or three grades.

In all work of this kind the observer must look directly at the star he is observing at the moment, and never try to compare two stars by a simultaneous inspection of both. After examining one star until he has a distinct impression of its average brightness, freed from the momentary changes due to atmospheric disturbance, he should observe the other in the same manner. Alternate observations of the two stars, each observation lasting for a few seconds, will give a truer impression than can be derived from a simultaneous observation in which the two images must be differently placed on the retina.

The principal objection to this method is the difficulty of determining the value of a grade, as it is liable to vary with the observer, the time, the condition of the air, and the brightness

of the stars. These difficulties are avoided by the following method. Select two stars for comparison; one, *a*, slightly brighter than the star to be measured, *v*, the other, *b*, slightly fainter. The interval between *a* and *b* should never exceed one magnitude. Estimate the brightness of *v* in tenths of the interval from *a* to *b*. Thus, if *v* is midway between *a* and *b* the interval will be five tenths, and we may write *a* 5 *b*. If *v* is nearly as bright as *a*, we may have *a* 1 *b* or *a* 2 *b*; if *v* is not much brighter than *b*, we may have *a* 8 *b* or *a* 9 *b*. An advantage of this method is that larger intervals in brightness may be used between the comparison stars, and accordingly less distant stars employed. An increase in distance of the stars always renders the comparison more difficult. We can also obtain many independent comparisons by using several comparison stars. If we have m stars brighter and n fainter, we shall only have $m + n$ independent measures by the method of grades, while we may have $m\,n$ comparisons by estimating tenths, since estimates may be made in terms of the intervals between each brighter and each fainter star. On the other hand, especially when observing stars not very near together, it is a decided advantage to have to compare two stars rather than three. Each method has its advantages, and that to be used should doubtless depend on the temperament of the observer.

Several precautions are needed to secure the best results. No observations should be made near the horizon; and, when the objects examined are at any considerable zenith distance, stars differing several degrees in altitude should be avoided. If the stars are bright and there is no choice, a correction may be made for the error due to the varying absorption at these different altitudes if the time of observation has been noted. When using a telescope or opera-glass, the stars should be brought in turn to the centre of the field, as when near the edge they will not appear of their true brightness. This is found to be better than placing them at equal distances from the centre. In selecting comparison stars, the proximity of a brighter star is very objectionable, causing a large error, which varies with the mag-

nifying power used. Double stars should be avoided if the power used is sufficient to show the companion. Comparing stars of different colors is also objectionable.

Any persons who desire to take part in these observations are requested to communicate with the writer, and send answers to the questions given below.

1. What is the location of your point of observation? In the city or in the country, on the ground, from a roof, or from a window? Is any part of your horizon obstructed, or can you observe in all parts of the sky?

2. What is the aperture, focal length, and name of maker of your telescope? also the lowest magnifying power and largest field of view you can obtain with it? Have you a field-glass or opera-glass?

3. Can you identify bright and faint stars from their designations or right ascensions and declinations? Have you Heis' Atlas Cœlestis Novus, the Uranometria Argentina, the Durchmusterung, or other maps and catalogues of the stars?

4. Would you prefer to observe the known or the suspected variables, or to divide your time between them?

For convenience in making the reductions and for future reference, it is essential that all the observations should be made according to the same system. Observers are accordingly requested to adopt the following form. Use half-sheets of letter paper (eight inches by ten), writing only on one side and leaving a margin of half an inch for binding. Begin with a new sheet every evening, and write the date and location (township and state) on the first line. Each sheet when completed should be signed, and all should be numbered consecutively. When several sheets are used on the same night, the date should be entered on each. The record should be made in pencil, and all subsequent remarks or corrections added or interlined with ink, taking especial care not to obliterate or render illegible the original record.

A general statement should be made each evening of the condition of the sky, as "clear," "hazy," "passing clouds," etc. The time of beginning and ending work should also be noted. One

line should be assigned to each comparison. The hour and minute should be written to the left, and the comparison next to it. The right-hand half of the line will be left blank for reducing the observation.

Certain evenings or portions of evenings must also be devoted to the selection of the comparison stars of suspected variables. If they are contained in maps which are available, the letters assigned to each star may be marked on the maps and lines drawn to show with what suspected variable star they are associated. If preferred, a sketch may be made of the neighboring stars and the letters entered on them. This sketch with a proper description should be entered on the observing sheets described above, and a copy should be retained for reference. Every month the observations will be interrupted by moonlight, and accordingly, three or four days before the full moon, all the sheets that have accumulated should be mailed, addressed Harvard College Observatory, Cambridge, Mass. An acknowledgment will be sent at once, so that if this is not received a second notification should be sent.

To attain success it is particularly important that the plan should not be local or national. Observers in the southern hemisphere are much needed, and for some purposes those in various longitudes. It is hoped that among the many amateurs of Europe, and especially of England, may be found some ready to participate in this work. No restriction regarding the observations or publication is intended; but it is hoped that a large addition to our present knowledge of the variable stars may be secured, without interfering with what would otherwise be obtained. Copies of this pamphlet and further information will be furnished on application. Any persons desiring to participate are requested to address the writer, sending answers to the questions given above. The details will differ with each observer, and will be arranged by correspondence. Apart from the value of the results attained, it is believed that many amateurs will find it a benefit to accustom themselves to work in a systematic manner, and that they will thus receive a training

in their work not otherwise easily obtained outside of a large observatory. The lesson should be taught that time spent at a telescope is nearly wasted, unless results are secured worthy of publication and having a permanent value. Those who have once accomplished such work are likely in the future to appreciate its value, and will often continue to do useful work in some other department of practical astronomy, if not in that of variable stars. The education of a class of skilled observers would be a work of no less value than the results anticipated from the observation of the variable stars.

EDWARD C. PICKERING.

Harvard College Observatory,
Cambridge, Mass.

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STATEMENT OF WORK

DONE AT THE

HARVARD COLLEGE OBSERVATORY

DURING THE YEARS

1877–1882.

BY

EDWARD C. PICKERING,

Director of the Observatory.

CAMBRIDGE:
JOHN WILSON AND SON.
University Press.
1882.

HARVARD COLLEGE OBSERVATORY.

1877–1882.

In the autumn of 1878 an effort was made to secure for the Observatory an annual subscription of five thousand dollars for five years. It was claimed that, with this, the current work could be doubled, and that much progress might be made in reducing the large amount of past observations which had been accumulating for several years. It was also anticipated that the results attained would be sufficient to justify the continuance of this annual increase by means of an endowment. About seventy ladies and gentlemen of Boston and vicinity responded to this call, and by their generous aid have placed the Observatory in its present improved condition. As the subscription terminates this year, the last instalments being payable next March, the present seems to be a proper time to show how far the promised results have been attained.

An attempt will also be made to show that the condition of the Observatory has been so greatly improved by this subscription that it would be most unfortunate that it should relapse to its former condition. A subscription of one hundred thousand dollars is now in progress, and if secured will permit a permanent condition of activity. It will be noticed that the increased amount of work is quite out of proportion to the increase of income. This is to be expected, since a large part of the expenses are the same in either case, and the increase is therefore

directly available for the attainment of scientific results. The formation of a corps of skilled assistants also requires time, and a delay in securing a continuation of our present income would seriously reduce our capacity for attaining results with the greatest economy both of time and money.

As an increased expenditure was undertaken before the completion of the subscription, it is deemed best not to limit the present report to a period of exactly five years, but to include all the work undertaken since my first connection with the Observatory in February, 1877.

The effect of the subscription may be summarized in a few words. Without it, only one instrument, the meridian circle, was kept actively at work, the large telescope being comparatively idle. The reductions even of this one instrument could not be kept up, but every year fell more and more behindhand. With the subscription, the large telescope, the meridian circle, and the meridian photometer, are in constant use. A large number of the old observations have been published, while the remainder have been reduced, and before long will be ready for publication. One volume of the recent observations with the large telescope has already been published, another volume of meridian photometer observations is now passing through the press. The unfinished volumes of Annals were completed so that, as is shown below, our work is now known through twelve quarto volumes, while in 1876 but four had been given to the public. Eight more volumes of Annals will be needed to complete the publication of the observations already made. The increased rate of work ensues simply because the corps of assistants has been more than doubled.

Below will be given in order the various researches undertaken with the large equatorial, with the meridian circle, and with the meridian photometers. Then will be considered other researches, the distribution of standard time and the various publications that have been made. Fifteen assistants are at present attached to the Observatory. A large part of the work to be done consists of copying or computing of a very simple

character. By a proper division of labor, such work is done economically, the time of the more skilful assistants being reserved for the more difficult work. In this way researches can be carried out in a few years which are beyond the reach of the less favored observatories where the corps of assistants is small, or where such work can only be done if the astronomer is willing to devote the best years of his life to a single research. It should be observed that many of the investigations named below are of great extent, involving many thousand observations, and in many cases an amount of computation which would render it impracticable for a small observatory to prepare the work for publication without a delay of many years.

Large Equatorial.

The large telescope has been devoted mainly to photometry,— that is, to a measurement of the light of the stars. As but little work of this kind had been done with large telescopes, it was necessary to devise instruments for this purpose, have them constructed, and learn by experience the various errors to which each was subject. As over a dozen forms of photometers have been tried in this work, the labor involved in addition to that of observation has been very considerable. The principal investigations undertaken with the large telescope have been the following: —

Satellites. — Measurements of the light of the satellites of Mars were undertaken immediately after their discovery in 1877. These measures were repeated in the oppositions of 1879 and 1881. Similar measures were made of the satellites of Jupiter, Saturn, Uranus (outer satellites), and Neptune. A long series extending over one hundred and twenty-one evenings was made of Iapetus, the outer satellite of Saturn, to determine the law by which its light varies.

Eclipses of Jupiter's Satellites. — The importance of observing these phenomena has long been recognized. Hitherto the time

of disappearance or reappearance has alone been noted. Errors are thus introduced dependent on the size of the telescope, the sensitiveness of the eye of the observer, the haziness of the air, and, worst of all, a systematic error arises from the effect of twilight when Jupiter is observed near the sun. All of these errors are eliminated by comparing the satellite photometrically with another near it. Instead of single determinations we thus obtain whole series, each of which gives as many independent values of the true time of disappearance as it contains separate observations. By the aid of one or two assistants from seven to twelve settings per minute may be secured. These observations were begun in June, 1878, and since then one hundred and eighty eclipses have been observed. It is proposed to continue these observations for at least twelve years, during an entire revolution of Jupiter around the sun.

Double Stars. — The relative light of the components of over two hundred double stars was measured in 1879–80. Each star was observed in more than ten sets of four settings each.

Faint Stars. — A hundred very faint companions to bright stars were selected in 1878 and observed photometrically. Several asteroids were similarly observed. A map of the stars in close proximity to the pole was constructed, and careful measures were made of the brightness of these stars, to make them available as standards for other observers.

Planetary Nebulæ. — A photometer was devised for measuring the brightness of the nebulæ, and applied to all the planetary nebulæ. The spectrum of each was also examined, and the diameters of most of them determined.

New Planetary Nebulæ. — The simple plan of placing a direct vision prism in front of the eye-piece of a telescope appeared to be an easy method of detecting minute planetary nebulæ by their spectra. Some interesting objects of this class were found

which could not be distinguished from stars without the prism. Stars could thus be examined very rapidly, many thousand being viewed in a single evening. A systematic series of sweeps has been undertaken, zones 5° by 10° equally distributed over the sky being selected. From these the inference may be drawn that these nebulæ are not to be found far from the Milky Way. A number of sweeps have accordingly been made in the Milky Way, and have led to the discovery of eleven of these objects. One was so bright that it had been observed as a star, without a suspicion of its real character. Two objects were also found having a very singular spectrum of bright lines, one being quite unlike that of any other known star. In the course of the exploration two new variable stars were discovered.

Variable Stars. — Photometric observations of the variable stars have generally proved less satisfactory than those made by the eye alone. A new photometer, however, overcame the usual difficulties and gave light curves of several variables of the Algol class with great accuracy. About three thousand measures were made of each star, the observations generally extending over several hours. In one case nine hundred measures were made in a single night, extending without intermission from seven o'clock in the evening until the variable had attained its full brightness, at half past two in the morning.

Moon. — The light of different portions of the moon's surface is ordinarily estimated according to an arbitrary scale. The Selenographical society of England were invited to select a series of points as standards, and the promise was made to measure their light photometrically. The Society accordingly selected about fifty such points, which were measured here, and the results transmitted to the Society (*Selenographical Journal,* V. 57).

Bond Zones. — A large part of the work of the Observatory during the years 1852 to 1860 consisted in the determination of

the positions of about fifteen thousand stars between the equator and one degree north. Plans are now in progress for the revision of those between $+ 50'$ and $+ 1° 1'$, comprising about one fifth of the entire list. These stars will be observed with a modification of Professor Pritchard's wedge photometer, by which their light may be determined very rapidly during their transit across the field. This determination of the light of a large number of faint stars will afford a means of extending the present photometric scale of magnitudes. Convenient standards for comparison will thus be furnished for other faint stars. Incidentally the positions of the stars will be measured and a comparison made with their places, as observed more than twenty years ago. It is hoped that some interesting cases of variability or of large proper motion will thus be detected. In no other part of the heavens have we such precise determinations of so many very faint stars, and the observations of Professor Bond may thus acquire a value beyond that of simple catalogue positions.

Satellites of Mars. — The amount of photometric work accomplished has prevented the accumulation of many micrometric measurements. In special cases, however, an attempt has been made to supply observations not likely to be obtained elsewhere. In 1877, beside the photometric measures of the satellites of Mars, a series of measures of their positions was also made. The number of these observations was second only to that obtained with the great telescope at Washington. This was partly due to the use of a shade of red glass, which in the following opposition was generally adopted at other observatories. In 1879, thirteen hundred and forty-eight measurements of the satellites were made, Deimos being last seen at this Observatory as it gradually receded from the earth. This is remarkable as our telescope has entered into competition with the largest telescopes in the world, some of which admitted two or three times as much light.

Comets. — By securing the services of Mr. Chandler as an assistant, and by the co-operation of Mr. Ritchie of the Science Observer, a scheme has been developed which has made a great advance in the early announcement of comets. When a comet is discovered, notification is usually sent to this Observatory by telegraph. If the discovery was made in this country, a telegram is at once sent to the Dun Echt Observatory, and thence distributed throughout Europe. The following evening, if clear, an observation of the comet is taken, and the resulting position telegraphed to Europe. These early positions have in some cases proved of great value, and have been used again and again in each subsequent orbit. Great care has been taken to avoid all delay in sending them, with the result that occasionally these precise positions have become known abroad before the discovery itself had been announced by the usual method. As soon as these observations are obtained, the computation of the orbit is begun, and the work continued at all hours of the day or night until the results have been translated into the Science Observer cipher and cabled. About four days after the comet is discovered, its elements and ephemeris are generally printed and distributed in this country and in Europe. By an arrangement with the Signal Service and with other observatories, when cloudy weather is expected here observations are sometimes obtained elsewhere to avoid delay. Such arrangements have been made with the daily papers and with the Associated Press, that any important observation made here before midnight would probably be printed in the papers of the following morning in the principal cities of the country.

By the same system of co-operation a plan for sweeping for comets has been developed, in accordance with which a number of observers have undertaken to examine a certain portion of the heavens once or twice every month, and satisfy themselves that no comet within the reach of their telescope is to be found there. The results published in the Science Observer show that the entire heavens is now so thoroughly swept that a comet is not likely to be long visible without detection.

Miscellaneous Observations. — Among those may be named the determination of the focal length of the large telescope, the pitch of the screw of the micrometer, observations of asteroids, and a careful series of preliminary observations for determining stellar parallax.

Meridian Circle.

The observations carried on with this instrument during the period here discussed may be classified as follows: —

Zone Observations. — This series of observations was undertaken in 1870 as a part of a general scheme for the observation of all stars of the ninth magnitude or brighter, situated north of the equator. The work to be accomplished was shared by a number of observatories in Europe and America. The stars observed here are those situated in the zone from 50° to 55° north of the equator, with an extension of 10′ into the adjacent zones on both sides, in order to provide means of connecting the results obtained here and elsewhere. These observations were carried on by Professor Rogers from the autumn of 1870 to January, 1879, when they were completed except so far as the reductions may show the need of revision to remove discordances. The work has included two complete observations of each of about eight thousand three hundred stars, with numerous additional observations of fundamental stars from which the instrumental corrections are derived.

Coast Survey List. — A list of two hundred and fifty-eight stars requiring observation for geodetical purposes was drawn up by the United States Coast Survey, and an arrangement was made with this Observatory to furnish such observations. The plan involved six observations of each star. The work was begun in January, 1878, and completed in one year.

Standard Stars. — The principal work of the Meridian Circle after February 15, 1879, consisted in the determination of the

absolute co-ordinates of rather more than one hundred of the brighter stars. This work involves daily observations of the Sun and observations of the stars at all times of the night and day. It has occupied three years, but two years more will be required to complete it according to the original plan.

Miscellaneous Observations of Stars and Planets.—Observations have been frequently made at the request of other astronomers for special purposes. Mr. Gill, now Director of the Royal Observatory at the Cape of Good Hope, desired the places of a number of stars used in his observations for the parallax of Mars and of some asteroids. These places were accordingly determined here, and subsequent observations were made to investigate the difference apparently existing between bright and faint stars as determined at different places and by different observers. Observations of Mars and of neighboring comparison stars were also made in pursuance of a proposition made by Professor Eastman. Observations of certain stars were made at the request of Professors Davidson and Hall, of Captain Tupman, and of M. Bossert. The comparison stars used in determining the places of several asteroids were likewise observed, and two observations were made of Comet 1881, III.

Instrumental Constants.—One of the graduated circles of the instrument has been minutely examined with the aid of special contrivances for detecting errors of graduation. A long collimator has been put up on the grounds of the Observatory, by means of which small variations in the direction of the line of sight have been studied. Many experiments have been tried with levels of different kinds for the purpose of discovering errors, and observations of stars have been made in order to show whether the telescope revolves in a vertical plane. Its flexure, likewise, has been carefully discussed, with the unexpected result that the value of this quantity for each year is found to be about 0.″6 greater in January than in July.

Meridian Photometers.

In 1878 an instrument was devised by which any star when crossing the meridian could be compared with the Pole-star. The working of this instrument proved highly satisfactory, and many of the errors were avoided to which other photometers are subject. The images to be compared are precisely alike, and are viewed with the same magnifying power, aperture of telescope, and emergent pencil, on the same background, and in general under the same conditions. The stars are observed very rapidly, over a hundred having been measured (with four settings on each) by a single observer in one evening. A catalogue of all the stars visible to the unaided eye in the latitude of Cambridge was prepared, and each of these was observed on at least three evenings. Nearly half were observed twice as often, or on six evenings each, and some important stars still more frequently. The entire work was completed in less than three years, and furnishes a measure of the light of about four thousand and three hundred stars. Nearly one hundred thousand settings are involved in this work. The results are now being put in type, and will include a discussion of the constancy of the light of the Pole-star, the variation in the atmospheric absorption at various altitudes, and a comparison of the nomenclature of various authorities. An important part of the work will be a comparison of the scales of magnitudes employed by various authorities from the Almagest to the present time. A comparison will finally be made of the light of each star according to the separate authorities after applying a proper correction for this difference of scale.

To afford a better means of comparison with the results of the eye observations, each of the stars of the northern heavens has been compared by three observers with a series of standard stars. Two catalogues are thus obtained, one giving the light of the stars as seen with the naked eye, the other, the true light as indicated by the photometer. Interesting results are anticipated from a comparison, as showing the effect of the proximity of bright stars, of the background of the Milky Way, etc.

The success of the first meridian photometer led to the construction of a similar instrument of much larger size and of improved form. The apertures of the telescopes were four inches instead of an inch and a half, and various changes were made in the details. With this instrument the light of any star brighter than the ninth or tenth magnitude may be determined. It will be used for a still larger piece of work than that above described. In the revision of the stars of the northern heavens, referred to in the description of the work of the meridian circle, the zones of the various observatories overlapped by a small amount. Over eight thousand stars are contained in these overlapping portions, and therefore are each observed at two observatories. The light of all these stars is to be determined with the new meridian photometer, and the estimates of light of the various observers in the entire work of revision may thus be reduced to a single standard. Incidentally, the scale of various catalogues will also be determined. Besides this work, the standard stars of the Uranometria Argentina, the comparison stars for variables, and other objects of interest will be measured.

Miscellaneous Observations.

Meteorology. — A portion of the meteorological observations of the Observatory are still kept up, but less importance is attached to them than formerly, on account of the vast amount of material now being collected by the United States Signal Service. Attention is rather directed to the more unusual and less observed phenomena. In this connection may be named a careful study by Mr. Searle of certain zodiacal phenomena, especially that designated by Brorsen as "Gegenschein."

Longitude Determinations. — The attention now paid at Washington to the determination of longitude renders the prosecution of this work at Cambridge less necessary than formerly. In any important work of the kind it is, however, still desirable to determine the longitude from this Observatory, since for many years it was the standard meridian in this country to which all

others were necessarily referred. It also served as the basis from which the longitude of America from Europe has been found. The most important longitude campaign recently carried on here has been the determination of the difference in longitude of Detroit and Cambridge, which connects the system of the Lake Survey with that determined from this Observatory. The difference in longitude of New Haven and Cambridge has also been determined.

Variable Stars. — An important piece of bibliographical work has been undertaken in collecting all the observations hitherto made of the variable stars. This has led to the formation of a catalogue of over a thousand stars whose light has been supposed by various authorities to be variable. Of the known variables of long period the observed times of maxima and minima and their corresponding brightness have been in part already collected. Finally, it is proposed to select the more important observations, more especially those in which the variable is compared with one or more stars of constant brightness, and reduce them to a uniform scale. The comparison stars will probably be measured with the meridian photometer.

Preparations are now being made to enlist the aid of amateur astronomers and others in a scheme for determining the constancy or variability of all the suspected variables referred to above. Also for determining the light of all the variables of long period at intervals of a month or less throughout their entire variation from maximum to minimum. If many persons can be induced to take part in this work, a very important addition to our knowledge of the variable stars will be attained. No instrument is required but a telescope or even an opera-glass. Besides the more experienced observers it is hoped that many persons interested in astronomy but unaccustomed to such work will undertake it, as with care the needed skill is readily acquired. The education of a class of observers from those who have hitherto failed to add to our knowledge of the science would be in itself a most important work. Doubtless, should the plan

prove successful, many professional astronomers would be willing to co-operate in the matter, and thus greatly add to the value of the research.

Standards of Lengths. — An important comparison of the various standards of length has been conducted by Professor Rogers, in addition to his Observatory work. In connection with this work he visited London and Paris in 1880, and obtained some important comparisons with European standards. He has also devoted much attention to the construction of standard scales with equal subdivisions, and his work has proved essential in the observations of the Transits of Mercury and Venus.

Atmospheric Refraction. — For several years past investigations involving many thousand observations have been in progress, under the direction of the Rumford Committee. Measures have been made at temperatures varying from +92° to —12° to determine the effect of changes in this disturbing element.

Time Service.

For many years an important service rendered by the Observatory to the public has consisted in furnishing an accurate standard of time. This department has been improved and extended in various ways. Two new clock rooms have been constructed, and various devices introduced for increasing the regularity and accuracy of the signals furnished. With the co-operation of the Signal Service and of the Equitable Life Assurance Company, a time ball has been erected in Boston, and is dropped every day at noon. A full description of this ball will be found in the Professional Papers of the Signal Service, No. 5.

Publications.

The following tabular statement of volumes wholly or partly in print, belonging to the Annals of the Observatory, exhibits in

successive columns the general subject of each volume or part, the instrument chiefly used in the observations described in it, the epoch of the observations or of the events discussed, and the date of publication. An asterisk in the first column denotes that the distribution of the corresponding work took place during the period covered by this report. The publication and distribution of Vol. XIII., Part II., and of Vol. XIV., will probably occur in 1883.

Vol.	Part	Subject.	Instrument.	Epoch.	Date of Publication.
I	I	History and Description .		1840–55	1856
I	II	Zone Catalogue, 5,500 Stars	East Equatorial . .	1852–53	1855
II	I	The Planet Saturn . . .	"	1847–57	1857
II	II	Zone Catalogue, 4,484 Stars	"	1854–55	1867
III		Great Comet, 1858 . . .	"	1858	1862
IV	I	Catalogue of Standard Stars		1855	1863
IV*	II	Right Asc., 505 Stars . .	East Transit Circle	1862–65	1878
V		Nebula of Orion	East Equatorial . .	1847–65	1867
VI*		Zone Catalogue, 6,100 Stars	"	1859–60	1872
VII*		Solar Spots	West Equatorial .	1847–49	1871
VIII*	I	History and Description .		1856–76	1876
VIII*	II	Astronomical Engravings .	East Equatorial . .	1872–74	1876
IX*		Photometric Researches .	Zöllner Photometer	1872–75	1878
X*		Places of 1200 Stars . . .	Meridian Circle . .	1868–72	1877
XI*	I	Photometric Observations .	East Equatorial . .	1877–79	1879
XI*	II	" "	"	1877–79	1879
XII*		Places of 628 Stars . . .	Meridian Circle . .	1871–75	1880
XIII*	I	Micrometric Measurements	East Equatorial . .	1866–81	1882
XIII	II	Meteorology		1840–79	
XIV		Meridian Photometry . .	Mer. Photometer .	1879–82	

Material for about eight volumes of Annals, in addition to those above named, has been collected, and these volumes might be completed and published in the absence of any further observations, except with the large meridian photometer. The subjects of the volumes are described briefly below. A large part of the observations has been reduced, and several of the volumes can soon be prepared for printing.

Observations of fundamental stars made before 1879, and various series of similar determinations carried on between 1870 and the present time.

Observations of the stars between 50° and 55° north declination.

Catalogue of stars resulting from observations just mentioned.

Observations, in 1879 and later, of the standard stars mentioned above in describing the work of the Meridian Circle.

Miscellaneous photometric work, chiefly with the large equatorial, 1879 and later.

Photometric observations of the eclipses of Jupiter's satellites.

Photometric observations with the large meridian photometer.

Variable stars.

Besides the researches which have appeared in the Annals above mentioned, a large number of articles relating to astronomy and kindred subjects have been published, during the period here considered, by the astronomers connected with the Observatory. A list of these is given below.

By William A. Rogers.

New Elements of Iphigenia (117) from the Opposition Observations of 1870, 1872, 1873, 1877. (*Astronomische Nachrichten*, XCI. 107.)

On Standard Measures of Length. (*Am. Quarterly Microscopical Journal*, Jan. 1879.)

On two Forms of Comparators for Measures of Length. (Id. April, 1879.)

On the Limits of Accuracy in Measurements with the Telescope and the Microscope. (*Proc. Am. Acad. of Arts and Sciences*, XIV. 168.)

On the First Results from a new Diffraction Ruling Engine. (*Am. Journal of Science and Arts*, Jan. 1880; 3d Ser. XIX. 54.)

On the Present State of the Question of Standards of Length. Presented April 14, 1880. (*Proc. Am. Acad. of Arts and Sciences*, XV. 273.)

On Tolles's Interior Illuminator for Opaque Objects. (*Journal of Royal Microscopical Society*, III. 754.)

The Coefficient of Safety in Navigation. (*Proc. U. S. Naval Institute*, VII. No. 3. *Science*, April, 1881; II. 171.)

On a Convenient Method of Expressing Micrometrically the Relation Between English and Metric Units of Length on the Same Scale. (*Proc. Am. Assoc. for Advancement of Science*, Aug. 1881; XXX. 116.)

On a Method of Reducing Different Catalogues of Stars to a Homogeneous System. (Id. XXXI. 11.)

On the Performance of a New Form of Level Invented by Mr. John Clark of the U. S. Coast Survey. (Id. XXXI. 14.)

Plane *v.* Cylindrical Surfaces. (*Mechanics*, No. 31, p. 90.)

A Study of the Problem of Fine Rulings with Respect to the Limits of Naked Eye Visibility and Microscopic Resolution. (*Am. Monthly Microscopical Journal*, Sept. 1882, p. 165.)

A Comparison of the Harvard College Observatory Catalogue of Stars for 1875.0 with the Fundamental Systems of Auwers, Safford, Boss, and Newcomb. (*Memoirs of the Am. Acad. of Arts and Sciences*, X. 389–429.)

On the Conditions of Success in the Construction of Standards of Length and in their Subdivision into Equal Parts. Read before the American Society of Microscopists at the meeting held at Elmira, N. Y., Aug. 1882. (*Mechanics*, Oct. 27, 1882 and later.)

By Arthur Searle.

On Certain Zodiacal Phenomena. (*Astronomische Nachrichten*, XCIX. 91, 369; CII. 263. *Science Observer*, July, 1882; IV. 4.)

By Leonard Waldo.

Standard Public Time. (*Observatory Circular*, 1877.)

Engineer's Instruments and their Adjustments. Boston, pp. 40, l. 8°.

On the Longitude of Waltham, Mass., Nov. 1877. (*Proc. Am. Acad. of Arts and Sciences*, XIII. 175.)

A Lecture on Telling the Time. (*Bulletin of Essex Institute*, Feb. 1878; X. 40.)

Note on the Measurement of Short Lengths. Feb. 1878. (*Proc. Am. Acad. of Arts and Sciences*, XIII. 352.)

Observations of the Satellites of Mars, and of Double Stars. (*Astronomische Nachrichten*, XCII. 87.)

Meridian Observations of Mercury at its Transit. May 5–6, 1878. (Id. XCII. 361.)

Observations of the Satellites of Saturn. (Id. XCIV. 339.)

Report of the Observations of the Total Solar Eclipse, July 29, 1878, made at Fort Worth, Texas. Cambridge, 1879. 4°.

Articles "Transit," "Sextant," "Telescope," "Ruling Machine," "Time Signals," "Mural Circle," "Zenith Telescope." (*Johnson's New Universal Cyclopaedia*, New York, 1877–78.)

Description of a New Position Micrometer. (*Am. Journ. of Science and Arts*, July, 1880; 3d Ser. XX. 49.)

On the Adaptation of the Opera Glass to Extremely Myopic Eyes. (*N. Y. Ophthalmological Journal*, 1880.)

By Winslow Upton.

Observations of Minor Planets. [W. Upton and W. A. Rogers, observers.] (*Astronomische Nachrichten*, XCIII. 171.)

Determination of the Orbit of (185) Eunike. (Id. XCIV. 51.)

By O. C. Wendell.

Observations of Comet *c* 1879 (Swift). (*Astronomische Nachrichten*, XCVI. 21.)

Observations of Comets 1881 V. and 1881 VIII. (Id. CI. 231.)

Observations of Comets 1880 IV. and 1881 II. (Id. CI. 299.)

Observations of Comets 1881 III. IV. VI. (Id. CIII. 145.)

Comet (*b*) 1881. (*Science Observer*, Aug. 1881; III. 81.)

By J. Rayner Edmands.

Geodetic Formulae. (*Appalachia*, July, 1880; II. 135.)

Report of Councillor of Topography. (Id. July, 1880; II. 161.)

Geodetic Formulae. [Second Paper.] (Id. Dec. 1881; II. 351.)

The Mountains between Saco and Swift Rivers. [With map.] (Id. June, 1882; III. 57.)

By S. C. Chandler, Jr.

Elements of Comet *b* 1881. (*Astronomische Nachrichten*, C. 121.)

Elements and Ephemeris of Comet *e* 1881. (Id C. 319.)

Observations and Elements of Barnard's Comet, 1881 VI. (Id. CI. 57.)

On the Periodicity of Comet (Denning) 1881 V. (Id. CI. 93.)

On the Variability of DM. + 23° 1599. (Id. CII. 139.)

On Sawyer's Variable, DM. +1° 3408. (Id. CII. 371.)

On the Telegraphic Transmission of Astronomical Data. (*Science Observer*, Aug. 1881; III. 65.)

Letter to the Astronomische Gesellschaft on the Science Observer Code, (*Vierteljahrsschrift der Astronomischen Gesellschaft*, 1881, XVI. 344.)

Elliptic Elements of Comet (*f*) 1881—Denning. (*Science Observer*, Dec. 1881; III. 91.)

On a New Variable Star in the Constellation Cetus. (Id. March, 1882; III. 105.)

On Sawyer's Variable, DM. +1° 3408. (Id. July, 1882; IV. 11.)

On the Period of R *Hydrae*. (Will appear shortly.)

By Edward C. Pickering.

The following list contains the titles of similar works of my own during the same period, including those of an official character, describing the work of the Observatory, as well as those containing the results of special researches.

The Micrometer Level. (*Appalachia*, June, 1877; I. 138.)

Address of the Vice-President, Section A. (*Proc. Am. Assoc. for Advancement of Science*, Aug. 1877; XXVI. 63.)

Annual Report of the Director of Harvard College Observatory, presented to the Visiting Committee November 26, 1877. Cambridge, 1877. 8°.

Report on the Progress of the Zone Observations. (*Vierteljahrsschrift der Astronomischen Gesellschaft*, 1877, XII. 290.)

Annual Report of the Director of Harvard College Observatory, presented to the Visiting Committee November 14, 1878. Cambridge, 1879. 8°.

The Cosine Galvanometer. (*Nature*, Jan. 1879; XIX. 217.)

Stellar Magnitudes. (*Astronomische Nachrichten*, XCV. 29. *Nature*, May, 1879; XX. 14. *Astronomical Register*, XVII. 175.)

Thirty-fourth Annual Report of the Director of the Astronomical Observatory of Harvard College. Presented to the Visiting Committee December 5, 1879. Cambridge, 1880. 8°.

Report on the Progress of the Zone Observations. (*Vierteljahrsschrift der Astronomischen Gesellschaft*, 1879, XIV. 387.)

Observations of the Satellites of Mars. [E. C. Pickering, O. C. Wendell, A. Searle, and F. Waldo, observers.] (*Astronomische Nachrichten*, XCVII. 115, 145.)

Light of Webb's Planetary Nebula. (*Nature*, Feb. 1880; XXI. 346.)

Dimensions of the Fixed Stars, with especial reference to Binaries and Variables of the Algol Type. (*Proc. Am. Acad. of Arts and Sciences*, June, 1880; XVI. 1.)

Two New Planetary Nebulae. (*Nature*, Aug. 1880; XXII. 327.)

Novel Celestial Object. (Id. Sept. 1880; XXII. 483.)

New Planetary Nebulae. (*Am. Journal of Science*, Oct. 1880; CXX. 303. *The Observatory*, March, 1881; IV. 81.)

Thirty-fifth Annual Report of the Director of the Astronomical Observatory of Harvard College, presented to the Visiting Committee December 6, 1880. Cambridge, 1881. 8°.

Variable Stars of Short Period. (*Proc. Am. Acad. of Arts and Sciences*, Feb. 1881; XVI. 257. *The Observatory*, Aug.—Oct. 1881; IV. 225, 264, 284.)

Observations of Comet III. 1869. (*Astronomische Nachrichten*, XCIX. 95.)

Observation of the Solar Eclipse of Dec. 30, 1880. (Id. XCIX. 107.)

The Companion of Sirius. (Id. XCIX. 219.)

Photometric Magnitude of Jupiter's Satellite III. (*The Observatory*, April, 1881; IV. 113.)

Large Telescopes. (*Proc. Am. Acad. of Arts and Sciences*, April, 1881; XVI. 364. *Nature*, Aug. 1881; XXIV. 389.)

Photometric Measurements of the Variable Stars β *Persei* and DM. 81° 25, made at the Harvard College Observatory. [E. C. Pickering, Arthur Searle, and O. C. Wendell, observers.] (*Proc. Am. Acad. of Arts and Sciences*, April, 1881; XVI. 370.)

Objects remarkable for their Colors or Spectra. (*Astronomische Nachrichten*, XCIX. 375.)

New Variable Star in Puppis. (Id. C. 13.)

Comet 1881, III. (*Science*, July, 1881; II. 329.)

Report of the Committee on Standards of Stellar Magnitude. (*Proc. Am. Assoc. for Advancement of Science*, Aug. 1881; XXX. 1.)

Thirty-sixth Annual Report of the Director of the Astronomical Observatory of Harvard College, presented to the Visiting Committee, November 10, 1881. Cambridge, 1882. 8°.

Report on the Progress of the Zone Observations. *Vierteljahrsschrift der Astronomischen Gesellschaft*, 1881, XVI. 317.)

Reply to inquiries regarding Time Balls. (*Professional Papers of the Signal Service*, No. 5, p. 24.)

Stars with peculiar spectra, discovered at the Astronomical Observatory of Harvard College. (*Astronomische Nachrichten*, CI. 73.)

Order of Brightness of Stars. (*English Mechanic and World of Science*, Nov. 1881; XXXIV. 278.)

Remarkable Star Spectrum; New Planetary Nebula. (*Science*, Dec. 1881; II. 581. *Copernicus*, Dec. 1881; I. 242.)

The Pleiades. (*Astronomical Register*, Feb. 1882; XX. 40.)

Variable Stars. (*English Mechanic and World of Science*, Feb. 1882; XXXIV. 542.)

Photometric Observations of Planets and of Jupiter's Satellite III, made at the Harvard College Observatory. (*Astronomische Nachrichten*, CII. 151.)

Photometric Observations of the Satellites of Mars, 1881–82. [E. C. Pickering and O. C. Wendell, observers.] (Id. CII. 193.)

The Meridian Photometer. (*Monthly Notices of the R. Astr. Society*, June 1882; XLII. 365.)

Photometric Comparisons of Lunar Objects. (*Selenographical Journal*, July–Aug. 1882; V. 53, 57.)

Erratum in Observations of Comet Wells, 1882. (*Astronomische Nachrichten*, CII. 223.)

Photometric Measurements of Sawyer's Variable (DM. +1° 3408), and its Comparison Stars. (Id. CIII. 61.)

New Planetary Nebulae. (Id. CIII. 95, 165.)

Small Planetary Nebulae, discovered at the Harvard College Observatory. (*The Observatory*, Oct. 1882; V. 294. *The Sidereal Messenger*, Oct. 1882; I. 139.)

A Plan for Securing Observations of the Variable Stars. Cambridge, 1882. 8°.

Whether the next five years shall prove as fruitful of results as the last, or whether the Observatory shall relapse to the much less active condition in which it was obliged to remain before the subscription of 1878, will depend on the result of the effort now being made to increase its endowment by one hundred thousand dollars.

EDWARD C. PICKERING.

20

THIRTY-SEVENTH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE, NOVEMBER 22, 1882,
AND LAID BEFORE THE BOARD OF OVERSEERS,
JANUARY 10, 1883.

CAMBRIDGE:
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1883.

REPORT

OF THE

DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF HARVARD COLLEGE

For 1881–82.

To the President of the University:

Sir, — The progress made in the reduction of the past observations has been the distinguishing feature of the last year. The micrometric observations of the large telescope, to the beginning of 1882, have been published, and the volume has been distributed. A large part of it is occupied with the work of my predecessor, the late Professor Winlock, upon double stars, nebulæ, and other objects, including spectroscopic results. The reductions of the zone observations made with the meridian circle from 1871 to 1879 are now approaching completion. The meteorological observations from 1840 to 1880 have been brought together and are nearly ready for publication. The most important piece of photometric work as yet undertaken here has been completed, and the catalogue giving the results of the measures of four thousand stars is now in the hands of the printer. It thus appears that if the present rate of work is maintained, it will not be long before the accumulated observations of the past are reduced and printed, and we may anticipate a publication of our future observations without a delay greater than is needed for their reduction.

The work of the various instruments will now be considered in detail.

East Equatorial.

Eclipses of Jupiter's Satellites. — The observations of these eclipses photometrically, begun in 1878, have been continued during the past year. One hundred and eighty-five eclipses have been observed, thirty-two since Nov. 1, 1881.

Objects having singular Spectra.—The search for these objects in previous years has been mainly in regions selected without special system. The heavens have now been divided into sections of equal area, and a rectangle of 5° by 10° in the centre of each has been selected for examination. The twelve regions having centres at declination + 15° and right ascension 1^h, 3^h, 5^h, etc., have all been swept over in this way. Of the twelve corresponding regions having a declination of — 15° six have so far been examined. From this systematic search, we may learn in what portion of the heavens we may expect to find objects of interest, and thus justify a more extended examination. The absence of such objects in other regions will also show that a further search in their vicinity is unadvisable. Several minute planetary nebulæ have been found near the Milky Way, but none at a distance from it. A special search was therefore made last summer in the Milky Way from — 30° to + 20°. This resulted in the discovery of seven new planetary nebulæ. Most of these objects, although moderately bright, are so minute that they could not be distinguished from stars by an ordinary eyepiece. One of them has hitherto been mistaken for a star, and is given in the catalogues as DM. + 1° 3979.

Satellites of Mars.—At the last opposition of Mars the satellites were looked for and were repeatedly seen by Mr. Wendell and myself between December 16 and February 24. It thus seems probable that these objects may be seen with our telescope at any opposition of the planet. The observations made of them here on this occasion were entirely photometric. The results were in general confirmatory of those obtained at the oppositions of 1877 and 1879, but the difference between the brightness of Deimos when preceding and when following Mars, which had been formerly noticed, was not now apparent.

Lunar Objects.—The photometric determination, for the Selenographical Society, of the brightness of various points on the Moon, mentioned in the last report, was completed and the results communicated to the Society. They appear in the Selenographical Journal, V. 57. It appeared from this investigation that the scale of brightness in common use by observers of the Moon might be very closely expressed in terms of stellar magnitudes, each degree of the scale answering to the ratio of light equivalent to six tenths of a magnitude.

Double Stars. — As a part of the bibliography noted below, a list has been prepared of the double stars, the variability of whose components has been suspected. The photometer employed in the observations discussed in Volume XI. Part I., is especially suited to observing such objects. The usual methods of detecting variability fail when applied to the double stars. One of the most striking instances in this list is the star θ *Serpentis*, one component of which has been thought to be variable by Dr. Gould. According to our observations on three evenings in 1878, the respective differences in magnitude of its components were 0.5, 0.5, and 1.4. It has been repeatedly measured during the past year with the results 0.4, 0.4, 0.4, 0.5, 0.4, on different evenings. In general each of these numbers is the mean of sixteen settings. If really variable, the star probably belongs to the Algol class, and this conclusion seems unavoidable unless the wrong star was observed in the last measures in 1878. The measures show the great precision attainable with this form of photometer.

Wedge Photometer. — A modification has been made of Professor Pritchard's wedge photometer which seems to render it especially suitable to the measure of the light of faint stars in zones. The wedge is placed so that the diurnal motion of the stars carries them from its thin to its thick portion, and the time of disappearance is noted. A bar in the unobstructed part of the field serves to determine the position of each star. From 1852 to 1860 the measurement of the positions of the stars from the equator to one degree north formed an important part of the work of the large telescope. The stars in the northern 10′ of this zone have been arranged in catalogue form, and the necessary preliminary observations have been made by Mr. Searle. Besides determining the light of these objects it is hoped that some interesting cases of proper motion may be detected.

Sawyer's Variable. — A careful study of this interesting star by Mr. Chandler proved that it belonged to the Algol class, and also that its period was only about twenty hours. This gives it the shortest period of any variable star as yet discovered, and only a little more than one third of the period of any other variable of the same class. The variation of light is about three fourths of a magnitude. A long series of observations of the light curve and of successive minima has been made by Mr. Chandler, and gives the period $20^h\ 7^m\ 41^s.6$, with a probable error of $1^s.3$.

The comparison stars were measured on ten nights with the meridian photometer, and furnished a means of reducing these observations to absolute light ratios. The light curve has also been determined photometrically by Mr. Wendell and myself. Observations were made on eighteen nights and give a very precise measure of the changes of light.

Comets. — The telegraphic system for the speedy transmission of various data respecting comets, which was mentioned in the last report, has been maintained and extended. The first observation of a new comet which is obtained at this Observatory is now telegraphed to Europe in advance of the elements, and is often found useful by European computers. It has happened in two cases, those of the comets discovered by Messrs. Wells and Barnard, that the receipt of the position obtained here was the first intimation abroad of the discovery. This led in the case of the Wells comet to the supposition that the discovery had been made at this Observatory, and to guard against similar errors in future, the despatches are now made fuller, giving the facts of discovery as well as the positions. Arrangements have also been made with Mr. Swift, director of the Warner Observatory at Rochester, in accordance with which he forwards information received by him of new discoveries, either for immediate transmission to Europe, or for a previous investigation here.

The four comets, discovered respectively by Swift in 1881, by Wells and Barnard in 1882, and by various southern observers (the great comet of the year), will show the working of this system. In each case, an observation was obtained at this Observatory on the night following the receipt of intelligence of the discovery, and an orbit was computed, telegraphed to Europe, and published by the Science Observer in this country, during the next four days. In two cases, the accurate position obtained here, and telegraphed in advance of the orbit, was published abroad on the day following that on which the news of the discovery reached us. The orbit of Swift's comet sent from this Observatory first announced its discovery to European astronomers, as likewise happened in two cases already mentioned with regard to the first observation obtained here. These first observations were extensively used in the computations made in Europe. Cloudy weather prevented sufficient data for an orbit of the great comet from being secured here in the first few days

of its appearance; but, on receipt by telegraph from Europe of one more position, the orbit was computed and telegraphed back within seven hours.

MERIDIAN CIRCLE.

After continuous observations extending over twelve years, Professor Rogers has found it necessary to take a prolonged rest from night work. The interrupted series will be resumed in February, 1883. Meanwhile the instrument has been used in determining the local time. Its constants, including the collimation, the level, the flexure, the reading of the long collimator, and the index error of the circle, have been determined every week. Excellent progress has been made with the reduction of the observations from 1870 to 1879. These observations will occupy about twelve hundred printed pages, and will fill three volumes of the Annals. The subject will probably be divided according to the following plan. The first volume will contain an introduction giving a description of the processes of observation and of reduction, and a discussion in detail of the instrumental constants for the entire period. The tabular values of these constants will be given for each date. The greater portion of this volume will be devoted to a table which will give the mean times of transit, the circle readings, the constants needed in the reduction, and the resulting right ascension and declination of each of the fundamental stars observed. The second volume will contain all the zone observations in journal form. The quantities used in the reduction of the zone stars will also be given in this volume. The third volume will contain the observations of the secondary polar stars observed during the years 1872–73, the observations of a list of stars made at the request of the United States Coast Survey in 1878, and all the miscellaneous observations made previous to 1879. The second part of this volume will contain the final catalogues of the primary stars, of the secondary stars, and of the zone stars reduced to 1875.0

The introduction to the first of these volumes, occupying one hundred and fifty pages of manuscript, is completed and is ready for publication. The data for the body of the work are also complete except the introduction of the newly determined instrumental constants depending directly upon the system given in Publication XIV. of the Astronomische Gesellschaft. The

preparation of the copy for publication has recently been commenced.

Of the second volume, the reduction of the zone observations to the beginning of the year of observation has been completed, and the copy containing the original data has been prepared for the printer. It occupies about six hundred pages of manuscript. The reduction of the Durchmusterung places from 1855 to 1875 is nearly completed. A large part of the reductions of the zone observations are to be examined either by cross checks or by recomputation. All doubtful observations are to be investigated, and the positions are to be reduced from the beginning of the year of observation to 1875.0 by means of manuscript tables.

The only portion of the third volume which is ready for publication is that containing the observations of the Coast Survey Catalogue. This occupies one hundred pages of manuscript. The reduction of the Polar Catalogue with the newly determined instrumental constants, and the formation of the final catalogues, still remains to be made.

Meridian Photometer.

The measurement of the light of the stars visible to the unaided eye was completed last summer. Over ninety thousand measures were made on about four thousand stars, most of them being observed on from three to six nights. The more important stars were measured more frequently. Four settings were made every evening on each object. A series of estimates by the unaided eye have also been made by three observers for purposes of comparison. The entire work will involve the discussion of several problems of general interest in connection with the light of the stars. Among these may be mentioned the atmospheric absorption. A discussion of about fifteen thousand observations available for this investigation shows that we may assume that the absorption at any altitude, exceeding 15°, equals in stellar magnitudes one quarter of the secant of the zenith distance. This agrees very nearly with the empirical law deduced by Seidel, especially if we apply a correction for the low barometric pressure due to his great elevation. The average deviation of the two laws does not exceed a thirtieth of a magnitude. A special series of eye estimates serves to extend this law to the horizon.

The constancy of the Pole Star is established by the same observations with the photometer. A series of eye estimates was also made to compare the light of the Pole Star with that of other stars of nearly equal brightness near it, and shows that the relative position of two stars to be compared has an important influence in their apparent brightness. After applying a proper correction, the average deviation of the results is reduced to six one hundredths of a magnitude. An extended comparison of the scale of magnitudes employed by previous observers has been made. A reduction of the observations of Sir William Herschel has been effected, and has led to interesting results. His observations of the light of the stars are not only far superior to any similar work preceding it, but are more precise than most of the subsequent determinations. Their neglect hitherto is partly due to the want of a suitable system of magnitudes by which they might be reduced. This want is supplied by the photometric measures now under consideration. There seems also to have been an impression that the intervals employed by Herschel were so large as to render the observations uncertain. Our reduction shows that the intervals he designated as a period, comma, and dash, do not exceed one, two, and four tenths of a magnitude; and that the average deviation of a single comparison of two stars, expressed in magnitudes, is only 0.25. As this includes the error of our measurements of each star, the difference of each as seen by the eye from its true brightness, and the variation each has undergone during the past century, it is obvious that the errors of Herschel's observations must be very small. We have thus an accurate measure of the brightness of a large part of the lucid stars a hundred years ago. This will be of the utmost value in determining any changes of long period that may take place in their light.

The large meridian photometer announced in my last report has been completed, and work with it begun. Sheets have been written for about ten thousand stars to be observed with this instrument. Over seven thousand measures have so far been made, mainly of the stars in the over-lapping portions of the zones assigned to different observatories engaged in the revision of the Durchmusterung. The measurement of the stars adopted as standards for the Uranometria Argentina is also in progress.

Publications.

Volume XIII. Part I., of the Observatory Annals, was published and distributed in October, 1882. It contains the previously unpublished results of micrometric observations made here to the end of 1881, mentioned in detail in the report of last year.

The printing of Volume XIV. is now in progress. This volume is to contain the results obtained with the first meridian photometer. The part of the work first printed consists of a catalogue of stars visible to the naked eye in this latitude, with their magnitudes as determined by the photometer, by the recent estimates of the northern stars likewise made here, and by various older series of observations.

The papers mentioned below have appeared during the year as communications from the officers of the Observatory. A few are added to the list which were overlooked in the preparation of the last report.

Report of the Committee on Standards of Stellar Magnitude. By Edward C. Pickering and others. Proc. Am. Assoc. for Advancement of Science, August, 1881; xxx. 1.

On a Convenient Method of expressing micrometrically the Relation between English and Metric Units of Length on the same Scale. By W. A. Rogers and G. F. Ballou. Id. xxx. 116.

On a Method of reducing Different Catalogues of Stars to a Homogeneous System. By W. A. Rogers. Id. xxxi. 11.

On the Performance of a New Form of Level invented by Mr John Clark of the United States Coast Survey. By W. A. Rogers. Id. xxxi. 14.

New Variable Star in Puppis. By Edward C. Pickering Astronomische Nachrichten, c. 18.

Elements of Comet *b* 1881. By S. C. Chandler, Jr. Id. c. 121

Elements and Ephemeris of Comet *e* 1881. By S. C. Chandler, Jr. Id. c. 319.

On the Telegraphic Transmission of Astronomical Data. By S. C. Chandler, Jr., and J. Ritchie, Jr. Science Observer August, 1881, iii. 65.

Comet *b* 1881. By O. C. Wendell. Id. iii. 81.

Report on the Progress of the Zone Observations. By Edward C. Pickering. Vierteljahrsschrift der Astronomischen Gesellschaft, 1881, xvi. 317.

Letter to the Astronomische Gesellschaft on the Science Observer Code. By S. C. Chandler, Jr., and J. Ritchie, Jr. Id. xvi. 344.

Order of Brightness of Stars. By Edward C. Pickering. English Mechanic and World of Science, November, 1881, xxxiv. 278.

Remarkable Star Spectrum; New Planetary Nebula. By Edward C. Pickering. Science, December, 1881, ii. 581. Copernicus, December, 1881, i. 242.

Geodetic Formulæ. [Second Paper.] By J. Rayner Edmands. Appalachia, December, 1881, ii. 351.

Elliptic Elements of Comet *f* 1881 — Denning. By S. C. Chandler, Jr. Science Observer, December, 1881; iii. 91.

Reply to Inquiries regarding Time Balls. By Edward C. Pickering. Professional Papers of the Signal Service, No. 5, p. 24.

Observations and Elements of Barnard's Comet, 1881, VI. By S. C. Chandler, Jr. Astronomische Nachrichten, ci. 57.

Stars with Peculiar Spectra, discovered at the Astronomical Observatory of Harvard College. By Edward C. Pickering. Id. ci. 73.

On the Periodicity of Comet (Denning) 1881, V. By S. C. Chandler, Jr. Id. ci. 93.

Observations of Comets, 1881, V., and 1881, VIII. By O. C. Wendell. Id. ci. 231.

Observations of Comets, 1880, IV., and 1881, II. By O. C. Wendell. Id. ci. 299.

The Pleiades. By Edward C. Pickering. Astronomical Register, February, 1882, xx. 40.

Variable Stars. By Edward C. Pickering. English Mechanic and World of Science, February, 1882, xxxiv. 542.

On the Variability of DM. + 23° 1599. By S. C. Chandler, Jr. Astronomische Nachrichten, cii. 139.

Photometric Observations of Planets and of Jupiter's Satellite, III., made at the Harvard College Observatory. By Edward C. Pickering. Id. cii. 151.

Photometric Observations of the Satellites of Mars, 1881–82. [E. C. Pickering and O. C. Wendell, observers.] By Edward C. Pickering. Id. cii. 193.

Erratum in Observations of Comet Wells, 1882. By Edward C. Pickering. Id. cii. 223.

On Certain Zodiacal Phenomena. By Arthur Searle. Id. cii. 263.

On Sawyer's Variable, DM. + 1° 3408. By S. C. Chandler, Jr. Id. cii. 371.

On a New Variable Star in the Constellation Cetus. By S. C. Chandler, Jr. Science Observer, March, 1882; iii. 105.

The Mountains between Saco and Swift Rivers. [With map.] By J. Rayner Edmands. Appalachia, June, 1882; iii. 57.

The Meridian Photometer. By Edward C. Pickering. Monthly Notices of the R. Astr. Society, June, 1882, xlii. 365.

Photometric Comparisons of Lunar Objects. By Edward C. Pickering. Selenographical Journal, July–August, 1882; v. 53, 57.

On Certain Zodiacal Phenomena. By Arthur Searle. Science Observer, July, 1882, iv. 4.

On Sawyer's Variable, DM. + 1° 3408. By S. C. Chandler, Jr. Id. iv. 11.

Photometric Measurements of Sawyer's Variable (DM. + 1° 3408) and its Comparison Stars. By Edward C. Pickering. Astronomische Nachrichten, ciii. 61.

New Planetary Nebulæ. By Edward C. Pickering. Id. ciii. 95, 165.

Observations of Comets, 1881, III., IV., VI. By O. C. Wendell. Id. ciii. 145.

Small Planetary Nebulæ, discovered at the Harvard College Observatory. By Edward C. Pickering. The Observatory, October, 1882, v. 294. The Sidereal Messenger, October, 1882; i. 139.

Plane *v.* Cylindrical Surfaces. By W. A. Rogers. Mechanics, No. 31, p. 90.

A Study of the Problem of Fine Rulings with respect to the Limits of Naked Eye Visibility and Microscopic Resolution. By W. A. Rogers. Am. Monthly Microscopical Journal, September, 1882, p. 165.

A Comparison of the Harvard College Observatory Catalogue of Stars for 1875.0 with the Fundamental Systems of Auwers, Safford, Boss, and Newcomb. By W. A. Rogers. Memoirs of the Am. Acad. of Arts and Sciences, x. 389–429.

On the Conditions of Success in the Construction of Standards of Length and in their Subdivision into Equal Parts. Read before the American Society of Microscopists at the meeting held at Elmira, N. Y., August, 1882. By W. A. Rogers. Mechanics, Oct. 27, 1882, and later.

A Plan for Securing Observations of the Variable Stars. By Edward C. Pickering. Cambridge, 1882. 8°.

On the Period of R Hydræ. By S. C. Chandler, Jr. Astronomische Nachrichten, ciii. 225.

Variable Stars. — The bibliography of the variable stars undertaken by Mr. Chandler last year has been nearly completed, so far as the preparation of the list of references is concerned. The catalogue of stars suspected of variability, and the remarks relating to each, are nearly complete. A plan has been prepared for securing co-operation in the observation of these objects. A pamphlet has been published relating to this matter, and will be furnished to all persons making application for it. It is hoped that many astronomers will be inclined to aid in this work, as observations which if detached might have little value would be most useful as part of an extensive system of observations. The aid of amateurs is especially invited, since the necessary skill is soon acquired and the habit of making observations of permanent utility would often have a value as great as the direct results anticipated. The aid of lady observers is also desired, since much useful work could be done by them at their own homes. Among the many ladies owning telescopes are doubtless some who have the time and inclination, and might, if properly directed, make observations of great value to science. The observations of the light curves of the variable stars of long period have been much neglected, and observations with this object in view are also needed. A number of observers have promised their aid, and by next year I shall hope to report a large amount of useful work accomplished.

Miscellaneous.

Cambridge was not selected by the United States Commission as a station for observing the transit of Venus. This was in some respects unfortunate, as a complete series of observations were made here in 1878 of the transit of Mercury. This was done at the request of the Naval Observatory, with the expectation that we should thus be prepared to observe the transit of Venus at the present time. On the other hand it is extremely doubtful whether the results obtained during the transit will add materially to our knowledge of the distance of the Sun,

and the chance of a cloudy day is twice as great as that of a clear one. I regard the expenditure of money on large pieces of routine work, where a result of value is certain to be secured, as more advisable than any large expenditure for observing occasional phenomena, where clouds may prevent the attainment of any result. If clear, the contacts will be observed, with such other facts as can be noted without much previous expenditure of time or money. Photographs might have been taken without the aid of the United States Commission; but, as they would necessarily have been made according to a somewhat different system, it is doubtful whether they would have added to the value of the whole.

During the past year, no change has occurred in the corps of assistants, which remains as described in the last Report. The buildings and grounds have been kept in good order without alteration, except that an additional flight of steps has been placed near the southwestern corner of the building, to make the path along its southern side more readily accessible. The West Equatorial has been removed from its pier for use in experiments on the horizontal mode of mounting telescopes, and its place is temporarily supplied by an excellent six-inch refractor belonging to Mr. Chandler.

The time service, in general charge of Mr. Edmands, has been carried on successfully as in previous years. The time ball was dropped correctly on three hundred and fifty-nine days, three hundred and seventeen by telegraph and forty-two days by hand. On three days it failed to drop at twelve o'clock, and according to the rule was dropped precisely five minutes later. On two days it was impossible to obtain the signals, and on one day an accident to the machinery rendered it impossible to raise the ball.

Financial Condition.

Having thus considered the scientific work of the Observatory, its present critical financial condition may be stated briefly. The subscription of 1878, which has given us an increase in income of five thousand dollars, expires with the present year. With this, three instruments instead of one have been kept at work, the corps of assistants has been doubled, excellent progress has been made in reducing the observations of the past, and a

large number of volumes of annals and brief publications have been issued. An attempt is now being made to secure a fund of one hundred thousand dollars to render permanent this increased rate of work. Over thirty thousand dollars have already been promised conditionally, the principal limitation being that at least seventy-five thousand dollars shall be secured before next September. A failure in this subscription will involve a return to the condition of comparative inactivity which we were previously obliged to maintain. With success, we shall have the means of accomplishing the amount of work to be expected from the standing of this University, and demanded by the advanced views regarding literary and scientific work held in this part of the country.

EDWARD C. PICKERING, *Director.*

XIII.

THE WEDGE PHOTOMETER.

BY EDWARD C. PICKERING.

Presented May 10, 1882.

MUCH attention has recently been directed to the use of a wedge of shade glass as a means of measuring the light of the stars. While it has been maintained by various writers that this device is not a new one, the credit for its introduction as a practical method of stellar photometry seems clearly to belong to Professor Pritchard, Director of the University Observatory, Oxford. Various theoretical objections have been offered to this photometer, and numerous sources of error suggested. Professor Pritchard has made the best possible reply to these criticisms by measuring a number of stars, and showing that his results agreed very closely with those obtained elsewhere by wholly different methods. His instrument consists of a wedge of shade glass of a neutral tint inserted in the field of view of the telescope, and movable so that a star may be viewed through the thicker or thinner portions at will. The exact position is indicated by means of a scale. The light of different stars is measured by bringing them in turn to the centre of the field, and moving the wedge from the thin towards the thick end until the star disappears. The exact point of disappearance is then read by the scale. The stars must always be kept in the same part of the field, or the readings will not be comparable. By a long wedge the error from this source will be reduced. A second wedge in the reversed position will render the absorption uniform throughout the field. Instead of keeping the star in the same place by means of clockwork, the edges of the wedge may be placed parallel to the path of the star, when the effect of its motion will be insensible. To obtain the best results the work should be made purely differential, that is, frequent measures should be made of stars in the vicinity assumed as standards. Otherwise large errors may be committed, due

to the varying sensitiveness of the eye, to the effect of moonlight, twilight, &c., and to various other causes.

A still further simplification of this photometer may be effected by substituting the diurnal motion of the earth for the scale as a measure of the position of the star as regards the wedge. It is only necessary to insert in the field a bar parallel to the edge of the wedge and place it at right angles to the diurnal motion, so that a star in its transit across the field will pass behind the bar and then undergo a continually increasing absorption as it passes towards the thicker portion of the wedge. It will thus grow fainter and fainter, until it finally disappears. It is now only necessary to measure the interval of time from the passage behind the bar until the star ceases to be visible, to determine the light. Moreover all stars, whether bright or faint, will pass through the same phases, appearing in turn of the 10, 11, 12, &c., magnitude, until they finally become invisible. For stars of the same declination, the variation in the times will be proportioned to the variations in the thickness of the glass. But since the logarithm of the light transmitted varies as the thickness of the glass, and the stellar magnitude varies as the logarithm of the light, it follows that the time will vary as the magnitude. For stars of different declinations, the times of traversing a given distance will be proportional to the secant of the declination. If δ, δ' are the declinations of two stars having magnitudes m and m', and t, t' are the times between their transits over the bar and their disappearances, it follows that $m' - m = A\ (t \sec \delta - t' \sec \delta')$. For stars in the same declination calling $A \sec \delta = A'$ we have $m' - m = A'\ (t - t')$. Accordingly the distance of the bar from the edge of the wedge is unimportant, and, as in Professor Pritchard's form of the instrument, it is only necessary to determine the value of a single constant, A. Various methods may be employed to determine this quantity. Professor Pritchard has recommended reducing the aperture of the telescope. This method is open to the objection that the images are enlarged by diffraction when the aperture is diminished; constant errors may thus be introduced. Changing the aperture of a large telescope requires some time, and in the interval the sensibility of the eye may alter. These difficulties are avoided by the following method, which may be employed at any time. Cover the wedge with a diaphragm in which are two rectangular apertures, and place a uniformly illuminated surface behind it. Bring the two rectangles into contact by a double image prism, and measure their relative light by a Nicol. From

the interval between the rectangles and the focal length of the telescope the light in magnitudes corresponding to one second, or A, may be deduced. Perhaps the best method with a small telescope is to measure a large number of stars whose light has already been determined photometrically, and deduce A from them.

The great advantage claimed for this form of wedge photometer is the simplicity of its construction, of the method of observing, and of the computations required to reduce the results. It may be easily transported and inserted in the field of any telescope like a ring micrometer. The time, if the observer is alone, may be taken by a chronograph or stop-watch. Great accuracy is not needed, since if ten seconds correspond to one magnitude, it will only be necessary to observe the time to single seconds. The best method is to employ an assistant to record and take the time from a chronometer or clock. If the stars are observed in zones, the transits over the bar serve to identify or locate them as well as to determine their light. A wedge inserted in the field of a transit instrument will permit the determination of the light of each star observed without interfering with the other portion of the observation. If the stars are all bright, time may be saved by dispensing with the thin portion of the wedge. In equatorial observations of asteroids the light may be measured photometrically with little additional expenditure of time. Perhaps the most useful application would be in the observation of zones. When the stars are somewhat scattered it would often happen that their light might be measured without any loss of time. By this instrument another field of usefulness is opened for the form of horizontal telescope advocated at a former meeting of this Academy (Proc. Amer. Acad. XVI. 364). Very perfect definition would not be required, since it would affect all the stars equally. To an amateur who would regard the complexity of an instrument as a serious objection to it, a means is now afforded of easily reducing his estimates of magnitude to an absolute system, and thus rendering them of real value.

REPORT OF COMMITTEE A. A. A. S.

SECOND REPORT OF THE COMMITTEE ON STANDARDS OF STELLAR MAGNITUDES.

THE first report of this Committee (*Proc. Amer. Assoc. XXX*, p. 1) included a plan for the determination of standards for stars fainter than the tenth magnitude. Twenty-four bright equatorial stars were chosen and the standards were to be selected from the regions following them from two to six minutes of time and not differing in declination from the leading stars by more than five minutes of arc. The observations described below have been made at the Harvard College Observatory unless otherwise stated. The light of each of the leading stars has been determined on from seven to eighteen nights with the meridian photometer. Charts have been constructed of all the stars visible with the fifteen inch telescope, in all but three of the regions from which the standards are to be selected. Most of these charts have been submitted to a careful scrutiny with the fifteen inch telescope of the Washburn Observatory. An important test of the completeness of the charts is thus afforded.

In the following table three successive columns give the names of the twenty-four leading stars and their approximate right ascensions and declinations for 1880. The next two columns give the number of nights on which they were observed with the meridian photometer, and the resulting magnitude. The details of these measures and a comparison with various other determinations of their light will be found in the *Harv. Observ. Annals*, Vol. XIV. The last columns give the number of stars in each of the charts, and the corresponding number of stars contained in the same portions of the Durchmusterung.

Stars suitable for standards must next be selected by the help of the charts. The light of these stars should then be measured in as many different ways as possible. The Committee will be much indebted for aid that may be rendered them in this portion of their work. The early publication of the charts now becomes a matter of importance, as it would permit their immediate use for various purposes.

NAME.	R. A. 1880.		Dec. 1880.		No. Nights.	Phot. Mag.	Stars on Chart.	D. M. Stars.
	h.	m.						
γ Pegasi	0	7.1	+ 14°	31′	13	3.04	49	3
θ′ Ceti	1	18.0	− 8	48	12	3.77	27	—
α Piscium	1	55.9	+ 2	11	12	3.99	—	1
α Ceti	2	56.0	+ 3	37	11	2.68	—	2
γ Eridani	3	52.4	− 13	51	10	3.05	30	—
α Tauri	4	29.0	+ 16	16	16	1.00	19	3
ε Orionis	5	30.1	− 1	17	16	1.76	42	6
γ Geminorum	6	30.8	+ 16	30	17	2.00	150	5
α Canis Minoris	7	33.0	+ 5	32	15	0.46	96	3
ε Hydrae	8	40.4	+ 6	51	7	3.58	64	4
α Leonis	10	2.0	+ 12	33	15	1.42	39	1
θ Leonis	11	7.9	+ 16	5	10	3.47	24	3
η Virginis	12	13.8	+ 0	0	10	4.05	23	2
α Virginis	13	18.9	− 10	32	13	1.23	30	—
α Bootis	14	10.2	+ 19	48	13	0.03	25	1
β Libræ	15	10.5	− 8	56	14	2.74	39	—
δ Ophiuchi	16	8.1	− 3	23	11	2.77	48	—
η Ophiuchi	17	3.5	− 15	34	10	2.62	100	—
η Serpentis	18	15.1	− 2	56	10	3.35	7	—
δ Aquilæ	19	19.4	+ 2	53	14	3.46	—	3
θ Aquilæ	20	5.1	− 1	11	10	3.39	110	2
β Aquarii	21	25.2	− 6	6	10	3.14	52	—
α Aquarii	21	59.6	− 0	54	10	3.16	48	0
α Pegasi	22	58.8	+ 14	34	18	2.61	29	3

Respectfully submitted,

EDWARD C. PICKERING, Chairman.
LEWIS BOSS.
S. W. BURNHAM.
ASAPH HALL.
WILLIAM HARKNESS.
EDWARD S. HOLDEN.
SIMON NEWCOMB.
C. H. F. PETERS.
ORMOND STONE.
C. A. YOUNG.

Mountain Observatories.

By Edward C. Pickering.

Read March 14, 1888.

Much attention has recently been directed to the question whether the conditions are more favorable to astronomical observations on the summit of a lofty mountain than at the level of the sea. The evidence so far collected is somewhat contradictory, although there can be no doubt that for certain investigations great elevations are almost essential. It therefore appears to be of interest to consider what advantages are to be expected, what work should be undertaken, and what instruments are best adapted to securing results of value. Astronomical science is at present limited not so much by the imperfections of our instruments as by meteorological conditions. The first cause of error is the change due to the effect of variations of temperature in the instruments themselves. This is especially noticeable in large reflecting telescopes, where a slight irregularity of temperature — such as would be produced by placing the hand upon the mirror — would entirely destroy the sharpness of the image. In any precise measures care must be taken that all portions of the instrument are at the same temperature, or serious deviations will ensue. The second and more important source of error is that due to the atmosphere. Owing to variations of temperature, the density, and consequently the index of refraction of the air, is constantly changing. This effect is magnified by a telescope, so that we always perceive the fluctuations in the images of a

large telescope, such as are sometimes noticed by the unaided eye when looking through the column of heated air rising from a chimney. The effect of the absorption of the air is not very serious for objects near the zenith. At the level of the sea, about one quarter of the light of a star in the zenith is absorbed by the air. At the greatest elevations at which an observatory could be erected, only about one half of the air would be surmounted. The brightness of zenith stars will therefore only be increased a little more than a tenth of a stellar magnitude, — an amount which is scarcely perceptible by the most careful comparisons. In observing the sun, the intense light of the sky in its immediate vicinity often seriously interferes with the results. The observations of our fellow-member, Professor Langley, and of Professor Young, have conclusively shown the advantages, in such researches, of very elevated stations. The light surrounding a bright star or planet is largely due to the aberration, internal reflections, and imperfect transparency of the object-glass. It is not certain, therefore, that any great benefit would be gained by an increased altitude in observing the fainter satellites, or companions to bright stars. The principal advantage we should anticipate would therefore consist in the increased steadiness of the images. Should this result be realized, the importance of a mountain observatory would amply repay the inconvenience of conducting it. Not only would the accuracy of all measurements be increased, but close double stars would be more easily separated, and the structure of the surfaces of the planets would be more distinctly shown. Stars too faint to be seen under ordinary conditions would become visible, owing to the concentration of their light in a single point.

We do not yet know whether more hours of good "seeing" can be obtained at a great height than at a properly selected point near the level of the sea. There can be no doubt that great advantages would accrue from a proper location; but it almost always happens that political or personal reasons determine the place where a large telescope is to be erected, independently of the best climatic conditions.

The difficulties of maintaining a large observatory at a great elevation are very serious. Among them may be named the expense of transportation of all the supplies needed, the unwil-

lingness of observers to lead so isolated a life, its probable unhealthiness, and the difficulty of performing much work of any kind in a rarefied atmosphere.

The expenditure required for an observatory may be divided into the original cost of buildings and instruments, and the current expenses for making the observations, reducing them, and publishing the results. If architectural effect is not aimed at, the building expenses need not be large. The impression is prevalent that the principal expenditure should be made on the instruments. In establishing many observatories this has proved to be a fatal mistake. If much work is to be done, by far the largest appropriation should be made for current expenses, mainly for the salaries of a large corps of assistants. At the Harvard College Observatory, the current expenses for two years would cover the entire cost of the instruments. In other words, estimating the rate of interest at five per-cent, ten times as much is expended on the observations, reductions, publications, and other current expenses, as on the instruments. A similar remark applies to the observatories at Greenwich and Washington. In most forms of routine work, such as is done at these observatories, the time required to prepare the observations for printing may be estimated at five to ten times that required to make them. The relative expenditures would not be so great, since much of the reduction consists in copying and in simple computations, which can be done by less expensive assistants. Apart from the high cost of living on the summit of a mountain, high salaries must be paid to assistants, to induce them to make the sacrifices required in such a life. Obviously, then, great economy may be attained by restricting the work on the mountain to that which can be done there only, and conducting the greater portion at some lower elevation, where the best facilities exist for completing it. Among other advantages will be the convenience for supervision and direction of the work, rapidity in publication, and ready communication with other observatories.

The principal question to be determined appears to be whether the increased steadiness of the air will give a real advantage. For this purpose two similar instruments, which need not be of very large size, should be erected, one on the mountain, the other at some convenient point below. Similar work should

be done with each for one or more years. The results would show the relative advantages of the two stations.

To proceed now to details. A form of telescope described in the Proceedings of the American Academy[1] has especial advantages for mountain observations. The telescope *A B*, Fig. 1, is mounted horizontally, pointing east or west, and has a plane mirror, *C*, inclined at an angle of 45° to its axis, placed in front of it. This mirror can turn around an axis coinciding with the axis of the telescope, so that any object when crossing the meridian can be brought into the field of view.

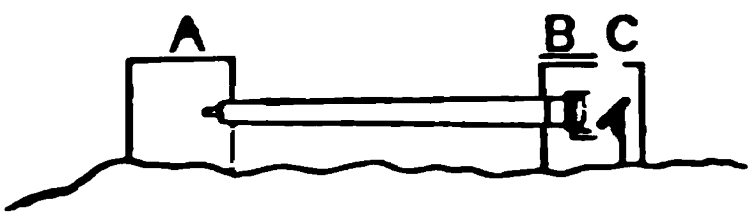

Fig. 1.

The position of the mirror can be controlled and determined by long rods extending to the eye end. Objects can be observed only when within an hour or two of the meridian, but in any large piece of work there are always enough objects so situated. The usual method of erecting a telescope of the same size in a dome is shown for comparison in Fig. 2.

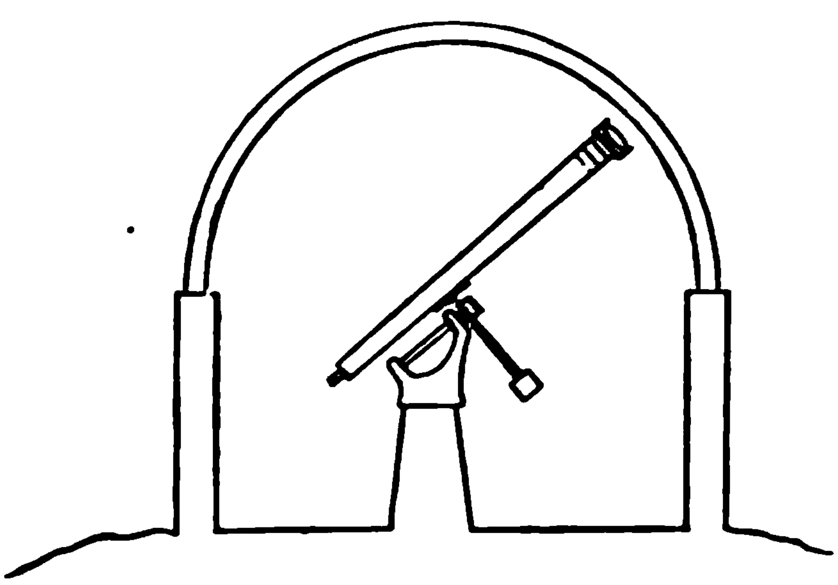
Fig. 2.

Among the advantages of the horizontal mounting may be named the following. A much smaller portion of the rock surface need be prepared for its erection. Only a small shed, which should be a part of the dwelling-house of the observer, and a pier at *B C*, covered to protect the lens and mirror from the weather, are needed. The intermediate space below the tube may be left in its natural condition. A dome requires a prepared circular floor of a diameter somewhat greater than the length of the telescope. To derive the full benefit from a dome, the view should be unobstructed. The observatory must be on the summit of the mountain, where it is greatly exposed to storms, and where fuel and water are difficult to obtain. The horizontal telescope, on the other hand, only requires a clear southern horizon. It may, therefore, often be mounted where

[1] Vol. xvi. p. 64.

it will be protected from the most violent storms. The wind is a serious source of danger to a large dome. The figures show the relative surfaces exposed to it. Snow and ice are liable to render it impracticable to use a dome; while with a horizontal telescope this difficulty is reduced to a minimum, since it is only necessary that the shutter, over *C*, may be opened. A most important advantage of the horizontal telescope is the convenience to the observer. The consequence is that the hours of observation may be prolonged, and far more work accomplished. In a rarefied air, where every movement is fatiguing, the labor of moving a dome would be very great. With a horizontal telescope, as the eye-piece does not move, the observer sits in comfort, always looking horizontally. One of the greatest advantages is that the observer may work in a warmed room. The object-glass is so far distant, horizontally, that the heat at the eye-piece will not affect the definition. Work may thus be carried on with perfect comfort on the coldest nights. The application of this to observations during the prolonged nights in the arctic regions is obvious. Portability is often a serious consideration in work at great altitudes. No portion of the horizontal telescope need be so heavy but that it could be carried by a horse, or even by a man. The tube need only support its own weight, and may be made of tin. A heavy steel tube is required to give sufficient stiffness with an equatorial mounting. The horizontal telescope has especial advantages in steadiness, as mirror, object-lens, and eye-piece are all close to the supporting pier. This is an important consideration on windy nights.

Various researches suitable to a horizontal telescope have been suggested in the article named above. In general, the greatest proportionate saving will be effected, when the least skill and time are required in the observations as compared with their reduction. Photography offers an especially promising field in this respect. The improvements in dry plates render it not improbable that photographs will replace star-maps, for the brighter, and perhaps for the fainter, stars. A skilful photographer, even if he had but little knowledge of astronomy, might obtain in a short time a collection of photographs which, by a proper discussion in a more convenient station, would yield a vast collection of valuable results. It cannot be long

before a daily photograph of the solar protuberances, with occasional photographs of the solar corona, will form a part of the routine work to be expected from astronomers. The advantages of a great elevation in such work are undoubted.

The working hours of the assistants should be devoted almost entirely to observations. But little of the reductions should be attempted, as this can be much better done at the central office. A sufficient description of the work should of course be appended, to render the results intelligible to another person, but almost all the clerical work can be better done below.

Of course many meteorological observations could be taken with advantage in a mountain observatory, and by self-registering instruments the time of the observer could be saved. The maximum force of the wind in storms, and the minimum temperature in winter, should be especially noted. An important investigation would be the measurement of the variations in the atmospheric refraction by the micrometer level (APPALACHIA, I. 63, 138). The apparent altitude of a number of the most distant points should be observed from time to time, and the atmospheric refraction deduced. Observations of the thermometer and barometer should be obtained at the same time. The value of the work would be greatly increased by simultaneous observations at one or more of the points observed. Such observations might at least be obtained at some point in the country below. Barometric observations at such a point, where the difference in height was as great as possible and the horizontal distance small, would have great value in determining the relation of the barometer to the height, as affected by other meteorological conditions.

In selecting a proper location, many preliminary observations would be necessary. We cannot depend altogether on the meteorological observations already made, as they commonly give the rainfall rather than the amount of cloudy weather. If a point could be found where the sky was almost always clear, at least in certain seasons, a great additional advantage would be gained for many purposes. Much time is now spent in preparing to observe certain occasional phenomena, and no result obtained on account of clouds. With a certainty of clear weather, one observatory could often make all the observations needed of such phenomena.

The portability of the horizontal telescope is a strong argument in favor of its use in the preliminary observations. The first trial might be made with a telescope of the usual form, small enough to be carried by a horse. As high a magnifying power as it would bear should be employed, and images of bright stars, of close doubles, and of faint companions to bright stars should be studied as they approach the western horizon. By noting the time at which the images become unsatisfactory, we can compute the altitudes at which they are equally good on different nights. We have thus a quantitative test of the steadiness of the atmosphere in different places. The amount of air looked through is proportional to the secant of the zenith distance. Suppose that at a given location we could see a star to within 30° of the horizon, as well as we could see one in the zenith at the sea-level. Since the zenith distances in the two cases are 60° and 0°, and the secants 2 and 1, we should infer that the relative steadiness of the air at these places was as two to one. Repeating the observation with a number of stars, we should soon establish a habit of observing which would give a much better determination of the condition of the air than any ordinary estimate. The transparency of the air may also be well determined by the eye alone. It is only necessary to compare a bright star, while rising or setting, with others near the zenith apparently equal to it in brightness, and note the time of such comparison. A watch and star-map are all that are needed for these observations. The difference in the absorptions will then equal the true difference in brightness of the stars. The reduction may be made subsequently by computing the altitude of each star, and deducing the law of absorption by combining all the observations made at nearly the same altitude. These observations are so easily made that they are recommended to any traveller in a region where the atmosphere appears to be particularly clear.

So much time will inevitably be occupied by the preliminary observations, that some years must elapse before we can expect to find a mountain observatory in full operation. An important step would be taken by the Club if it could lessen this interval by aiding in the preliminary work. No mountain in this vicinity is sufficiently high to be used for such an observatory as is here contemplated. This objection would not, how-

ever, apply to the preliminary observations. Mt. Washington would not be suitable on account of the prevalence of clouds and the severity of the winds and storms. The same criticism may be made to the selection of Mt. Lafayette, Mt. Adams, or Mt. Jefferson. Their pointed summits, and the difficulty and expense of occupying these stations, also render them undesirable. Mt. Moosilauk on the other hand, although less elevated, has a large flat summit, well adapted to work of this kind. It is moreover within convenient reach of the railroad, and has a carriage road to the top, over which the instruments could easily be transported. The house on the summit would afford abundant accommodation for observers, and has already been occupied during the winter by a member of this Club. Probably no point could be found, having an equal elevation, where the preliminary observations could be conducted at less expense. Several boarding-houses at the base of the mountain would furnish additional accommodation, if needed, and greater comfort in case of accident or illness. Should any unforeseen difficulty present itself, nearly equal advantages would be possessed by Mt. Mansfield, the highest of the Green Mountains. A series of observations at one of these points would clearly show the difficulties to be anticipated at a more elevated station. For the latter it is doubtful if we could find a better point than Mt. Whitney, the station selected by Professor Langley. The geographical location and meteorological conditions of this mountain are particularly favorable. It is one of the highest summits in the United States, and is so situated as to be unusually free from haze, clouds, and storms. Although at present somewhat difficult of access, it is probable that a railroad will soon be constructed to within a short distance of its base.

In conclusion, it is still uncertain whether any important advantage would be gained in most astronomical work by a great elevation, as compared with the best possible location at a moderate height. The question should, however, be decided, and it is believed that by the plan here proposed a sufficient test could be obtained at small expense. In any case valuable results would be attained, even if the observations in the observatories on the mountain summit were not much better than those at a less altitude.

A Critical Study of the Action of a Diamond in Ruling Lines Upon Glass.

By Prof. W. A. Rogers, A. M., F. R. M. S., Cambridge, Mass.

In offering a communication upon the subject indicated by the title of this paper, I am not unmindful of the fact that I enter a field in which I acknowledge a master. Since the death of the incomparable Nobert, Mr. Fasoldt, of Albany, stands easily first in the art of fine ruling. I desire to repeat here the reply which for the past three years I have invariably made to inquiries for test-plates from my own machine—viz., that with Mr. Fasoldt's special facilities for this class of work he can, I have no doubt, produce far better results than it would be possible for me to obtain by chance efforts. I have thought it better to confine my attention to another equally important problem—viz., an attempt to obtain copies of the Imperial Yard and of the Metre des Archives, at the temperatures at which they are standard, to subdivide these units into aliquot parts and then to obtain a microscopical unit whose subdivisions should be so nearly equal that the microscope would fail to reveal the difference. The first part of this work has been mainly completed. Two independently-obtained copies of the Imperial Yard yield nearly identical values for the length of this standard unit. Three independent comparisons with the Metre des Archives agree within very narrow limits in defining the absolute length of the metric unit, both at 32 and 62 degrees Fahrenheit. The subdivision of these units into aliquot parts—the yard into inches and the meter into centimeters—has been so far completed that any er-

rors which may remain will not affect the microscopical unit sought. With regard to the exact subdivision of these units, I can only report progress.

Notwithstanding this abandonment of attempts to produce test bands of the Nobert pattern, I have recently taken up the subject again, somewhat with the view of testing the claim of Mr. Fasoldt that he has succeeded in ruling lines one million to the inch, and especially by the claim that the existence of a spectrum in the bands is an evidence of the reality of the separate lines. The latter claim does not appear to be well founded. Aside from being at variance with theory, it can easily be disproved experimentally.

Before proceeding further with this investigation, I beg to refer to a theory proposed by the writer in a paper presented to the American Academy of Arts and Sciences, in 1875, in relation to the method which Nobert may possibly have employed in the production of his test-plates. Briefly stated, this theory is that the lines composing Nobert's bands are produced by a single crystal of the ruling diamond, whose ruling qualities improve with use. In the light of subsequent experience this theory may be stated in the following way: When a diamond is ground to a knife edge, this edge is still made up of separate crystals, though we may not be able to see them, and a perfect line is obtained only when the ruling is done by a single crystal. When a good knife edge has been obtained the preparation for ruling consists in finding a good crystal. Occasionally excellent ruling crystals are obtained by splitting a diamond in the direction of one or more of the twenty-four cleavage planes which are found in a perfectly-formed crystal. A ruling point formed in this way is, however, very easily broken, and soon wears out. Experience has shown that the best results are obtained by choosing a crystal having one glazed surface and splitting off the opposite face. By grinding this split face, a knife edge is formed against the natural face of the diamond which will remain in good condition for a long time. When a ruling crystal has been found which will produce moderately heavy lines of the finest quality, it is at first generally too sharp for ruling lines finer than 20,000 or 30,000 to the inch, even with the lightest possible pressure upon the surface of the glass. But gradually the edges of this cutting crystal wear away by use

until at last this particular crystal takes the form of a true knife edge which is parallel with the line of motion of the ruling slide. In other words, when a diamond has been so adjusted as to yield lines of the best character its ruling qualities improve with use. If Nobert had any so-called "secret," I believe this to have been its substance.

The problem of fine ruling consists of two parts—first, in tracing lines of varying degrees of fineness; and, second, in making the interlinear spaces equal. The latter part of the problem is purely mechanical, and presents no difficulties which can not be overcome by mechanical skill.

It will be the aim of the present paper to describe the more marked characteristics of lines of good quality ruled upon glass, and to illustrate these characteristics by corresponding specimens. To one who is familiar with Nobert's bands, a perfect line need not be described. It is densely black, with at least one edge sharply defined. Both edges are perfectly smooth. Add to these characteristics a rich black gloss, and you have a picture of the coarser lines of a perfect Nobert plate. How are those lines produced? In the study of the action of a diamond in producing a breaking fracture in glass, the microscope seems to be of little service, but we can call it to our aid in the study of its action in ruling smooth lines.

One would naturally suppose that a line of the best quality would be produced by the stoppage of the light under which it is viewed by the opaque groove which is cut by the ruling diamond. Without doubt this is the way in which lines are generally formed. But it is not the only way in which they can be produced. An examination under the microscope will reveal the fact that in some instances, at least, a portion of the glass is actually removed from the groove cut by the diamond; and that the minute particles of glass thus removed are sometimes laid up in a windrow beside the real line, as a plow turns up a furrow of soil. On the finest plate I have ever produced every line remained in perfect form for about two months. I then first noticed a tendency on the part of some of the single lines to disintegrate, while the lines ruled in closer bands seemed to retain their good qualities. This disintegration finally became so marked that, as an experiment, I removed the cover and cleaned

one-half of the surface of the glass by rubbing with chamois skin. The difference in the appearance of the two halves is now very marked. Above, the dense black lines remain. Below, a ragged abrasion of the surface of the glass has taken place. Above, the furrowed lines as originally formed are preserved; below, there is a coarse scratch. It may be said that the action in this case is accidental and abnormal. In reply I can say I have prepared plates which show that the particles of glass removed take four characteristic forms: (a) They appear as chips scattered over the surface of the glass. (b) They appear as particles so minute that when laid upon a windrow and forming an apparent line they cannot be separated under the microscope. (c) They take the form of filaments when the glass is sufficiently tough for them to be maintained unbroken. (d) They take a circular form.

I regret that three of the most striking specimens were broken in mounting. In one, a perfect line about one thirty-thousandth of an inch in width was formed with a clear space between it and the groove cut by the diamond. There was not a single break in these filaments from beginning to end, but at nearly equal intervals of about one-hundredth of an inch half-knots were formed similar to those formed in a partially-twisted cord. By rubbing the surface at one end, these filaments were broken up. For the most part they assumed a semi-circular form, but some of them maintained their thread-like form and became twisted together in the most intricate fashion.

In the third specimen, which was broken in mounting, the glass removed took a spiral form like the spiral chips from steel when turned in a lathe. A projecting crystal of the diamond caught these spirals and carried them unbroken to the end of each line, leaving them a tangled mass of threads. Even after they were protected by a cover-glass cemented to the surface, many of these spirals remained intact. Judging by the difference in focus of the various parts, the height of the mass, before the plate was covered, must have been one five-hundredth of an inch.

The same ruling crystal may produce smooth lines or either chips or threads, according to the motion of the diamond, as may be seen by examination of the accompanying rulings. In these

plates one-half of the lines of the bands are ruled by a forward motion and one-half by a backward motion of the diamond. Chips may be formed in ruling bands of very fine lines, as illustrated in the bands of lines twenty-four thousand to the inch.

It must not, however, be supposed that lines of the best quality always present the appearance described above. While it is exceedingly rare that lines appear as well after the surface of the glass has been rubbed as before, many instances have occurred within my experience in which the difference, especially in fine lines, was not particularly noticeable. According to the limited evidence at hand, the coarser lines of Nobert's bands present some of the characteristics which I have described. I have restored two of these plates, in which the lines had become nearly obliterated by some kind of condensation under the cover-glass. In one, the quality of the lines was not much affected by the operation of cleaning, but in the other the dark gloss which characterizes the heavy lines of nearly all of Nobert's plates was entirely destroyed. The finer lines, however, were much less affected than the coarse ones.

Lines of the character thus far described are evidently unsuited to the ordinary works of the microscopist. It is my experience that lines which are the most symmetrical in form and the most beautiful in appearance are produced indirectly rather than by the direct action of the diamond in cutting a groove in the glass. They can be protected to a certain extent by a cover-glass, but they are liable to undergo changes which will affect their original structure. Except for purposes of investigation, therefore, there is no advantage to be gained by ruling lines of this character.

Three conditions must be fulfilled in the production of lines having a permanently good character:

1. The glass must be tough. There is a marked difference in the character of the filaments produced, and, to a certain extent, of the lines themselves, yet the conditions under which the lines in the series of plates illustrating this paper were ruled were the same in nearly all of the plates—i. e., the same diamond was used; its setting remained unchanged, and there was no change in the pressure of the diamond upon the surface of the glass. I may add also, that I have in my collection several other plates which were ruled

especially to test the question of the requisite quality of the glass. They all agree in giving evidence that glass of a given quality will always yield lines of nearly the same quality—the ruling crystal remaining the same and in the same position.

2. The greatest difficulty encountered in setting a ruling crystal is to obtain one which will rule lines of the required quality which will retain their form after the surface of the glass is rubbed. The crystal with which nearly all the plates of this series were ruled was only obtained after a search continued at intervals through several weeks. Sometimes a diamond which will rule good light lines will not produce good heavy lines, and *vice versa*. According to my experience it is better to have a special diamond for each class of lines desired, though the diamond with which the present series of plates was ruled seems well adapted to every kind of work required except, perhaps, the production of the finest bands. An examination of plates illustrates the wide difference in the character of lines ruled with the same diamond, after the edges of the ruling crystal have been worn smooth. In one there are two sets of lines, side by side, in one of which the surface has been rubbed, and in the other of which the lines have been left undisturbed. The difference is very marked. It may be said here that the surface of a ruled plate should always be cleaned by rubbing in the direction of the lines only, never at right angles to the lines. It will often happen after sharp rubbing that the lines appear ragged, when the difficulty is that the chips have not all been removed from the grooves. Rubbing with Vienna lime, moistened with alcohol, will usually complete the cleaning satisfactorily.

3. After a crystal has been found which will fulfill the conditions of producing a line which will bear cleaning there still remains a difficulty which will only be revealed after the lapse of considerable time. This is well illustrated in one plate, in which the lines were as perfect as could be desired for several days after they were ruled. The lines of the band are now completely broken up. Evidently they were under a strain, which finally became so great that resistance to rupture became impossible. This, however, is an extreme case. Generally the lines simply enlarge at certain points. Usually the termination of the enlargement occurs at irregular dis-

tances along the lines, and it is nearly always very sharply defined. The most curious action of this kind which has ever come under my notice is where the lines have broken up into a form something like the strand of a heavy rope.

The process of setting a diamond is as follows: The holder has the means of adjustment in three planes: (a) An adjustment in a horizontal plane; (b) an adjustment in a vertical plane; (c) an adjustment in a plane at right-angles to the ruled lines. It is my practice to begin by giving the knife edge of the diamond considerable inclination to the line of motion of the ruling slide. I then rule a series of single lines at different known angles of inclination, care being taken to pass the line of parallelism. An examination of the character of the lines thus ruled will enable one to determine within narrow limits near which one the knife edge is set parallel with the slide. After a fair line has been obtained in this way, a sharp crystal is generally found by tilting the diamond in a vertical plane, though it will often be found necessary to make the third adjustment mentioned. Sometimes the cutting crystal is lost after ruling a few lines, but generally good results can be obtained after a constant service of weeks, and even months. A crystal is lost either by being broken off or by being worn out. When a crystal has been lost it need not be concluded that the diamond needs sharpening. It is only necessary to find a new crystal, an operation requiring patience rather than skill.

It should be stated that while this theory of individual-cutting crystals seems to be the true one, I have never been able to detect them by an examination with the microscope. It is only by their behavior that their existence can be recognized.

One of the most severe tests of the ruling qualities of a crystal consists in producing, without fracture, heavy lines which cross each other at a small angle of inclination and which will receive graphite without interruption of continuity at the intersection. Lines ruled at right-angles and forming small squares afford a better test than parallel lines. In one plate presented the curved lines formed by the intersection of straight lines are nearly perfect in form, and they hold the graphite quite as well as the original lines. In another plate I have attempted the representation of the nucleus of a

comet. The filling is not quite as perfect as in the other plate, but this is due to the quality of the glass. Attention is called to the granular structure under a moderately high power. I have found rulings of this form to be an excellent test of the quality of the glass required for receiving the best lines. In general, the first filling of the lines is the most perfect. One plate affords an illustration, exceedingly rare, of lines which receive the lines equally well after repeated fillings. Lines as fine as fifty thousand to the inch very readily receive the graphite. The limit beyond which it seems impossible to go may be placed at about one hundred thousand to the inch.

A few words may properly be added here with regard to the protection of ruled lines. When lines are formed by a true groove in the glass, it is better that they should remain unprotected. But when the lines are formed in the manner illustrated by the plates of this series, the quality of the lines in the end is pretty sure to deteriorate whenever there is an actual contact of the cover-glass with the slide. I have made serious efforts to overcome this difficulty, but with only partial success. Slides mounted with gutta-percha rings generally remain in good condition for a long time, especially if, after expelling the air as far as possible by heat, a ring of white wax cements the rim of the cover-glass to the slide. But even with this precaution there is no certainty of final preservation. If it should be found that the brass slides of this series are convenient in manipulation, their adoption can be recommended, since they entirely obviate this difficulty. They are made in the following way: A hole having been made in the center, a flange is left one two-hundredths of an inch in thickness. The cover-glass is then cemented to the surface of the brass, and the rulings are made on the under side. The protection is made by dropping upon the ledge of brass a rather thick circle of cover-glass, which is held in position by a circular brass wire.

After this digression, I return to the consideration of the credibility of Mr. Fasoldt's claim that he has succeeded in ruling lines one million to the inch. At this point it is only fair to say that until recently I have shared in the general incredulity with which Mr. Fasoldt's claim has been regarded. Indeed, I still think he has

placed the limit just a *trifle* too high. But if the limit is reduced one-half, I am by no means sure but that it may be reached. Possibly it may have been already reached.

But what evidence have we that it is possible to see single lines of this degree of fineness, granting that it is possible to produce them? The answer to this question involves another inquiry—viz., has the microscope reached its highest visual possibilities? Here again it is necessary to draw a sharp distinction between visibility and resolution. In the matter of limit of resolution it must be admitted that little or no progress has been made since the resolution of Nobert's nineteenth band. The distinguishing feature of Nobert's lines is a certain boldness which enables them to be photographed, and it is to photography, supplemented by the statement of the maker, that we owe the certainty of the resolution of the nineteenth band. But all attempts to go beyond this band, even with Nobert's later plates, have proved failures. I cannot learn that anyone has yet succeeded in photographing a Fasoldt plate as high as 100,000 to the inch. Certainly various attempts which have been made with bands of my own ruling higher than about 70,000 have not been successful. There are several Nobert plates of the new pattern in this country. They run as high as 240,000 lines to the inch, but who has gone beyond the number of lines in the nineteenth band? With great respect for the honest belief of several microscopists who claim to have resolved Fasoldt's bands as high as 152,000 to the inch, I must yet hold to the opinion that in no case has the resolution been proved by a test which will be generally accepted by microscopists. There is one test, and only one, which is absolutely decisive—viz., the one originally proposed by Nobert, that of ruling a definite number of lines in a band of given fineness, and keeping the number secret until the microscopist could give the correct count, not merely in one instance but in several. Even here we must depend upon the honesty of the maker in revealing the correct count. Has the correct count been made in any Fasoldt plate as high as 100,000 to the inch? I think not. Has it been done with any band of my own ruling of the same degree of fineness? No. Let us marshal the evidence, pro and con, offered by experience.

Mr. Fasoldt's finest bands present a perfectly smooth and uniform surface. They have well-defined limits and the width of the bands is what it should be by the number of lines claimed to be ruled. (b) According to present experience single lines can be ruled several degrees finer than I have been able to detect under the microscope. About four years since I sent to Professor J. Edwards Smith a ruled plate with a statement of the number of bands, accompanied with a description of the same. Soon after, I received a letter from Professor Smith saying there must be some mistake in the description, as he was unable to find two of the bands. I replied that the bands were certainly ruled, and that I thought I could convince him of that fact. I therefore requested him to re-examine the plate with the greatest care, and if he was still unable to find the bands to return the plate to me. After a vain endeavor to discover them the plate was sent to me. I removed the cover, filled the lines with graphite, remounted the slide, and returned it to Professor Smith. Not only had the invisible bands become visible, but the separate lines, with an interlinear space of 1-80,000 of an inch, were easily seen. Now, when Professor J. Edwards Smith, an acknowledged expert in the manipulation of the microscope, is unable to find lines which are really in the center of the field of the microscope, I suspect that other observers may find a similar difficulty. Among the plates presented is one series which were ruled to illustrate the possibility of producing lines which really exist, but which are invisible under the microscope. On one plate there are two sets of lines, one set on the slide and the other on the under side of the cover. Between the bands, 10,000 and 24,000 to the inch, the entire intervening space is filled with a continuous series of bands, 24,000 to the inch. I have not been able to see the lines of the last band. In another plate there are a series of bands containing twenty-one lines each, the entire linear space being 1-2,000 of an inch. The first eleven lines are ruled with a forward motion of the diamond, and the second ten lines are ruled with a backward motion. The last two bands are preceded by heavy finding lines. Each of the last three bands are followed by bands 24,000 to the inch. I think it will be found difficult to see the lines of the last two bands under any illumination at present in use, and yet I am

confident that the lines exist. I found my belief upon two bits of evidence: First, the pressure of the diamond upon the glass was sufficient to produce the lines. With considerable less pressure there would still have been a constant contact between the diamond and the glass. Second, I saw them ruled through the sense of hearing. When a diamond does its very best work it produces a sharp, singing tone, which is audible at a distance as great as twelve inches. This singing tone I distinctly heard for every line ruled. It is even more marked in ruling the finest lines than in coarse ones. I have two singing diamonds, or rather two diamonds with singing crystals, and these two are the ones with which I have done my best work.

The argument against the visibility of single-ruled lines which can not be seen with the present means at command, even if within the limits of possibility, considered in a physiological sense, is in one respect a sufficient answer to the evidence offered in favor of their existence. This evidence, while not exactly negative in its character, is yet not sufficiently conclusive to be regarded as coming under the head of proof through the medium by which the existence of any fact is attested—viz., the medium of some one of the senses. But may it not be true that we have not yet reached the fulfillment of the conditions necessary to visibility? It certainly cannot yet be safely asserted that it is impossible to see a material particle which has, in one direction, a magnitude not exceeding 1-500,000 of an inch. Photography offers the evidence, somewhat negative in its character, that the limit of visibility is reached with lines having a width of about 1-200,000 of an inch. Lines of this width are the finest that have ever been photographed. But the most conclusive evidence against the certainty of being able to produce lines as fine as 500,000 to the inch consists in the fact, repeatedly proven in my own experience, that lines which appear to be excessively fine often have a real width two or three times as great as they appear to have, as has been proved conclusively by filling the lines with graphite, which brings out the real limit. This phenomenon will come out again in connection with the subject of resolution.

I have already stated my belief that the limit of resolution has been so nearly reached that, though it is quite possible under a combination of favorable circumstances to obtain a resolution a lit-

tle beyond 113,000 to the inch, the uncertainty which must always attend observations of this character is so great that the certainty of resolution can not be safely asserted. In consideration of this uncertainty and of the fact that so little progress has been made in resolution compared with the recent advance in the construction of objectives, I beg to propose as a test the visibility of single-ruled lines in place of the resolution of these lines in close combination. Instead of bands of lines of the Nobert pattern, I propose a series of bands, each having the same interlinear unit, but with the lines of each successive band finer than those of the preceding band. The space between the lines should not be so great as to interfere with their easy detection, nor so small as to require any effort in resolution. One mikron is a convenient unit. A heavy line should precede the band in order to facilitate finding it.

According to my own experience there are four facts which must always throw grave doubt upon any reported case of difficult resolution:

1. It is well known that by the manipulation of the light, every other condition remaining the same, it is possible to vary the apparent number of lines in a given band of coarse rulings. Can any-one offer a reason why there should not be the same difference with bands of fine lines closely ruled?

2. I have many times ruled bands of lines with the interlinear spaces distinctly marked, but in which each line was in reality considerably wider than the space between the lines, as I have proved by extending single lines beyond the others and filling them with graphite. The only explanation of this singular fact which I can suggest is that the diamond may possibly cut square down at one edge of the line and for the remainder of the line produce only an abrasion of the surface of the glass, which is so slight as not to interfere with throwing up a furrow upon the remaining portion.

3. Lines of a given depth appear finer when closely ruled in bands than they do in single lines.

4. I add another observation with some hesitation, since I have not been able to prove its truth beyond peradventure. I have often, but not always, found that when single lines, apparently invisible, are placed in close combination in bands, they not only form a

visible band, but a band capable of apparent resolution into separate lines. Can anyone offer a reason why we can see in combination what we can not see as separate parts? Of course I shall be at once reminded by the astronomer that it is much easier to pick up a cluster than to see scattered stars of the same magnitude. But when it is once found, the separate stars composing it are no more easily seen than stars of the same magnitude more widely scattered. I offer this observation in a tentative way, since it has, if true, an important bearing upon the question of the ultimate limit of resolution. Among the accompanying plates is one that illustrates the statement here made. This plate consists of a series of bands, 12,000 to 24,000 to the inch, each preceded by a heavy finding line. The lines of each successive band are finer than the preceding. The last two bands were ruled with the same pressure of the diamond as the fourth band preceding. The intervals at which they were ruled are 1-80,000 and 1-200,000 of an inch. I do not by any means vouch for the existence of the separate lines, yet the bands are smooth, and there is a distinct difference in the appearance of the two halves of the 80,000 band, the first having been ruled with a forward and, the second with a backward motion of the diamond. The corresponding single lines of the fourth band preceding are wholly invisible. This plate seems to show that the visibility of the lines in bands depends somewhat on the narrowness of the interval between the lines, since the lines of the same degree of fineness with an interval of 1-24,000 of an inch can not be seen.

It is obvious that this whole question of resolution needs the most careful consideration and investigation, since it bears an intimate relation to the limit of visibility of single particles of matter. Mr. Hitchcock, in a recent number of his *Journal*, has made the claim that resolution has to a certain extent ceased to be a test of the quality of an objective. I suspect that this claim will be found to have some foundation in fact. For the last ten years we have only the assertion of resolution, without doubt honestly made, but yet unaccompanied with the proof. It is time that the proof should accompany the assertion. I insist that simple vision does not afford the required proof.

Now we must face this question as honest inquirers after truth. There is a limit which theory places to resolution with objectives

of given resolving power, not to visibility, as has been frequently stated. Before we can safely assert that observation has gone beyond theory, we must be prepared to offer evidence which can be placed upon record, can be discussed deliberately, can be weighed impartially in the balance with counter evidence, and can still stand unimpeached. Do you say that is hardly worth the trouble? I reply that the issue here raised comes to the surface in one form or another at almost every point in physiological and pathological investigations. It will do no harm to recall the number of times it has at this meeting stood as a sentinel at the entrance to the temple whose mysteries we are seeking to explore. Has not the question so tersely put by Dr. Gleason at the Elmira meeting of this Society: "Do we see what we see, or don't we see what we see, or do we see what we don't see?" been the stopping-place of more than one important issue raised at this meeting? I hope I do not need to say that I have no personal ends to serve in an inquiry in which I happen to be a personal factor. Let us then have a test which will forever set at rest this vexed question of resolution. I submit for your consideration the following outline of a test which I venture to think will be sufficient and conclusive: Let Mr. Fasoldt rule three plates under as nearly the same conditions as possible, except in the number of lines in the different bands of each plate. Let him label each plate and accompany it with a full description of the number of lines in each band. Let these plates be sent to any gentleman in whom the great body of microscopists have confidence as eminently qualified to conduct an investigation of this sort, such as Prof. H. L. Smith of Geneva or Col. J. J. Woodward of Washington. Let whoever receives the plates remove the labels of Mr. Fasoldt and put in their place labels whose signification is known only to himself. Then let the gentlemen who think they have resolved 152,000 lines to the inch take the plates, make their count of the lines in each band, and send in their report. Let the plates also be photographed, and let the number of lines be counted; then let the results of these investigations be published. If all substantially agree in the count, this will end further discussion.

The limit of visibility of a single particle of matter under the microscope bears an intimate relation to the limit of naked-eye visibility.

My attention was first called to the smallness of this limit by an accidental circumstance. I had ruled a micrometer upon a thin cover-glass, consisting, as I supposed, of moderately coarse lines. After several vain attempts to discover traces of the lines ruled, I chanced while holding the glass at a certain angle with respect to the source of light to breathe upon it. At the instant the flim of moisture was passing off, I was surprised to be able to see all the lines which were ruled, one hundred to the inch, with the greatest distinctness. I then carefully filled the lines with graphite, when they were, after the closest inspection, found to be as fine as any I have ever ruled. According to the nearest measurement I could make, their width was about one-sixth of a mikron. Repeated observations gave in every case satisfactory evidence of visibility. In order to ascertain what effect the thickness of the glass might have upon the visibility, the cover-glass was lightly cemented to a glass slide with gutta-percha, when it was found that the lines were by no means as distinctly visible as before. The cover was then removed, when the original observation was easily confirmed. The lines of this plate were readily seen by Prof. Pickering and by several assistants connected with the observatory. Unfortunately the glass was broken in an attempt to mount it upon a brass slide.

While it is a simple matter to rule lines which are easily visible by the unaided eye, especially in sunlight, having a width not exceeding one-fifty thousandth of an inch, I have never since succeeded in obtaining a plate quite as good as the one described. Clearly the ruling crystal has been broken off before this particular plate was ruled, and, as often happens, a minute and delicate crystal remained, which produced the lines which were really traced. In the course of subsequent experiments I found that while the visibility was increased by the flim of moisture, exceedingly fine lines could be seen without this aid to vision when the proper angles of inclination to the source of light are obtained. To get the best results the ruled surface should have an angle of about 15 degrees with the source of light and the lines themselves should have nearly the same angle of inclination. Everything depends upon getting the exact angles of inclination required. More striking results are obtained by sunlight than by artificial light. Highly polished

metals, especially tempered steel and iridium, yielded better results than glass. I will not undertake to say how fine lines traced upon metal can be seen, but I suspect that the limit of naked-eye visibility is far beyond the capacity of ruling. I have a plate of highly polished and nearly pure iridium upon which there are traced a series of lines which are discernible by the eye in sunlight, but which I have never yet been able to see under the microscope by direct light. Yet these lines are easily seen with a low power objective under certain conditions.

I do not propose to offer any theory to account for the facts which I have observed, not even the one which would naturally be the one first suggested,—viz., that of visibility by reflection. I admit that the apparent width of the lines would be increased if the real and reflected lines could be seen side by side. It can be easily shown that the lines in one of the accompanying plates are visible under conditions in which it is impossible for reflection to take place. For the present I content myself with stating the facts of observation illustrated by the ruled plates by which these observations can be repeated.

I close this paper with the suggestion that the increase in the efficiency of the microscope will probably come from the better manipulation of the light under which an object is viewed. At present the unaided eye is a not very unequal competitor of the microscope in the matter of simple vision. In fact, there are certain phenomena connectd with this question which can be better studied by the unaided eye than under the microscope. I believe it to be possible to see under the action of sunlight what can not be seen under any objective. There has been produced upon my ruling-machine, upon a polished surface of tempered steel, a band of ten thousand lines, covering a space of four inches. I have tested the equality of the spacing for aliquot parts of a revolution of the screw in every possible way by direct measurement. Other observers have done the same thing. I can hardly be wrong in the assertion that the spaces indicated by even tenths of a revolution are exactly equal as far as any tests of direct measurement can be applied. Yet, by holding this bar in a certain position with respect to the source of light, the limits of each revolution of the screw can be distinctly

seen. These waves of light and shade indicate an error which can be seen by the unaided eye, but which can not be measured with certainty.

Finally, if the visibility of ruled lines is so erroneously increased by the position which they occupy with respect to the source of light, why may not the visibility under the microscope be increased in nearly the same proportion by some mechanical device which shall enable the observer to find *exactly* the proper angle of inclination at which the light should be thrown upon the object in order to secure the best possible results ?

Micrometer Scale A, 1882.

The following history of the National Committee on Micrometry, the report of that Committee at the Chicago meeting, the report of of Prof. Rogers on the standard Micrometer Scale A, 1882, and the rules for the custody and use of the bar are brought together here in order that the value of the Micrometer may be better appreciated and its use more generally understood.

HISTORY OF THE NATIONAL COMMITTEE ON MICROMETRY.

At its session in Indianapolis, in August, 1878, the American Society of Microscopists adopted a resolution referring to the various microscopical societies certain questions pertaining to micrometry. It soon became evident that satisfactory progress toward a general agreement could not be reached without securing concerted action by the appointment of a committee representing the views of various societies. In accordance with many requests, the undersigned being then president of the American Society Microscopists and also president of the Troy Scientific Association, one of the oldest of the societies interested, brought the matter before the Troy society, which thereupon addressed a communication to all the microscopical societies in the country inviting their co-operation in the appointment of a national committee on micrometry. Nearly all the active societies. as well as many distinguished specialists in this branch of science, returned a prompt reply, approving the project and furnishing valuable hints and advice as to its execution. Many of the societies nominated members to represent them upon the committee, which ultimately became organized as follows:

Prof. Wm. Ashburner, San Francisco Microscopical Society; Prof. F. A. P. Barnard, L. L. D., *Chairman*, American Metrological Society; Lester Curtis, M. D., State Microscopical Society of

Illinois; Geo. E. Fell, M. D., Buffalo Microscopical Club; Henry Jameson, M. D., Indiana Microscopical Society; Prof. S. A. Lattimore, Rochester Academy of Sciences; Rev. Samuel Lockwood, State Microscopical Society of New Jersey; Prof. Ed. W. Morley, Microscopical sub-section American Association Advancement of Science; Joseph G. Richardson, M.D., American Postal Microscopical Club; Prof. Wm. A. Rogers, American Society of Microscopists; Prof. Stephen P. Sharpless, Boston Microscopical Society; Prof. H. L. Smith, Hobart College, Geneva, N. Y.; Prof. A. H. Tuttle, Microscopical Section Tyndall Association, Columbus, O.; C. M. Vorce, Cleveland Microscopical Society; R. H. Ward, M. D., *Secretary*, Troy Scientific Association; J. J. Woodword, M. D., U. S. A. Medical Museum, Washington, D. C.

Finding nearly all the societies to be in favor of adopting the *micron* ($\mu = \frac{1}{1000}$ m.m.) for our unit in Micrometry, the $\frac{1}{1000}$ millimeter being more convenient than the $\frac{1}{100}$ as well as more in accordance with usage abroad, this unit was adopted, and a communication was tendered to the Buffalo meeting of the American Society of Microscopists, proposing the withdrawal of its recommendation of $\frac{1}{100}$ m.m. as a micrometric unit. This report was accepted, its proposal adopted, and the subject referred back to the the committee with instructions to report further at the next annual meeting. This Society subsequently nominated a member as its special representative on the committee.

The other branch of the committee work, the selection or preparation of a standard micrometer, presented greater difficulties and caused greater delays. After much conference and correspondence, it was decided to procure, from a source capable of giving it originally an official character, a new scale as nearly indistructable as possible and of carefully determined value. The U. S. Bureau of Weights and Measures, through the kindness of Prof. J. E. Hilgard, undertook to prepare and authenticate such a standard, and, after delays unavoidable in such work, a scale excellently ruled on a platin-iridium bar and verified with great care by Prof. C. S. Pierce was placed at the disposal of the committee in August, 1882. A sub-committee on testing this micrometer was appointed, on whose behalf Prof. W. A. Rogers subjected the plate to a prolonged

and elaborate study which was not completed until August, 1883. It then seemed inexpedient to incur the great further delay of a repetition of the measurements by each member of the sub-committee, since the plate, being known to be as accurately prepared as could be hoped for at the present time, and being offered with the official sanction of the U. S. Bureau of Weights and Measures, was all that could be asked under the circumstances, and since discussions as to the ratio of its spacings might be completed, if capable of ever being completed beyond the possibility of further question, as well after its adoption as before. The committee therefore accepted the plate and unanimously tendered it to the American Society of Microscopists.

R. H. WARD,

Secretary of the National Committee on Micrometry.

Report of the National Committee on Micrometry.

Dr. Lester Curtis, Secretary *pro tem.*, for the National Committee on Micrometry, presented the following report:

The National Committee on Micrometry would respectfully submit the following report to the American Society of Microscopists:

We would recommend the adoption as a standard by the Society of the bar placed in our possession by Dr. R. H. Ward, Secretary of the committee, and adopt without change the accompanying report of Prof. Hilgard, Superintendent of the United States Coast Survey, and Director of the Bureau of Weights and Measures, in accordance with the following motion prepared by Dr. Ward:

Resolved, That the bar prepared by the United States Bureau of Weights and Measures as a standard for micrometry be accepted, and that a sub-committee of three be appointed to secure copies on glass for such societies as may desire them.

Resolved, That the following report be tendered to the American Society of Microscopists: The national committee on micrometry having received from the United States Bureau of Weights and Measures an excellently ruled bar designed and tendered by the Bureau as a standard for micrometry, and believing that such a standard should be subject to the approval and sanction of the Society, hereby tenders the standard to the Society with the recommendation that it be accepted and adopted as a basis for future studies and discussions in micrometry.

PROF. HILGARD'S REPORT.

Centimeter Scale A, 1882.

This scale is divided into ten millimeters, each division being marked by three lines distant from one another ten microns and the measurement is to be made from the mean position of one triplet of lines to that of another. The first millimeter is again divided in the same manner into tenths of millimeters. The first tenth of a millimeter is subdivided into ten spaces of ten microns each. There are thirteen of these lines at the beginning of the centimeter, the first tenth of a millimeter being measured from the mean of the first three

to the mean of the eleventh, twelfth and thirteenth. The scale is engraved on a piece of platin-iridium made by Matthey, and containing 20 per cent. of iridium. The coefficient of expansion of this metal from 0 to 100 degrees C., according to Ste. Clair Deville and Mascart [Annales de l'Ecole Normale, second series, tome VIII., p. 9], is 8.778μ per degree centigrade per meter. But at ordinary temperatures it is no doubt somewhat smaller. The whole centimeter has been found to be 2.18μ shorter than Glass Centimeter No. 1 (Dec. 25, 1878), at 65 degrees F. and also at 70 degrees F., the two centimeters having sensibly the same coefficient of expansion at that temperature. This Glass Centimeter No. 1 (1878, Dec. 25,) is 1.61 microns longer than a mean centimeter of glass, decimeter scale No. 4. This mean centimeter is provisionally taken to be 2.80μ too long at 70 degrees F. We can refer this length to a wave length of light more accurately than to the meter—namely, the correction just given makes the wave length of Kirkhoff, 1,200.6, to be 0.5624918μ in air at 30 inches reduced pressure and 70 degrees F. (This value is subject to future correction).

The corrections to the different divisions of the scale A, 1882, at 70 degrees F. are as follows:

First line to second line, too long	0.08μ
Second line to third line, too short	0.34
Third line to fourth line, too short	0.05
Fourth line to fifth line, too short	0.09
Fifth line to sixth line, too long	0.41
Sixth line to seventh line, too short	0.20
Seventh line to eighth line, too short	0.39
Eighth line to ninth line, too long	0.19
Ninth line to tenth line, too long	0.05
Tenth line to eleventh line, too short	0.20
Eleventh line to twelfth line, too long	0.18
Twelfth line to thirteenth line, too short	0.23

TENTHS OF MILLIMETERS.

First, too short	0.54μ	Sixth, too short	0.52μ
Second, too short	0.33	Seventh, too short	0.37

Third, too short.... ....0.65μ	Eighth, too short..........0.48μ
Fourth, too long.........0.10	Ninth, too short...........0.41
Fifth, to short...........0.07	Tenth, too long...........0.16

MILLIMETERS.

First, too short..........0.31μ	Sixth, too long............0.87μ
Second, too short........0.27	Seventh, too long..........0.53
Third, too short 0.27	Eighth, too long...........0.49
Fourth, too long (F. 55)..0.28	Ninth, too long............0.28
Fifth, too long..........0.45	Tenth, too long...........0.19

A Study of the Centimeter, Marked "A," Prepared by the U. S. Bureau of Weights and Measures for the Committee on Micrometry.

At its last session, this Society conferred the honor of appointing me as its representative upon the general committee upon micrometry, representing the various microscopical societies of this country. This committee has, through its chairman, President Barnard, of Columbia college, and its Secretary, Dr. Ward, of Troy, obtained from Professor J. E. Hilgard, Superintendent of the United States Coast Survey and Director of the Bureau of Weights and Measures, a standard centimeter ruled upon a platin-iridium surface which appears to satisfy every requirement essential in a standard unit of measurement. This standard was sent to me by Dr. Ward at the beginning of the present year, with the request that I should compare it with the one-hundredth part of the meter in my possession, the relation of which to the Metre des Archives at 62 degrees Fahrenheit has been definitely established. Inasmuch as no special investigation of the coefficient of expansion of this particular plate of platin-iridium, attached by silver solder to a plate of brass had been made, I was requested by Dr. Ward to undertake this investigation.

The standard was received by me January 20 of the present year. In its examination with a half-inch objective supplied with a Tolles' opaque illuminator, it became at once apparent that the defining lines are of the most beautiful character. I do not think I have ever succeeded in producing lines upon a metal surface quite equal to the lines upon this plate. The surface of the platin-iridium does not appear to be quite as well prepared as it is possible to prepare a surface of tempered steel, but from some experiments which I have

since made with pure iridium it is evident that this metal, with all its other good qualities, is not well adapted in this respect. This defect is, however, apparent rather than real, since it only affects the definition of the graduations in certain parts. I found a few scratches upon the plate, especially near the third millimeter. It should be noted that it is extremely difficult to polish this metal without leaving traces of the polishing material. While this plate was in my possession, its surface was not touched, even to remove the particles of dust which accumulated upon it, except with a camel's hair brush. The graduated surface is not quite parallel with the lower surface of the brass plate. I found it necessary to cement to one end five thicknesses of tissue-paper.

Before proceeding to give an account of the results obtained in this investigation it will be necessary to allude briefly to the original unit with which this standard has been compared. The original basis of this unit is a meter upon copper prepared for me by Professor Tresca, of the Conservatoire des Arts et Metiers at Paris. This meter was transferred from meter No. 19 of the Conservatory, at 2 o'clock on the morning of Feb. 6, 1880, and its relation to this standard was determined by a sufficient number of comparisons.

According to the report of Professor Tresca, this meter was found to be, by comparison, with the Metre des Archives 118.9μ too long at 13.70° C.

Since it seems desirable that all units of measurement shall be referred to a temperature near the mean temperature at which scientific observations are usually made, I have selected 62° Fahrenheit or 16.67° C., since this is the temperature at which the Imperial Yard is a standard. In order, therefore, to determine the length of the Tresca meter at 16.67° C., it became necessary to determine its coefficient of expansion with great care. It will not be necessary to describe the various unsuccessful attempts which were made to determine this coefficient with precision. It is sufficient to say that I have been unable to obtain satisfactory results by immersing the bar in a liquid. The method which was finally adopted seems to meet every difficulty. The line meter was compared at extreme temperatures with an end-measure meter immersed in melting ice. The details of this investigation will be found in the forthcoming volume of the Proceedings of the American Academy of Arts and Sciences.

The observations extend from February 7 to May 8, 1883, under temperature ranging from —11° C to plus 29° C. From the solution of 74 equations of condition the coefficient of expansion was found to be 16.18μ for each degree centigrade, or 8.99μ for each degree Fahrenheit. With this coefficient of expansion this meter was transferred to a bronze bar of the same dimensions and composition as the Imperial Yard, allowing also for the error at 13.70° C. Rigorous comparisons were then instituted between these meters, from which it appeared finally that at 16.67° C—

The Tresca meter is 167.0μ too long.

The bronze meter is 1.3μ too long.

The coefficient of the bronze bar was found to be 17.17μ for each degree centigrade.

In February of the present year, I received from Paris a meter which has been compared with great precision with the standard prototype of the International Bureau of Weights and Measures. This bar was prepared many years ago by the U. S. Bureau of Weights and Measures, and presented to the Stevens Institute, Hoboken. It has defining lines representing both the yard and the meter upon silver plugs inserted at the bottom of wells, sunk to the plane of the neutral axis of the bar. By the kindness of President Morton of Stevens Institute, I was allowed to take this bar to Europe in 1880 in order to obtain a comparison with the original standards. The yard upon this bar was compared directly with the Imperial Yard by Mr. Chaney, the Warden of the Standards, in 1880. The bar was then sent to Breteuil, near Paris, and Dr. Pernet kindly undertook a definitive comparison of the meter with a prototype of the Metre des Archives. Rigorous comparisons were made near 1, 7 and 12 degrees centigrade, from which it was found that the meter on this bar is 310μ shorter than the Metre des Archives at 0° C.

Since the coefficient of expansion of the platinum standard is known with the greatest precision, these observations furnish the data for an accurate determination of the coefficient of the Coast-Survey bar, but the obtained relations have not yet been communicated. It became necessary, therefore, to make a definitive determination of this coefficient. The method employed is the same as

that used with the Tresca bar. From seventy equations of condition the coefficient was found to be 17.60μ for each degree centigrade. The meter was found with this coefficient to be 16.6μ too short at 16.67° C.

The comparison with the meter on the bronze bar gave a result identical with the result obtained from the Tresca bar. Of course this exact coincidence is accidental, since it is impossible to rely upon the accuracy of the observations to the extent indicated by this agreement. The coincidence is, however, of value in showing that the meter which I have prepared after several years of investigation, can not differ more than 2μ or 3μ from the true length at this temperature. This conclusion is confirmed by a comparison with an end-measure meter which I purchased of the celebrated mechanician, M. Froment, of Paris, which has been compared with the metre des archives through the medium of a meter belonging to the observatory of Kazan. According to this comparison, the Froment meter is 8.4μ too long at 0° C. The coefficient of this bar has been found by the method already described to be 10.11μ, and the agreement of the length with the meter on the bronze bar at 16.67 degrees is very satisfactory, the difference being only 2μ. The coincidence is still further confirmed by a series of comparisons with the Coast-Survey meter No. 49, which I was enabled to make by the kindness of Professor Hilgard. No. 49 has been compared by Dr. Förster of Berlin, with a meter which has been compared directly with the Metre des Archives, and indirectly through the meter of the Conservatory.

Since glass is the material which the microscopist employs in investigations involving accurate measurements, it seems to be desirable that the platin-iridium centimeter should be compared with the one-hundredth part of a meter traced upon glass. Through the kindness of Mr. Chaney, I have obtained a glass bar having the dimensions 41 by 1.6 by 1.6 inches. Several similar bars were made for the Standards Department by Chance & Sons in 1870, and it is supposed that by this time they have assumed their normal condition, if, indeed, they suffer any change in their structure by age, which I do not believe. One surface of this bar was made a plain surface by Alvan Clark & Sons, when the bar is supported at points about

four inches from each end. The particular meters employed in this investigation were two provisional transfers from my steel standard, before the coefficient of expansion of the glass bar was well determined. By a comparison with the standards just described, it was found that these meters are each about 21μ too short at $16.67°$ C. The subdivisions of the méter upon both the bronze and the glass bars are made according to the method described in my paper upon this subject published in the proceedings of the meeting at Elmira; they need not, therefore, be repeated here.

The following are the relative errors of the separate subdivisions, both for the bronze and glass meters. A plus sign indicates that the measured space is too short:

SUB-DIVISIONS OF THE BRONZE STANDARD.

METER.

Halves.

I =	—0.7μ
II =	+0.7

Dm. Spaces.

		Σ
I =	—4.4μ	—4.4μ
II =	—0.4	—4.8
III =	+1.2	—3.6
IV =	+3.3	—0.3
V =	+0.3	+0.0

Cm. Spaces.

		Σ
I =	—3.4μ	—3.4μ
II =	+1.9	—1.5
III =	+0.6	—0.9
IV =	—1.1	—2.0
V =	+0.4	—2.4
VI =	+0.1	—2.3
VII =	—0.6	—2.9
VIII =	+0.0	—2.9
IX =	—0.3	—3.2
X =	+3.2	+0.0

YARD.

Halves.

I =	+1.0μ
II =	—1.0

Six-Inch Spaces.

		Σ
I =	+4.0μ	+4.0μ
II =	—3.7	+0.3
III =	—0.3	+0.0

Inch Spaces.

		Σ
I =	+0.1μ	+0.1μ
II =	+0.2	+0.1
III =	+0.3	+0.6
IV =	—0.4	+0.2
V =	+2.0	+2.2
VI =	—2.2	+0.0

SUB-DIVISIONS OF THE GLASS STANDARD.

There are two independent sets of graduators upon this bar, the lines of the first set being rather coarse, and of the second set, being rather fine. The first are designated B and the latter C. B_5 and C_5 indicate the fifth centimeters counting from the middle defining line of the meter towards the centimeter end.

METERS.

Scale B.		Scale C.	
Halves.		*Halves.*	
I+3.1μ		I+3.6μ	
II—3.1		II—3.6	
Dm. Spaces.			
I+1.1μ	+1.1μ	I+0.3μ	+0.3μ
II—2.5	—1.4	II—2.2	—1.9
III—0.4	—1.8	III—1.7	—3.6
IV—0.4	—2.2	IV+1.5	—2.1
V+2.2	+0.0	V+2.1	+0.0
Cm. Spaces.			
I+0.57μ	+0.37μ	I+0.93μ	+0.93μ
II+0.73	+1.10	II—1.47	—0.54
III+0.12	+1.22	III+ .30	—0.24
IV+0.45	+1.67	IV+ .30	+0.06
V—1.67	+0.00	V— .06	+0.00
5 Cm. Spaces.			
I+0.0μ		I+0.1μ	
II—0.0		II—0.1	

The comparisons of the centimeter A, were made with centimeters Nos. 8 and 9 of the bronze bar. It is therefore necessary to determine the error of these spaces in terms of the Metre des Archives. We have:

	No. 8.	No. 9.
Correction for relative errors..................	+0.00μ	—0.30μ
Correction for error in the whole length.......	— .01	— .01
Correction for error of first half...............	— .01	— .01
Correction for error of the fifth decimeter......	+ .03	+ .03
	+0.11	—0.29

For the mean of 8 and 9 we have therefore the correction—0.11μ. In the same way the correction to the fifth centimeter of meter B of the glass bar was found to be —1.48μ, and to the fifth centimeter of the scale C of the same bar was found to be +0.26μ.

The comparisons with centimeters 8 and 9 of the bronze bar were made with three different objectives, a one-inch, in which the value of one division of a spider line micrometer is 0.503μ; a one-half inch, in which the value of one division is 0.222μ, and a one-fourth inch, in which the value of one division is 0.110μ. The temperatures were obtained through a Yale college centigrade standard thermometer designated Y 61. I give below all the observations which have been made:

WITH ONE-HALF INCH OBJECTIVE.

Date.	Y61.	(A—9).	(A—8).
Jan. 26	−1.7°	−1.6μ	....μ
Jan. 26	−1.7	−1.2	....
Jan. 26	−1.7	−1.1	....
Jan. 26	−3.4	−2.4	−2.0
Jan. 28	+25.9	+0.7	....
Jan. 28	+23.3	+0.4	....
Jan. 28	+29.2	+0.4	....
Jan. 28	+29.8	+1.5	....
Jan. 29	+18.7	+0.4	+0.1
Jan. 29	+19.3	+0.3	+0.3
Jan. 30	+26.7	+1.0	+1.3

WITH ONE-QUARTER INCH OBJECTIVE.

Jan. 24	+ 8.6°	−0.2μ	−0.2μ
Jan. 24	+ 8.6	−0.3	−0.1
Jan. 25	— 2.7	−0.8	−1.0
Jan. 25	— 1.5	−0.9	−0.8
Jan. 25	+ 3.4	−0.6	−0.4
Jan. 26	— 9.3	−2.3	−2.2
Jan. 26	— 0.5	−1.7	....
Jan. 26	— 0.5	−1.8	....
Jan. 26	— 0.3	−2.2	....

Jan. 27	+12.4°	—0.4μ	....μ
Jan. 28	+25.9	+0.7	....
Jan. 28	+23.4	+0.6	....
Jan. 28	+29.3	+0.6	....
Jan. 28	+29.8	+1.2	....
Jan. 28	+29.8	+0.9	....
Jan. 29	+18.7	—0.2	+0.2
Jan. 29	+19.3	—0.4	—0.1
Jan. 29	+19.5	—0.2	....
Jan. 30	+26.4	+0.6	+1.0

WITH ONE-INCH OBJECTIVE.

Jan. 22	+18.6°	+0.4μ	+0.4μ
Jan. 22	18.6	—0.2	—0.7
Jan. 22	—12.4	—3.5	—4.6
Jan. 24	—12.4	—3.1	—3.6
Jan. 24	— 9.1	—3.8	—3.6
Jan. 24	— 5.1	—1.3	—1.3
Jan. 24	— 2.5	—1.3	—1.7
Jan. 24	— 1.7	—2.3	—2.4
Jan. 26	— 0.5	—1.5	....
Jan. 26	— 1.7	—1.0	....
Jan. 26	— 0.3	—1.7	....
Jan. 27	+12.4	—0.1	....
Jan. 28	+25.8	+0.4	. .
Jan. 28	+23.3	+1.4	....
Jan. 28	+29.3	+0.1	....
Jan. 28	+29.8	+1.6	....
Jan. 28	+29.8	+1.1	....
Jan. 29	+18.7	+0.7	+0.8
Jan. 29	+19.2	—0.1	+1.0
Jan. 30	+26.7	+1.1	+0.9

In order to obtain a provisional value of the relative coefficient of expansion between the centimeter A and a centimeter of the bronze bar, the following equations of condition were found from the above data, for the mean of 9 and 8. They are as follows:

Jan. 22,	$+0.0\mu = a + 18.6\,b$
Jan. 24,	$-3.7 = a - 11.3\,b$
Jan. 24,	$-1.7 = a - 3.1\,b$
Jan. 24,	$-0.2 = a + 8.6\,b$
Jan. 25,	$-0.8 = a + 2.1\,b$
Jan. 25,	$-0.5 = a - 3.4\,b$
Jan. 26,	$-1.8 = a - 1.5\,b$
Jan. 27,	$-0.3 = a + 12.4\,b$
Jan. 28,	$+0.8 = a + 27.6\,b$
Jan. 29,	$+0.1 = a + 19.2\,b$
Jan. 30,	$+1.0 = a + 26.7\,b$

The determined normal equations are:

$$- 7.1\mu = 11.0\,a + 98.5\,b$$
$$+95\ 0 = 98.5\,a + 2572.5\,b$$

Whence b = the relative coefficient of expansion = $+.0938\mu$

a = the difference in length at 0° C. = -1.48μ

For the temperature 16.67° C. = 62°.0 Fah., we obtain with this value of b

$$A - \frac{8+9}{2} = +0.07\mu.$$

We are now prepared to reduce all the comparisons to 16°.67 C., employing for this purpose the relative coefficient 0.094μ. The following are the results:

Relations between scale A and centimeters Nos. 8 and 9 of the bronze standard at 16.67° C.

WITH ONE-HALF INCH OBJECTIVE.

Date. 1883.	(A—9).	(A—8).
Jan. 26	$+0.1\mu$	....
Jan. 26	$+0.5$	....
Jan. 26	$+0.6$	....
Jan. 26	-0.5	-0.1μ
Jan. 28	-0.2	....
Jan. 28	-0.5	....
Jan. 28	-0.8	....
Jan. 28	$+0.3$	....

Jan. 29	+0.2μ	—0.1μ
Jan. 29	+0.1	+0.1
Jan. 30	+0.1	+0.4
Means	—0.01	+0.07

WITH ONE-QUARTER INCH OBJECTIVE.

Jan. 24	+0.6μ	+0.6μ
Jan. 24	+0.4	+0.6
Jan. 25	+1.0	+0.8
Jan. 25	+0.8	+1.0
Jan. 25	+1.0	+1.2
Jan. 26	+0.1	+0.2
Jan. 26	+0.9	....
Jan. 26	+0.8	....
Jan. 26	—0.6	....
Jan. 27	+0.0	....
Jan. 28	—0.2	....
Jan. 28	+0.0	....
Jan. 28	—0.6	....
Jan. 28	—0.1	....
Jan. 28	+0.3	....
Jan. 29	—0.4	—0.6
Jan. 29	—0.6	—0.8
Jan. 29	—0.5	....
Jan. 30	—0.3	+0.1
Means	+0.14	+0.34

WITH ONE INCH OBJECTIVE.

Jan. 22	+0.6μ	+0.6μ
Jan. 22	+0.0	—0.5
Jan. 24	—0.8	—1.9
Jan. 24	—0.4	—0.9
Jan. 24	—1.4	—1.2
Jan. 24	+0.7	+0.7
Jan 24	+0.5	+0.1
Jan. 24	—0.6	—0.7

Jan. 26	$+0.1\mu$	....μ
Jan. 26	$+0.7$	....
Jan. 26	-0.1	....
Jan. 27	$+0.3$	....
Jan. 28	-0.5	....
Jan. 28	$+0.8$	....
Jan. 28	-1.1	....
Jan. 28	$+0.4$	...
Jan. 28	-0.1	....
Jan. 29	$+0.5$	$+0.6$
Jan. 29	-0.3	$+0.8$
Jan. 30	$+0.1$	-0.1
Means	-0.03	-0.23

We have therefore the following relations with respect to $\frac{A_o}{100}$:

From Comparisons with one-half inch Objective.	From Comparisons with one-quarter inch Objective.	From Comparisons with one inch Objective.
$A+0.01\mu=$ No. 9	$A-0.14\mu=$ No. 9	$A+0.03\mu=$ No. 9
$\frac{A_o}{100}+0.29=$ No. 9	$\frac{A_o}{100}+0.29=$ No. 9	$\frac{A_o}{100}+0.29=$ No. 9
$A-0.28=\frac{A_o}{100}$	$A-0.43=\frac{A_o}{100}$	$A-0.26=\frac{A_o}{100}$
$A-0.07=$ No. 8	$A-0.34=$ No. 8	$A+0.23=$ No. 8
$\frac{A_o}{100}+0.01=$ No. 8	$\frac{A_o}{100}+0.01=$ No. 8	$\frac{A_o}{100}+0.01=$ No. 8
$A-0.08=\frac{A_o}{100}$	$A-0.35=\frac{A_o}{100}$	$A+0.22=\frac{A_o}{100}$

Collecting results we have the following values of (A—9) and (A—8):

	With 1 inch.	½ inch.	¼ inch.
From centimeter No. 9	-0.28μ	-0.43μ	-0.26μ
From centimeter No. 8	-0.08	-0.35	$+0.22$
Means	-0.18	-0.38	-0.02

And finally at 62°.0 Fahr.:

$$A-0.19\mu=\frac{A_o}{100}.$$

It will be noticed that the difference between the results from centimeters (9) and (8) are nearly constant for the observations with different objectives, indicating a slight error in the determination of the relations of these spaces with respect to the entire meter.

The observed data for the relations between centimeter A and centimeters B_5 and C_5 of the glass bar are as follows:

Date.	Y61.	B_5—A.
Feb. 1	— 8.3°	+0.9μ
Feb. 1	— 8.3	+1.0
Feb. 3	+20.6	+1.2
Feb. 3	+20.6	+1.6
Feb. 3	+20.6	+1.8
Feb. 4	+28.3	+2.6
Feb. 4	+28.3	+2.8
Feb. 4	+23.0	+2.5
Feb. 4	+31.6	+2.1
Feb. 4	+31.6	+2.2
Feb. 4	+34.2	+2.6
Feb. 4	+34.2	+3.3
Feb. 4	+32.0	+2.5
Feb. 4	+32.0	+2.8
Feb. 5	— 0.8	+1.5
Feb. 5	— 0.8	+1.6
Feb. 6	— 8.6	+1.9
Feb. 6	— 0.7	+2.1
Feb. 7	+ 0.3	+1.5
March 22	+11.2	+1.8
March 23	+ 2.0	+1.1
March 25	+ 4.4	+1.3
March 26	+ 8.7	+0.7
March 29	+ 0.7	+1.5
March 30	+ 4.5	+1.8
April 1	+ 4.0	+2.0
April 2	+ 2.2	+1.3
April 3	+ 4.5	+1.3
April 4	+ 5.2	+1.2
April 5	+13.1	+1.1
April 8	+13.4	+1.1
April 10	+ 5.3	+1.4
May 13	+ 7.8	+1.7
May 13	+ 7.8	+1.3
May 16	+12.4	+1.3
May 17	+16.7	+1.3
May 19	+22.3	+1.6
May 20	+15.6	+1.5
May 31	+16.6	+1.2

Date.	Y61.	C_5—A.
Feb. 3	+20.6°	—0.3μ
Feb. 3	+20.6	—1.1
Feb. 3	+20.6	—0.6
Feb. 4	+23.0	—0.4
Feb. 4	+31.6	—0.2
Feb. 4	+31.6	—0.6
Feb. 4	+34.2	—1.4
Feb. 4	+24.2	—0.6
Feb. 4	+32.0	—1.3
Feb. 4	+32.0	—1.4
Feb. 5	— 0.8	—0.7
Feb. 5	— 0.8	—0.3
Feb. 6	— 8.6	—1.1
Feb. 6	— 0.7	—0.5
Feb. 7	+ 0.3	—0.4
March 22	+11.8	—1.2
March 23	+ 2.0	—1.1
March 25	+ 4.4	—0.7
March 26	+ 8.7	—1.0
March 29	— 0.7	—0.6
March 30	+ 4.5	—0.3
April 1	+ 4.0	—0.7
April 2	+ 2.2	—0.5
April 3	+ 4.5	—0.4
April 4	+ 5.2	—0.6
April 5	+13.1	—1.2
April 8	+13.4	—0.3
April 10	+ 5.3	—0.7
May 13	+ 7.8	—0.3
May 13	+ 7.8	—0.4
May 16	+12.4	—0.1
May 17	+16.7	+0.3
May 19	+22.3	—0.2
May 20	+15.6	+0.1
May 31	+16.6	—0.5

Collecting these observations and arranging them with respect to the temperatures 0° and 16.67° C, we have as follows:

Date.	Y61.	C_5—A.	B_5—A.
Feb. 1....	— 8.3°	+ 1.0μ	....
Feb. 5....	— 0.8	+ 1.6	—0.5μ
Feb. 6....	— 8.6	+ 1.9	—1.1
Feb. 6....	— 0.7	+ 2.1	—0.5
Feb. 7....	+ 0.3	+ 1.5	—0.4
March 22.	+11.2	+1 8	—1.2
March 23.	+ 2.0	+ 1.1	—1.1
March 25.	+ 4.4	+1.3	—0.7
March 26.	+ 8.7	+0.7	—1.0
March 29.	— 0.7	+ 1.5	—0.6
March 30.	+ 4.5	+ 1.8	—0.4
April 1...	+ 4.0	+2.0	—0.7
April 2...	+ 2 2	+1.3	—0.5
April 3...	+ 4.5	+1.3	—0.4
April 4...	+ 5.2	+ 1.2	—0.6
April 10..	+ 5.3	+ 1.4	—0.7
May 13...	+ 7.8	+1.5	—0.4

Date.	Y61.	B_5—A.	C_5—A.
Feb. 3....	+20.6°	+1.5μ	—0.7μ
Feb. 4....	+26.5	+2.6	—0.4
Feb. 4....	+32.8	+2.6	—0.9
March 22.	+11.2	+1.8	—1.2
April 5...	+13.1	+1.1	—1.2
April 8...	+13.4	+1.1	—0.3
May 13...	+ 7.8	+1.5	—0.4
May 16...	+12.4	+1.3	—0.1
May 17...	+16.7	+1.3	+0.3
May 19...	+22.3	+1.6	—0.2
May 20...	+15.6	+1.5	+0.1
May 31...	+16.6	+1.2	—0.5

We have therefore the two equations:

$$+1.47\mu \ldots -0.68\mu = a + 3.12\,b.$$

$$+1.59\mu \ldots -0.46\mu = a + 17.42\,b.$$

Whence $+0.12\mu \ldots\ldots +0.22\mu = 14.30\,b.$

And $b = \dfrac{+.008\mu + 015\mu}{2} = +.012\mu.$

For 16.67°

$$a = +1.44\mu \text{ for } B_5.$$

$$a = -0.73\mu \text{ for } C_5.$$

For the relations between A_0 B_5 and C_5 we have·

$$B_5 + [+.21 + .06 + .22 - 1.67]\mu = \frac{A_0}{100}.$$

$$C_5 + [+.21 + .07 + .21 - .06]\mu = \frac{A_0}{100}.$$

$$\text{But } B_5 - 1.44\mu = A.$$

$$C_5 + 0.73\mu = A.$$

$$\text{Hence } A + 0.26\mu = \frac{A_0}{100} \text{ from } B_5.$$

$$A - 0.30\mu = \frac{A_0}{100} \text{ from } C_5.$$

Adopting the mean relation we have:

$$A - 0.02\mu = \frac{A_o}{100}.$$

But from the bronze bar we have:

$$A - 0.19\mu = \frac{A_o}{100}.$$

By combination we have finally:

$$A - 0.10\mu = \frac{A_o}{100}.$$

In order to determine the definitive coefficient of expansion of the plate A, the following method of reduction was employed, in preference to the derivation of this quantity, from the equations of condition formed from the relations between A and the bronze bar. The normal relations between A and centimeters 8+9 divided by 2 were formed by reducing all the observations near 0° to the equivalent value for 0°, and all the observations near 16.67° to the equivalent values for 16.67°, employing for this purpose the coefficient already found, viz: 0.094μ. In this way comparative freedom from errors due to an erroneous coefficient is secured.

The following results were obtained from the data given above:

From observations near 0°.

Date.	Y61.	$A - \frac{8+9}{2}$	$A - \frac{8+9}{2}$ At 0°
Jan. 24	−11.3°	−3.6μ	−2.6μ
Jan. 24	+ 8.6	−0.2	−1.0
Jan. 24	− 3.1	−1.7	−1.4
Jan. 25	− 0.3	−0.8	−0.8
Jan. 26	− 1.4	−1.5	−1.3
Jan. 26	− 9.3	−2.2	−1.4
Jan. 27	+12.4	−0.2	−1.3
Means			−1.40μ

From observations near 16.67° C.

Date.	Y61.	$A - \frac{8+9}{2}$	$A - \frac{8+9}{2}$ At 16.67°
Jan. 22	+18.6°	+0.0μ	−0.2μ
Jan. 26	+ 8.6	−0.1	+0.6
Jan. 27	+12.4	−0.1	+0.3
Jan. 28	+27.4	+0.8	−0.2
Jan. 29	+19.0	+0.1	−0.1
Jan. 30	+26.7	+1.0	+0.1
Means			+0.08μ

The relative coefficient between the limits 0° and 16.67° C is:

$$\frac{+1.40+.08}{16.67}\mu = .0888\mu.$$

But the coefficient of the bronze bar is 17.17μ.

The absolute coefficient of A, derived from these observations is, therefore:

$$.0829\mu.$$

It is obvious from the observed relations between A and the glass bar that for any practical purpose, they have the same coefficient. These observations are not, however, well adapted to secure an accurate value of the coefficient. Indeed it can hardly be expected that a precise value could be obtained from such a short unit. It will be noticed that the value of the relation between A and the mean of 8 and 9 of the bronze bar, from only those observations made near 16.67° C, is nearly the same as that derived from the whole series.

It is to be noted that these measures are from the middle line of each band of three defining lines and not from the mean of the three lines, as is the case with the determination of the error given in the official report which accompanies this standard. I have purposely avoided ascertaining the relation between the middle line and the mean of the three, in order that my investigation might be made without the bias which previous knowledge sometimes gives, quite unconsciously, to an observer. I leave the investigation of this relation to other observers. It can be easily made with the ordinary appliances of the microscope. But for any purpose required by the microscopist the agreement is practically perfect without this reduction. I assume that 0.2μ is the limit of precision in microscopic measures, beyond which it is impossible to go with *certainty*. Certainly that is the result of my own experience. *If we admit this limit, it follows that the middle defining lines of this standard require no correction at 62 degrees Fahrenheit.*

This conclusion is substantiated by a comparison with a centimeter upon glass, prepared by the writer in 1881, and since presented to the Royal Microscopical Society of London. This standard consists of 1001 lines in one centimeter. No error in the whole length at 62° Fahr. could be detected at the time of its construction.

After the greater part of the observations detailed above had been completed, the following comparisons were made between these independently determined standards:

1883, May 23,	Glass Centimeter	-0.04μ	= A.
May 24,	" "	-0.29	= A.
May 25,	" "	-0.09	= A.

Hence:

$$\text{Glass Centimeter} - 0.14\mu = A.$$

But;

$$A - 0.10\mu = \frac{A_o}{100}.$$

Whence:

$$\text{Glass Centimeter} - 0.04\mu = \frac{A_o}{100}.$$

With regard to the errors of subdivision, I have considered it worth while to investigate only the millimeter spaces, since the remaining subdivisions can be investigated by any observer with the aid of a filar or an eye-piece micrometer. The following are the relative errors of these subdivisions, the measures being made from the middle defining line of the bands of three lines:

SUBDIVISIONS OF SCALE A.

MM. Spaces.

1883.	May 31.	June 1.	June 3.	June 3.	June 4.	June 5.	June 6.	Means	Σ
	μ	μ	μ	μ	μ	μ	μ	μ	μ
I	+0.63	+0.53	+0.57	+0.64	+0.61	+0.56	+0.84	+0.63	+0.63
II	+ .03	— .13	— .04	— .22	+ .07	+ .22	+ .04	+0.00	+0.63
III	— .30	— .62	— .42	— .29	— .35	— .29	— .22	—0.36	+0.27
IV	— .65	— .09	— .15	— .62	— .67	— .16	— .29	—0.39	—0.12
V	— .33	+ .18	— .20	— .44	— .67	— .37	— .37	—0.31	— .43
VI	+ .87	+ .68	+ .59	+ .82	+ .81	+ .53	+ .74	+0.72	+ .29
VII	+ .31	+ .29	+ .24	+ .27	+ .20	+ .33	+ .09	+0.25	+ .54
VIII	+ .21	+ .60	+ .22	+ .29	+ .26	+ .29	+ .37	+0.32	+ .86
IX	—1.41	—1.48	—1.22	—1.05	—1.20	—1.26	— .48	—1.30	— .44
X	+ .63	+ .25	+ .40	+ .70	+ .90	+ .14	+ .28	+0.47	+ .03

It appears from this examination that the most serious error is in the ninth space. It also appears from the summed series that there is no marked evidence of periodicity in the graduation.

Finally we may conclude:

(*a*) That centimeter A, defined by the middle lines of the terminal bands, requires no sensible correction at 62.0° Fahr.

(*b*) That the second millimeter is exactly one-tenth part of the standard unit, and therefore requires no correction.

WM. A. ROGERS.

Harvard College Observatory, Aug. 4, 1883.

Rules for the Control of the Standard Micrometer.

The following rules for the government and control of the Standard Micrometer, "Centimeter A, 1882," have been prepared by the Committee appointed for this purpose:

1. The Standard Micrometer, "Centimeter A, 1882," shall be held in the custody of the Treasurer and Custodian of the Society, subject to the general supervision of the Committee on Standard Micrometer; and shall ordinarily be kept in the vaults of an approved Safety Deposit Company.

2. The Standard Centimeter shall not pass out of the hands of the Custodian, except to parties of eminent ability, for the comparison and verification of their standards, and only by the permission of the Committee, President and Secretary of the Society.

3. A series of three or more copies of the Standard Centimeter shall be prepared by some competent maker and carefully compared with the Standard; and when thus standardized a copy may be loaned by the Custodian to any reliable party to prepare copies and make comparisons of their micrometers with the same. Such parties to deposit $10 with the Treasurer as security for its safety, and pay all charges of transportation to and from the Treasurer's Office.

4. It shall be the duty of the Custodian, or of some competent person to be secured by him, to compare with the Standard any micrometer sent him for that purpose, and certify to the degree of correctness of the same within practical limits, charging for the same a reasonable fee.

Subsequent to the adoption of the preceding rules, the following resolutions were adopted by the Committee:

1st. *Resolved,* That Prof. W. A. Rogers be requested to prepare three verified copies of the Standard Micrometer on glass.

2nd. That the copies so prepared be numbered respectively 2 A, 1884; 3 A, 1884; 4 A, 1884; and be distributed in accordance with the preceding rules, to the various makers who may wish to prepare copies of the Standard.

ALBERT McCALLA,
GEO. E. FELL,
LESTER CURTIS,
Committee on Standard Micrometer.

THE GERMAN SURVEY OF THE NORTHERN HEAVENS.

ADDRESS

BY

WILLIAM A. ROGERS,

VICE PRESIDENT, SECTION A.

[From the PROCEEDINGS OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Vol. XXXII, Minneapolis Meeting, August, 1883.]

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ADDRESS

BY

WILLIAM A. ROGERS,

VICE PRESIDENT, SECTION A.

THE GERMAN SURVEY OF THE NORTHERN HEAVENS.

THE illustrious Argelander was accustomed to say, in the quaint form of speech which he often employed, "The attainable is often not attained if the range of inquiry is extended too far." In no undertaking is there greater need of a judicious application of this sound maxim than in the systematic determination of the exact positions of all the stars in the visible heavens which fall within the reach of telescopes of moderate power.

The first subject which engaged the attention of the Astronomische Gesellschaft, at its formation in 1865, was the proposition to determine accurately the coördinates of all the stars in the northern heavens down to the ninth magnitude. To this association of astronomers (at first national, but since become largely international, in its character and organization) belongs the credit of arranging a scheme of observations by which, through the coöperation of astronomers in different parts of the world, it has been possible to accomplish the most important piece of astronomical work of modern times. With a feasible plan of operations, undertaken with entire unity of purpose on the part of the observers to whom the several divisions of the labor were assigned, this great work is now approaching completion. While it is yet too early to speak with confidence concerning the definite results which the discussions of all the observations are expected to show, we may with profit consider the object sought in the undertaking, the gen-

eral plan of the work, the difficulties which have been encountered, and the probable bearing which the execution of the present work will have upon the solution of a problem concerning which we now know absolutely nothing with certainty; a problem of which what we call universal gravitation is only one element, if, indeed, it be an element; a problem which reaches farther than all others into the mysteries of the universe, — the motion of the solar and the sidereal systems in space.

Our first inquiry will be with respect to the condition of the question of stellar positions at the time when this proposal was made by the Gesellschaft in 1865. All the observations which had been made up to this time possess one of two distinct characteristics. A portion of them were made without direct reference to any assumed system of stellar coördinates as a base, but by far the larger part are differential in their character. This remark holds more especially with reference to right ascensions. Nearly all of the observations of the brighter stars made previous to about 1818 were referred to the origin from which stellar coördinates are reckoned, by corresponding observations of the sun; but since that date it has been the custom to select a sufficient number of reference stars, symmetrically distributed both in right ascension and declination, and whose coördinates were supposed to be well known. The unequalled Pulkowa observations for the epoch 1845 form, I believe, the only exception to this statement. From the assumed system of primary stars are derived the clock errors and instrumental constants which are employed in the reduction of all the other stars observed. The positions of these secondary stars, therefore, partake of the errors of the assumed fundamental system, in addition to the direct errors of observation.

The following list comprises the most important of the catalogues which have been independently formed; viz., Bradley for 1755, the various catalogues of Maskelyne between 1766 and 1805, D'Agelet for 1783, Piazzi for 1805, Auwers' Cacciatore for 1805, Bessel for 1815, a few of the earlier catalogues of Pond, Brinkley for 1824, Bessel for 1825, Struve for 1824, Bessel for 1827, Argelander for 1830, and Pulkowa for 1845.

An analysis of the important catalogues of secondary stars published previous to 1865 reveals four important facts:—

1. That nearly all of the observations relate to bright stars, at least to stars brighter than the eighth magnitude.

2. That in a large number of cases the same star is found in different catalogues, but that no rule is discoverable in the selection.

3. That with the exception of the polar catalogues of Fedorenko, Groombridge, Schwerd, and Carrington, the double-star observations of Struve, and the zone observations of Bessel and Argelander, the observations were not arranged with reference to the accomplishment of a definite object.

4. That each catalogue involves a system of errors peculiar to the observers, to the character of the instrument employed, and to the system of primary stars selected, but that thus far there had been no attempt to reduce the results obtained by different observers to a homogeneous system. In estimating the value of these observations it will be necessary to refer to the researches which have been made subsequent to 1865.

The systematic deviations of different catalogues in right ascension, *inter se*, were noticed at an early date by several astronomers; but the first attempt to determine the law of these variations seems to have been made by Safford in a communication to the Monthly Notices of the Royal Astronomical Society in 1861 (xxi, 245), "On the Positions of the Radcliffe catalogue." I quote the equation derived by Safford, since it appears to be the first published account of a form of investigation almost exclusively followed since that time in the treatment of this problem. It is as follows:—

Diff. of R. A. (Greenw. 12 Year Cat. — Rad.) $= -\overset{s}{.038} + \overset{s}{.032} \sin (\alpha + \overset{h}{5}\ \overset{m}{32})$. Extending this expression to terms of the second order, it may be put under the form, $\Delta =$ a constant $+ (m \sin a + n \cos a) + (m' \sin 2a + n' \cos 2a)$.

Safford also seems to have been the first to notice the connection between the observed residuals, and the errors in position of the primary stars employed. He remarks, "In investigating the causes which give rise to such systematic discrepancies, I was struck with the fact that the same or nearly the same variations were apparent in the assumed places of the time stars for the years since 1845; that, if the correct positions of the time stars had been assumed, the resulting positions would have been free from these small errors." That the relation given by Safford should have been observed at all is the more remarkable, since the primary stars upon which the Radcliffe positions depend, are nearly the same as

those employed at Greenwich. In reality, the systematic errors of both catalogues have since been found to be considerably greater than is here indicated, and the deviation pointed out by Safford is in the nature of a second difference, but the existence of periodic errors in the Greenwich catalogue was not, however, overlooked by him. The speaker has shown (*Proc. Amer. Acad.*, 1874, 182) that the weight of the errors of the provisional catalogue assumed fell between the first and third quadrants in the Radcliffe observations for 1841–42, on account of the omission of certain clock stars which were used at Greenwich.

Since the discordances which exist between two catalogues may arise from errors either in one or in both, it is clearly impossible either to determine the nature of the errors, or to assign their true cause, until a fundamental system has been established which is free both from accidental and from periodic errors, — from accidental errors, since a few abnormal differences may easily invalidate the determination of the errors which are really periodic; from periodic errors, because a relative system can only become an absolute one, when one of the elements of which it is composed becomes absolute.

We owe to the researches of Newcomb, published in 1869–70, a homogeneous system of stellar coördinates in right ascension, which are probably as nearly absolute in their character as it is possible to obtain from the data at present available. He determined the absolute right ascensions of thirty-two stars of the first, second, and third magnitudes, and comprised between the limits —30° and + 46° declination. A comparison of the places of these stars for a given epoch, with the same stars in any catalogue for the same epoch, enables us to determine with considerable precision the system of errors inherent in that catalogue. Several circumstances prevent the exact determination of this relation. Among them may be mentioned the fact that Newcomb's system cannot safely be extended far beyond the limits in declination of the stars composing the system, that the stars are not symmetrically distributed in declination, and that the system of errors derived from bright stars is probably not the same as that derived from stars of less magnitude.

To a certain extent all of these objections have been met in the later discussion by Auwers, to which reference will presently be made. The substantial agreement of these two systems, independ-

ently determined, furnishes satisfactory evidence that we have at last obtained a foundation system with which it is safe to make comparisons, from which we may draw conclusions with comparative safety. When the catalogues which were formed between 1825 and 1865 are compared with Newcomb's fundamental system, through the medium of these thirty-two stars, the following facts are revealed: —

a. The only catalogues in which there is freedom from both accidental and periodic errors are Argelander's Äbo catalogue for 1830, and the Pulkowa catalogue for 1845. One is reminded, in this connection, of the remark of Pond, that "we can hardly obtain a better test of our power of predicting the future positions of stars than by trying by the same formula, how accurately we can interpolate for the past. In a variety of papers which I have submitted to the Royal Society, I have endeavored to show, that, with us, the experiment *entirely* fails."

b. During this interval, the constant differences between the earlier catalogues and Newcomb's system vary between $+ 0.17^s$ for Pond, 1820; and -0.19^s for Pond, 1830: and for later catalogues, between $+ 0.07^s$ for Cambridge, 1860; and $+ .02^s$ for Greenwich, 1860.

c. All the right ascensions determined at English observatories, and especially those which depend upon the positions published by the British Nautical Almanac, are too large in the region of five hours, and too small in the region of eighteen hours. The general tendency of the constant part of the deviation from Newcomb's system is to neutralize the periodic errors in the region of five hours, and to augment them in the region of eighteen hours, where, in the case of a few catalogues, the error becomes as great as 0.10^s, — a quantity which can be readily detected from the observations of two or three evenings with an indifferent instrument, if it relates to a single star.

The right ascensions determined at French observatories exhibit systematic errors, which follow nearly the same law as those which characterize English observations.

Distinctively German observations are nearly free from systematic errors. As far as they exist at all, their tendency is to neu-

tralize the errors inherent in distinctively English and French observations.

d. In the case of several catalogues, residual errors of considerable magnitude remain after the systematic errors depending upon the right ascensions have been allowed for. These errors are found to be functions of the declination of the stars observed, and without doubt have some connection with the form of the pivots of the instruments with which the observations were made. This statement holds true, especially with respect to the observations at Paris, Melbourne, and Brussels, between 1858 and 1871; and to the Washington observations between 1858 and 1861.

e. The systematic errors which exist in observations previous to 1865 follow the same law, and have nearly the same magnitude, as the errors of the same class which are inherent in the national ephemerides of the country in which they were made.

The British Nautical Almanac and the Connaissance des Temps are largely responsible for the perpetuation of this class of errors. For a few years before and after 1860, the ephemerides of the Nautical Almanac were based upon the observations of Pond, which contain large periodic errors. It is found that the errors of this system have been transferred without sensible diminution to every catalogue in which the observations depend upon Nautical Almanac clock stars. At English observatories, it has been the custom to correct the positions of the fundamental stars by the observations of each successive year; but this has produced no sensible effect on the diminution of the periodic errors, which belong to the fundamental system. The periodic errors of the American Ephemeris follow nearly the same law as the errors of the Nautical Almanac, but their magnitude is somewhat reduced. The error of equinox is also less.

Wolfers' *Tab. Reg.*, upon which the Berliner Jahrbuch is based, has no well-defined systematic errors; and the correction for equinox is nearly the same in amount as in the Americam Ephemeris, but with the opposite sign. The accidental errors seem to be rather larger than in the system of the American Ephemeris.

f. A general estimate may be formed of the relative magnitudes of the errors of secondary catalogues by comparing the average

error for each star of the primary catalogue. The numbers given below represent the average deviation for each star, expressed in hundredths of seconds, after the various catalogues have been reduced to a common equinox.

•		Average error for each star.
Argelander	1830	1.1
Pulkowa	1845	1.1
Greenwich	1845	2.0
Greenwich	1860	2.0
D'Agelet (Gould)	1783	2.2
Cape of Good Hope (Henderson)	1833	2.2
Greenwich	1850	2.2
Greenwich	1871	2.2
Paris	1867	2.4
Washington	1846–52	2.5
Struve	1830	2.5
Cape of Good Hope	1856	2.8
Radcliffe	1860	3.1
Greenwich	1840	3.1
Bessel	1825	3.2
Pond	1830	3.7
Gillis	1840	3.8
Madras (Taylor)	1830	3.9
Cape of Good Hope (Fallows)	1830	3.9
Radcliffe	1845	4.5
Armagh	1840	5.0
Piazzi	1800	5.3
Bessel's Bradley	1755	7.9
Lalande	1800	13.2
Lacaille	1750	24.9

It is obvious from these relations, that previous to about 1825 the magnitude of the accidental errors of observation, combined with the errors of reduction, prevents any definite conclusions with respect to the periodic errors inherent in these early observations. It is probable, also, that early observations of stars of the eighth and ninth magnitudes are subject to a class of errors peculiar to themselves, the nature of which it is now well nigh impossible to determine.

The systematic errors in declination which belong to the various secondary catalogues named are even more marked than those in right ascension. The experience of Pond in 1833 is the experience of every astronomer who has attempted to compare observations of the same star made at different times, under different circumstances, with different instruments, and by different observers. He says, "With all these precautions, we do not find, by comparing the present observations with those of Bradley made eighty years ago under the same roof, and computed by the same table of

refractions, that we can obtain by interpolation any intermediate catalogue which shall agree with the observations within the probable limits of error."

We owe to the investigations of Auwers (*Astron. Nachr.*, nos. 1532–1536), the first definite system of declinations which is measurably absolute in its character. Yet the deviations of this system from that derived by the same author, but from much additional data in Publication xiv of the Gesellschaft, is no less than $1''.2$. The present difference outstanding between the Pulkowa and Greenwich systems at 10° south declination is $1''.7$.

Within the past five years, the labors of Auwers, of Safford, of Boss, and of Newcomb, have resulted in the establishment of a mean system of declinations from which accidental errors may be considered to be eliminated in case of a large number of stars; but the different systems still differ systematically, *inter se*, by quantities which are considerably greater than the probable error of any single position.

When the discussion of the question of a uniform determination of all the stars in the northern heavens to the ninth magnitude was taken up by the Gesellschaft at its session in Leipsic in 1865, Argelander, who was then president of the society, appears to have been the only astronomer who had a clear apprehension of the difficulties of the problem. He alone had detected the class of errors whose existence subsequent investigations have definitely established. He alone had found a well-considered plan by which these errors might be eliminated, as far as possible, from future observations.

Argelander, however, always claimed for Bessel the first definite proposal of the proposition under consideration (see *Astron. Nachr.*, vol. i, 257). It was in pursuance of this plan that the zones between —15° and +15° in declination were observed. These zones were to form the groundwork of the Berlin charts; and Argelander, in the execution of the Bonn Durchmusterung simply carried out the second part of Bessel's recommendation.

These two great works — the second being a continuation of the first, under a better and more feasible plan — are the only ones in existence which give us any knowledge of the general structure of the stellar system, with the exception of the observations of Cooper at Makree observatory, and the charts of Chacornac.

The observations of stars to the ninth magnitude, found in the catalogues of Bessel, Lalande, and Piazzi, form the groundwork of the Berlin charts. The coördinates in right ascension and declination of the stars found in these authorities were first reduced to the epoch 1800; the resulting right ascension being given to seconds of time, and the declination to tenths of minutes of arc. With these places as points of reference, all other stars were filled in, down to the ninth magnitude, by observations with equatorial instruments. The work was divided into zones of one hour each. Bremiker undertook five zones; Argelander and Schmidt, two; Wolfers, three; and Harding, two. The remaining zones were undertaken by different astronomers in widely separated localities.

The work seems to have been performed with somewhat unequal thoroughness, some zones containing nearly all the stars to the ninth magnitude, while in others a large number of stars having this limit in magnitude are wanting.

The Durchmusterung undertaken by Argelander at Bonn was a far more serious and well-considered undertaking. This unequalled work consists in the approximate determination of the coördinates of 324,198 stars situated between —2° and + 90° declination. It includes stars to the 9.5 magnitude, the coördinates being given to tenths of seconds of time, and the declinations to tenths of minutes of arc.

The first definite proposal of the work undertaken by the Gesellschaft, however, appears to have been made by Bruhns. In the course of a report upon the operations of the Leipsic observatory, he stated, that, in his view, the time had come for undertaking a uniform system of determinations of the places of stars to the ninth magnitude in the northern hemisphere by means of meridian circles but he proposed at the same time that the position of stars fainter than the ninth magnitude should be determined by means of differential observations with equatorial instruments. After explaining certain plans and arrangements relating particularly to his own observatory, he introduced the following resolution: —

"The Astronomische Gesellschaft regards it as needful that all the stars to the ninth magnitude, occurring in the Durchmusterung, should be observed with meridian circles, and commissions the council to arrange for the execution of the work."

This proposal occasioned a long and somewhat animated discussion, in which Argelander, Hirsch, Bruhns, Förster, Schönfeld, and Struve took part.

Argelander declared himself surprised at this proposal, which called for the rapid realization of a plan of organization which he had been considering for years with the greatest care, the difficulties of which he had maturely considered, and the execution of which still demanded the most careful deliberation and preparation. One of the necessary preliminary steps was a plan which he had already prepared, published and presented to the society in an informal way, which provided for contemporaneous and corresponding observations of the brighter stars. As president of the society, he felt unequal to undertaking the charge which the acceptance of the resolution proposed would involve; as this procedure seemed to him premature without previous preparation. He would admit, however, that every call to action of this kind tended to stimulate enthusiasm, and should therefore be encouraged; but he felt obliged to ask the society not to require from him the immediate execution of the plan, but to intrust the serious consideration of it, and the preparation for it, to his zealous friends in the council.

Upon the motion of Struve, the society, by a rising vote, expressed its confidence in the assurance of the president that he would bring forward his plan at the proper time, as soon as the means for its execution could be assured.

At the meeting held at Bonn in 1867, Argelander again brought up the subject in a communication which appears to have been an exhaustive discussion of the whole problem. This paper is not printed in the proceedings of the Gesellschaft; but at its conclusion a committee was appointed to take definite action with respect to the recommendations which it contained. The committee reported at the same session; and their report, which is published in the place of the paper presented by Argelander, is probably identical in substance with it. The plan proposed and adopted was finally published in the form of a programme, in which the details of the work are arranged with considerable minuteness. As this programme has been widely distributed, it seems unnecessary to give anything more than a general abstract of it. Since it differs in a few minor points from the first report

of the committee at the Bonn meeting, the essential features of this report will be given instead of an abstract of the programme itself.

They are as follows: —

a. The limits in declination of the proposed series of observations are — 2° and + 80°. The first limit was chosen on account of the lack of suitable fundamental stars south of the equator. It is probable, also, that Argelander had a suspicion of the fact, since proven, that the uncertainty with respect to the systematic errors of southern stars is, of necessity, considerably greater than for northern stars, and that on this account it would be better to defer this part of the work until further investigations in this direction could be made.

The limit + 80° was chosen because the repetition of Carrington's observations between 81° and 90° was considered superfluous, and Hamburg had already undertaken the extension of Carrington's observations from 81° to 80°.

b. Within these limits, all stars in the Durchmusterung to the ninth magnitude, and, in addition, all stars which have been more exactly observed by Lalande, by Bessel at Koenigsberg, and by Argelander at Bonn, are to be observed.

c. The observations are to be differential. The clock errors are not to be found from the fundamental stars usually chosen for this purpose, and the equator point corrections are not to be derived from observations at upper and lower culminations, but these elements are to be derived from a series of 500 or 600 stars, distributed as uniformly as possible over the northern heavens. The exact coördinates of these are to be determined at Pulkowa, thus securing the unity necessary in order to connect in one system the observations of different zones.

d. Every star is to be observed twice. If the two observations differ by a quantity greater than ought to be expected, a third observation will be necessary.

e. In order to facilitate the work, it will be desirable to use only three or four transit threads, and only one or two microscopes.

In order to facilitate the reductions to apparent place, the working-list of stars should be comprised within narrow limits.

f. Before the commencement and after the close of each zone, two or three fundamental stars are to be observed upon the same threads and with the same microscopes as were used in the zone observations. When the seeing is not good, and when for any other cause it seems desirable, one or more fundamental stars may be observed in the course of the zone. The number and selection of the stars will depend upon the character of the instrument employed. If it remains steady for several hours, and has no strongly marked flexure or division errors, or if these errors have been sharply determined, the fundamental stars may be situated ten degrees or fifteen degrees away from the zone limits. However, there must remain many things for which no general rule can be given, and which must be left to the judgment of the observer, aided by an accurate knowledge of his instrument.

g. With a Repsold or a Martin instrument, one microscope will be sufficient, if its position with respect to the whole four can be determined. It will be sufficient, if the change in position during the observations can be interpolated to $0.2''$.

h. It will be desirable to divide beforehand the zones into such time intervals that the observations can be easily made.

i. Zones exceeding one and one-half or at the most two hours are not advisable, first, because the zero points will be too far apart, and secondly, because a longer duration will involve too much fatigue physically and mentally.

At the conclusion of this report, all the astronomers present who were willing to take part in this work were requested to communicate with the council, stating the region of the heavens which they preferred to select for observation.

At this meeting, Berlin, Bonn, Helsingfors, Leipsic and Mannheim, signified their intention to share in the work. Leiden also expressed its intention of taking part as soon as the work already undertaken should be completed.

When the stars to be observed had been selected from the Durchmusterung, it was found that the number would not vary much from 100,000, requiring rather more than 200,000 observations.

Preparations for the work of observation were immediately commenced; and, by the time of the next report in 1869, considerable progress had been made.

In the report for this year, the provisional places of a catalogue of 539 fundamental stars were published. This catalogue is composed of two parts. The list of Hauptsterne consisting of 336 stars to the fourth magnitude, had been previously observed at Pulkowa by Wagner with the large transit instrument, and by Gyldén with the Ertel vertical circle. The list of Zusat-sterne consists of 203 stars fainter than the fourth magnitude. As the details of the work in the formation of the provisional places of the stars of this list are not given in the report, it is not quite clear upon what authority they rest. The work assigned to the Pulkowa observatory by the zone commission was the exact determination of the places of the stars of this list. The observations were undertaken by Gromadski with the Repsold meridian circle. In accordance with the plan adopted, each star was observed eight times,—four times in each position of the instrument. The observations were differential with respect to the Hauptsterne.

The results were published by Struve in 1876; and the places there given were used in the first reduction of the Harvard college observations for 1874–75, and perhaps in some other cases also.

About this time a change seems to have been made in the original plan with respect to the formation of the final catalogue of fundamental stars, of which I have been unable to find a clear account. The original intention was to make the positions depend entirely upon the observations at Pulkowa. The zone commission established by the Gesellschaft, however, committed the formation of this catalogue to Auwers; and it is to him that we owe the most complete and most perfect catalogue of fundamental stars yet published. The Pulkowa system for 1865 was adopted as the basis; but, in order to obtain greater freedom from accidental stars, the final catalogue was obtained by combining with the Pulkowa series, the Greenwich observations from 1836 to 1876, the Harvard college observations for 1871–2, the Leipsic observations, in declination only, between 1866 and 1870, and the Leiden observations in declination between 1864 and 1870. Before this combination was made, however, these observations were all reduced to the Pulkowa system.

The following observatories have taken part in the zone observations:—

Observatories.	Limits of zones in declination.	Observatories.	Limits of zones in declination.
Nikolaiew	$-2°$ to $+1°$	Lund	$+35°$ to $+40°$
Albany	$+1$ " $+5$	Bonn	$+40$ " $+50$
Leipsic	$+4$ " $+10$	Harvard College .	$+50$ " $+55$
Leipsic	$+10$ " $+15$	Helsingfors . . .	$+55$ " $+60$
Berlin	$+15$ " $+25$	Christiana . . .	$+65$ " $+70$
Cambridge (Eng.) .	$+25$ " $+30$	Dorpat	$+70$ " $+75$
Leiden	$+30$ " $+35$	Kasan	$+75$ " $+80$

The zone between $-2°$ and $+1°$ was originally undertaken at Palermo, that between $+1°$ and $+4°$ at Neuchâtel, that between $+4°$ and $+10°$ at Mannheim, and that between $+35°$ and $+40°$ at Chicago.

In the latter case, the great fire at Chicago crippled the resources of the observatory to such an extent, that Safford was compelled to relinquish the work, which was at that time quite far advanced.

The chief items in connection with this work are found in the accompanying tabular statement.

Attention was called, at an early date, to the importance of continuing the survey of the northern heavens beyond the southern limit fixed by Argelander. The preparation necessary for the execution of this work consisted in the extension of the Durchmusterung to the tropic of Capricorn. This was undertaken by Schönfeld.

In the report to the Gesellschaft at the meeting held at Stockholm in 1877, he has given an account of this work, in which he stated that it was sufficiently near completion to invite the consideration of the question of the meridian circle determinations of the places of stars to the ninth magnitude. The lack of southern fundamental stars whose positions were well determined was still a hinderance to the immediate commencement of the work. Relatively more stars of this class are required than in the northern observations, in order to eliminate the inequalities due to refraction. Schönfeld stated, that, while the burden of the determination of the places of these southern fundamental stars must rest mainly upon southern observations, it seemed necessary to connect them with the Pulkowa system by a connecting link (Mittel-

STATION. — ZONE.	Diameter of objective.	Focal length.	Magnifying power employed	No. of transit threads for zone stars.	No. of transit threads for fundamental stars.	Me obser
NIKOLAIEW. — 2°...+ 1°	107 mm.	164 cm.	120 ... 175	4...6	6...11	Chron
ALBANY. + 1°...+ 5°	8 inches.	10 feet.	—	—	—	Chron
LEIPSIC. + 4°...+ 15°	162 mm.	246 cm.	190	4...6	21	Chron
BERLIN. + 15°...+ 20°	108 mm.	163 cm.	171	5	15...25	Chron
BERLIN. + 20°...+ 25°	189 mm.	264 cm.	230	5	15...25	Chron
CAMBRIDGE, ENG. + 25°...+ 30°	8 inches.	9 feet.	120	3	7	Eye a
LEIDEN. + 30 ...+ 35	6 inches. (Paris.)	6 feet. (Paris.)	200 ... 130	5	15	Chron
LUND. + 35°...+ 40°	6 inches. (Paris.)	7 feet. (Paris.)	173	4	9	Eye an Dec Chron afte
BONN. + 40°...+ 50°	117 mm.	195 cm.	140	3...4	6...9	Eye a
HARVARD COLLEGE. + 50°...+ 55°	8¼ inches.	112 inches.	310	5	5...19	Chron
GOTHA HELSINGFORS. + 55°...+ 65°	5¼ inches. (Paris.)	8 feet. (Paris.)	180	3+	5+	Eye a
CHRISTIANA. + 65°...+ 70°	109 mm.	158 cm.	212	9	15	Eye a
DORPAT. + 70°...+ 75°	108 mm.	164 cm.	180	4	13	Eye a
KAZAN. + 75°...+ 80°	—	—	—	—	—	

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glied), through observations at some observatory well situated for this purpose. At this meeting Sande Bakhuyzen, at Leiden, gave notice of intention to take part in this work. Gyldén urged the importance of securing the coöperation of Melbourne; and Peters suggested the advantage of securing Washington as an additional "mean term" (V. J. S., 1877, p. 265).

The next reference to this work is contained in the Vierteljahrsschrift for 1881, xv, p. 270. A list of three hundred and three southern stars is here given, whose exact places were at that time being determined at Leiden and at the Cape of Good Hope. This list was selected by Schönfeld and Sande Bakhuyzen, in a way to meet the requirements referred to in previous discussions.

A final catalogue of eighty-three southern fundamental stars by Auwers appears in this number of the Vierteljahrsschrift. The places depend upon the same authorities as for the northern stars, with the addition of the Cape of Good Hope catalogue for 1860, Williamstown, Melbourne for 1870, and Harvard College (Safford) for 1864. For stars not observed at Pulkowa, the general catalogue of Yarnall (1858–61), and the Washington observations, with the new meridian circle between 1872 and 1875, were employed. As in the case of the northern stars, these observations are all reduced to the Pulkowa system for 1865. It is understood that the coördinates of the list of three hundred and three stars are to depend upon this extension of the general system of Publication xiv, to the limits required by the southern Durchmusterung of Schönfeld.

It would be surprising if all the conditions of success were fulfilled in the first execution of a work having the magnitude, and involving the difficulties, of the scheme of observations undertaken under the auspices of the Gesellschaft. The extent of the discordances which are to be expected between the results obtained by different observers can only be ascertained when the observations by which the different zones are to be connected have been reduced. Each observer extended the working-list of his own zone 10′ north and south; and it is expected that a sufficient number of observations of this kind have been made to determine the systematic relations existing between the coördinates of each zone with those of its neighbor.

It is probable, however, that the experience of Gill will be

repeated on a larger scale. In 1878 he solicited the coöperation of astronomers in the determination of the coördinates of twenty-eight stars, which he desired to employ in the reduction of his heliometer observations of the planet Mars for the purpose of obtaining the solar parallax. The results obtained at twelve observatories of the first class are published in vol. xxxix, p. 99, of the Monthly Notices of the Royal Astronomical Society. Notwithstanding the fact that the final values obtained at each observatory depend upon several observations, the average difference between the least and the greatest results, obtained by different observers for each star, is 0.24^s in right ascension and $2.3''$ in declination. In four cases the difference in right ascension exceeds 0.30^s, and in four cases the difference in declination exceeds $3.0''$.

Even after the results are reduced to a homogeneous system, the following outstanding deviations from a mean system are found :—

Authority.	$\Delta\ \alpha$	$\Delta\ \delta$	Authority.	$\Delta\ \alpha$	$\Delta\ \delta$
	s.	"		s.	"
Koenigsberg . .	+.005	—0.71	Leiden . . .	—.053	—0.19
Melbourne . . .	+.026	—0.49	Paris	+.055	+0.01
Pulkowa . . .	+.005	+0.36	Washington . .	—.120	+0.78
Leipsic	+.049	+0.40	Harvard College.	—.072	+0.09
Greenwich . . .	+.009	—0.56	Cordoba . . .	—.032	—0.20
Berlin	+.044	+0.67	Oxford . . .	+.076	+0.21

The observations of a second list of twelve stars, one-half of the number being comparatively bright, and the remaining half faint, showed no marked improvement, either with respect to the magnitude of errors which could be classed as accidental, or in regard to the systematic deviations from a mean system.

This discussion revealed one source of discordance which will doubtless affect the zone observations, viz.: the difference between right ascensions determined by the eye-and-ear method, and those determined with the aid of the chronograph.

The programme of the Gesellschaft makes no provision for the elimination of errors which depend upon the magnitude of the stars observed; but special observations have been undertaken at several observatories for the purpose of defining the relation between the results for stars of different magnitudes. At Harvard College observatory, the direct effect of a reduction of the magni-

tude has been ascertained by reducing the aperture of the telescope by means of diaphragms. Besides this, the observations have been arranged in such a manner that an error depending upon the magnitude can be derived from an investigation of the observations upon two successive nights.

At Leiden, at Albany, and perhaps at other observatories, the effect of magnitude has been determined by observations through wire gauze. But notwithstanding all the precautions which have been taken in the observations, and which may be taken in the reductions, it will undoubtedly be found that the final results obtained will involve errors which cannot be entirely eliminated.

In the experience of the speaker, two other sources of error have been detected. It has been found, that there is a well-defined equation between the observations, which is a function of the amount, and the character of the illumination of the field of the telescope. It has also been found that observations made under very unfavorable atmospheric conditions differ systematically from those made under favorable conditions. When the seeing was noted as very bad, it is found that the observed right ascensions are about $.08^s$ too great, and that the observed declinations are about $0.8''$ too great.

There are doubtless other sources of error which the discussion of the observations will bring to light. The effect of the discovery of these and other errors will probably be to hasten the repetition of the zone observations under a more perfect scheme, framed in such a manner as to cover all the deficiencies which experience has revealed, or may yet reveal. The labor involved in this supplemental survey would not be very great, since the object sought would be accomplished by a special programme of observation, involving perhaps one-tenth of the number of observations made in the present survey. One would not probably go far astray in naming the year 1900 as the mean epoch of the new survey. If the observations are again repeated in 1950, sufficient data will then have been accumulated for at least an approximate determination of the laws of sidereal motion.

What is the present state of our knowledge upon this subject? It can be safely said that it is very limited. First of all, it cannot be affirmed that there is a sidereal system in the sense in which we speak of the solar system. In the case of the solar system, we have a central sun about which the planets and their satellites revolve in obedience to laws which are satisfied by the hypothesis

of universal gravitation. Do the same laws pervade the interstellar spaces? Is the law of gravitation indeed universal? What physical connection exists between the solar system and the unnumbered and innumerable stars which form the galaxy of the heavens? Do these stars form a system which has its own laws of relative rest and motion? or is the solar system a part of the stupendous whole? Does the solar system receive its laws from the sidereal system? or did Newton indeed pierce the depths of the universe in the discovery of the law which gave him immortality? Are we to take the alternative stated by Ball, — either that our sidereal system is not an entirely isolated object, or its bodies must be vastly more numerous or more massive than even our most liberal interpretation of observations would seem to warrant? Are we to conclude, for example, that stars like 1830 Groombridge and *a* Centauri, "after having travelled from an infinitely great distance on one side of the heavens, are now passing through our system for the first and only time, and that after leaving our system they will retreat again into the depths of space to a distance which, for anything we can tell, may be practically regarded as infinite?" Can we assert with Newcomb, that in all probability the stars do not form a stable system in the sense in which we say that the solar system is stable, that the stars of this system do not revolve around definite attractive centres? Admitting that the solar system is moving through space, can we at the present moment even determine whether that motion is rectilinear or curved, to say nothing of the laws which govern that motion? How much of truth is there in the conjectures of Wright, Kant, Lambert, and Mitchel, or even in the more serious conclusions of Mädler, that Alcyone of the Pleiades is the central sun about which the solar system revolves?

These are questions which, if solved at all, must be solved by a critical study of observations of precision accumulated at widely separated epochs of time. The first step in the solution has been taken in the systematic survey of the northern heavens undertaken by the Gesellschaft, and in the survey of the southern heavens at Cordoba by Dr. Gould. The year 1875 is the epoch about which are grouped the data which, combined with similar data for an epoch not earlier than 1950, will go far towards clearing up the doubts which now rest upon the question of the direction and the amount of the solar motion in space; and it cannot be doubted that our knowledge of the laws which connect the si-

dereal with the solar system will be largely increased through this investigation. The basis of this knowledge must be the observed proper motions of a selected list of stars, so exactly determined that the residual mean error shall not affect the results derived; or, failing in this, of groups of stars symmetrically distributed over the visible heavens, sufficient in number to affect an elimination of the accidental errors of observation, without disturbing the equilibrium of the general system.

For an investigation of this kind, a complete system of zone observations, at widely separated intervals, will afford the necessary data, if the following conditions are fulfilled.

First: The proper motions must be derived by a method which does not involve an exact knowledge of the constants of precession. In every investigation with which I am acquainted, the derived proper motions are functions of this element.

Second: The general system of proper motions derived must be free from systematic errors. Errors of this class may be introduced either through the periodic errors inherent in the system of fundamental stars employed in the reduction of the zone observations, or in a change in the constants of precession. It is in this respect that the utmost precaution will be required. If from any cause errors of even small magnitude are introduced into the general system of proper motions at any point, the effect of these errors upon the values of the coördinates at any future epoch will be directly proportional to the interval elapsed. We can, therefore, compute the exact amount of the accumulated error for any given time.

When this test is applied to the stellar systems independently determined by Auwers, Safford, Boss and Newcomb, we find the following deviations *inter se* at the end of a century.

	Maximum mean deviation in a century.		Maximum systematic deviation in a century.	
	$\Delta\alpha$	$\Delta\delta$		
Auwers *minus* Safford . . .	*—0.22s.	+0.2"	0.23s.	1.1"
Auwers *minus* Boss	-	+0.8	-	2.1
Auwers *minus* Newcomb . .	—0.09	+0.8	0.06	2.2

* With Bessel's constants of precession.

It is the common impression, that both the direction and the amount of the motion of the solar system in space are now well established. The conclusions of Struve upon this point are stated in such explicit language that it is not surprising that this impression exists. He says, "The motion of the solar system in space is directed to a point in the celestial sphere situate on the right line which joins the two stars of the third magnitude π and ω Herculis at a quarter of the apparent distance between these stars measured from π Herculis. The velocity of this motion is such that the sun, with the whole cortége of bodies depending on him, advances annually in the direction indicated, through a space equal to 1.623 radii of the terrestrial orbit or one hundred and fifty-four millions of miles."

It must be admitted that there is a general agreement in the assignment by different investigators of the coördinates of the solar apex. This will be seen from the following tabular values:

Authorities.	Right ascension.	Declination.
Herschel, 1783	257° 00′	+25° 00′
Prevost	230 00	+25 00
Klugel, 1789	260 00	+27 00
Herschel, 1805	245 52	+49 38
Argelander, 1837	259 52	+32 29
Lundahl	252 24	+14 26
Struve	261 23	+37 36
Galloway	257 04	+34 18
Mädler	261 38	+39 54
Airy	256 54 261 29	+39 29 +26 44
Dunkin	261 14 263 44	+32 55 +25 00

In estimating the value which should be attached to these results, several considerations must be taken into account.

(*a*) All of the results except those of Galloway depend practically upon the same authorities at one epoch, viz., upon Bradley.

(*b*) The deviations *inter se* probably result, in a large measure, from the systematic errors inherent in one or both of the fundamental systems from which the proper motions were derived. For example, Lundahl employed Pond as one of his authorities, and it is in Pond's catalogue that the most decided periodic errors exist.

(*c*) Biot in 1812, Bessel in 1818, and Airy in 1859, reached the conclusion that the *certainty* of the movement of the solar

system towards a given point in the heavens could not be affirmed.

(*d*) The problem is indirect. In the case of a member of the solar system, exact data will determine the exact position in orbit at a given time; but here we have neither exact data, nor can we employ trigonometrical methods in the solution. We simply find that the observed proper motions are probably somewhat better reconciled under the hypothesis of an assumed position of the apex of the solar motion. The method of investigation employed by Safford, who has of late years given much attention to this subject, consists in assuming a system of coördinates for the pole of the solar motion, from which is determined the direction each star would have if its own proper motion were zero. Comparing this direction with the observed direction as indicated by the observed proper motion, equations of condidition are formed from which a correction is found to the assumed position of the apex, by the method of least squares.

It must always be kept in mind, that the quantities with which we must deal in this investigation are exceedingly minute, and that the accidental errors of observation are at any time liable to lead to illusory results. The weak link in the chain of Mädler's reasoning is to be found here. I think we can assume $0''.2$ as the limit of precision in the absolute determination of the coördinates of any star, however great the number of observations upon which it depends. Beyond this limit it is impossible to go, in the present date of instrumental astronomy. It is safe to say, that there is not a single star in the heavens whose coördinates are known with certainty within this limit. Do not misunderstand me. Doubtless there are many stars in which the error will at some future time be found to fall within this limit. But who is prepared to select a particular star, and say that the absolute position of this star in space cannot be more than $0''.2$ in error?

(*e*) At present an arbitrary hypothesis is necessary in the discussion of the problem. Airy assumed that the relative distances of the stars are proportional to their magnitudes; and he found slightly different results according to different modes of treatment. Safford in his investigation made use of a device which he had previously employed in the computation of the differential coëfficients of planetary elements, thereby reducing to a certain ex-

tent the magnitude of the residuals between observation and computation to quantities of the second order. The axis of Z was located in the direction of Argelander's determination of the apex of the solar motion or in right ascension 259° 50″.8 and declination +32° 29′.1. The axis of X was placed in the equator in right ascension 349° 50′.8 and that of Y in right ascension 79° 50′.8 and in declination 57° 30′.9. When the reductions to these three axes were made, it was found that nearly all of the motion was along the axis of Z as was to be expected from the assumption made. The tendency to move in this direction was therefore primarily assumed to represent the solar motion. Since it was found that the *average factor*, by which the proper motion must be multiplied in order to represent this tendency, changes only in a slight degree with the magnitude of the proper motion itself, the proper motion was assumed, as a first approximation, to be inversely proportional to this element.

Later investigations have been made by De Ball (Ueber die eigene Bewegung des Sonnensterns, Bonn, 1877) but the details have not yet come to hand. It is understood, however, that his results coincide in a general way with those previously obtained.

The most recent investigation is by Bolte. His paper may be considered as a numerical application of the formulæ by which Schönfeld has attempted to express the relation between the motion of the solar system and a systematic rotation of the fixed stars which is assumed to take place in a direction perpendicular to the plane of the Milky Way. The values of the precession constants derived are certainly interesting, but they can hardly be considered decisive, especially in view of the fact that the values derived from the observed declinations alone, do not agree especially well with the values derived from the right ascensions.

It is clear from this brief review, that we have here a field of investigation worthy of the highest powers of the astronomer. The first step has been taken in the survey of the heavens carried on under the auspices of the Gesellschaft. It remains for the astronomers of the present generation to solve the difficulties which now environ the problem, and to prepare the way for a more perfect scheme of observation in the next century.

THIRTY-EIGHTH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE, DECEMBER 19, 1883, AND LAID BEFORE THE BOARD OF OVERSEERS, JANUARY 9, 1884.

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1884.

THIRTY-EIGHTH

ANNUAL REPORT

OF THE

DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF HARVARD COLLEGE.

1882–83.

To the President of the University:

Sir, — During the past year the subscription of five thousand dollars annually for five years, which was secured in 1878, has expired. To replace it an attempt has been made to raise by subscription the sum of one hundred thousand dollars as a permanent fund. Up to the present time about fifty thousand dollars have been promised for this purpose, the income of which will permit a large amount of useful work to be carried on permanently by the Observatory. This sum will be sufficient to prevent the sudden restriction of scientific operations, which would otherwise have been inevitable on the expiration of the subscription of 1878. It is hoped that additional donations may eventually increase this fund to one hundred thousand dollars, and thus permit the same degree of activity to be maintained as during the past five years.

I give my best thanks to the ladies and gentlemen who by their generous gifts have aided the Observatory at this critical period in its history. Their names, with the amount of the subscription of each, are given below in alphabetical order. Asterisks indicate subscribers to the temporary fund of 1878, as well as to the present permanent fund.

List of Subscribers.

Name.	Amount.	Name.	Amount.
A Friend	$10,000	* H. H. Hunnewell	$1,000
* W. Amory	2,000	S. Johnson	250
Francis Blake	500	* H. P. Kidder	2,485
* J. I. Bowditch	5,000	A. A. Lawrence	500
* Martin Brimmer	500	A. T. Lyman	500
Shepherd Brooks	500	* T. Lyman	200
T. Q. Browne	500	* W. P. Mason	200
* J. A. Burnham	1,000	* F. H. Peabody	500
* C. F. Choate	500	* O. W. Peabody	250
Wm. Claflin	200	* E. C. Pickering	1,000
Alex. Cochrane	200	H. B. Rogers	1,000
* C. P. Curtis	1,000	* Stephen Salisbury	2,000
J. C. Delano	250	* Mrs. D. Sears	200
W. Endicott, Jr.	2,000	* N. Thayer	5,000
* J. M. Forbes	1,000	* C. E. Ware	500
* Geo. Gardner	100	* Miss Wigglesworth	200
* J. L. Gardner	500	* R. C. Winthrop	250
R. C. Greenleaf	500	R. C. Winthrop, Jr.	100
* Mrs. A. Hemenway	2,000	Mr. and Mrs. J. H. Wolcott	200
* G. Higginson	5,000		

I take this occasion to gratefully acknowledge the aid of the Visiting Committee, to whose efforts both this fund and the subscription of 1878 are due. It is hoped that the results attained will satisfy the friends of the Observatory that this increase in income has been a judicious application of money to scientific purposes. The increase in the number of assistants and of instruments in active use is especially to be noted.

In the present distribution of observatory work, my own observations are chiefly made with the large equatorial and the meridian photometer. Mr. Wendell takes part in the observations with both these instruments, and Professor Searle in those made with the equatorial. Messrs. Cutler and Eaton assist in recording these observations. Their reduction is carried on by Miss Farrar, Mrs. Fleming, and Mr. Cutler, with the aid and partly under the supervision of Mr. Wendell. Their preparation for the press and publication are conducted by Professor Searle and myself. The meridian circle remains in charge of Professor Rogers, who is aided in the observations by Mr. Pratt, and in their reduction also by Mrs. Rogers, Miss Saunders, Miss Bond, Miss Winlock,

and Mr. Eaton. The equatorial mounted in the small dome is actively employed by Mr. Chandler, mainly in the observation of variable stars. He is assisted in the reductions by Mr. Metcalf. The time signals are in charge of Mr. Edmands. The meteorological observations and the ordinary business matters of the Observatory are usually conducted by Professor Searle. The computation of cometary orbits is undertaken by Mr. Chandler, and the announcements of astronomical discoveries and results by telegraph or by published circulars are in charge of Mr. Ritchie.

East Equatorial.

Eclipses of Jupiter's Satellites. — Photometric observations of these eclipses have been continued upon the system adopted in 1878. In all, two hundred and forty eclipses have now been observed, fifty-five since the end of October, 1882. Some observations were also made last spring to determine the accuracy with which occultations and transits of Jupiter's satellites could be observed. The method heretofore in use has been to observe the times of contact, but these are not easily determined, from the indefinite outlines of the planet and its satellites. The observations just mentioned were made with a double image micrometer. An image of the satellite was placed midway between the two images of Jupiter, and the time of the setting was recorded. Each measure of this kind furnished a determination of the distance between the centres of the planet and satellite, and a large number of measures could be obtained on each occasion instead of a single estimate of contact. The result showed that the time of an occultation or transit could thus be determined with a probable error of about nine seconds, which is less than the probable error in observing an eclipse by the old method.

Revision of Zone Observations. — The stars between 50′ and 60′ north of the equator for the epoch 1860, and occurring either in the zones published in Volume VI. of the annals of this Observatory or in the Durchmusterung, have been twice observed by Professor Searle with the modified wedge photometer described in the last Report. The transit of each star over a bar, and its subsequent disappearance in the wedge were recorded by the chronograph, while its approximate declination was estimated with the aid of three transverse bars 5′ apart. The reduction of these observations is far advanced, and the examination of dis-

crepancies between them and those made under Professor Bond's direction has been begun, for the purpose of detecting any cases of considerable proper motion or variability which may exist in this zone of 10′ in width. Differences amounting in right ascension to $0^s.5$, and in declination to $0'.5$, are investigated by more accurate observations when the discrepancy is not explained by an error in reduction. A difference of three quarters of a magnitude in the two separate results obtained with the wedge photometer is also regarded as large enough for special examination. The number of stars which require re-observation does not thus far seem likely to be great. Even a merely negative result with respect to proper motion will have some interest, as it will tend to establish the comparative fixity of the fainter stars, which is at present assumed rather than known. As Professor Bond's zone observations were finished in 1860, the interval between them and the present revision is sufficient to bring into notice any case of annual proper motion much in excess of 1″, either in right ascension or in declination. In the provisional reduction of the photometric portion of the work, the scale of magnitude is made to depend upon the Durchmusterung, but it is expected that the meridian photometer will soon provide the means for a more complete reduction.

Objects having singular Spectra. — The search for these objects has been continued; the regions selected for examination have been chiefly in the Milky Way, but little time has been devoted to this work on account of the use of the large telescope for more important purposes.

Standards of Stellar Magnitude. — In accordance with a plan proposed in the Proceedings of the American Association for the Advancement of Science, xxx. 1, it is intended to form charts of the stars in small areas 4^m in extent in right ascension and 10′ in declination. The centre of each area follows one of a selected series of twenty-four bright equatorial stars at an interval of 4^m, in the same declination. Besides making the charts, it is proposed to determine the magnitudes of a sufficient number of faint stars in each region to exhibit a scale of magnitude to which reference may readily be made by observers who desire their estimates to be directly comparable with those of others.

In obtaining material for the proposed charts, various plans have been tried, with the view of economizing time so far as

practicable. Data have been collected for several stars, sufficiently complete to enable the construction of the charts to be undertaken.

Comets. — The present arrangements for the distribution of astronomical intelligence, to be further mentioned below, insure the early receipt at this Observatory of the news of cometary discoveries, so that the first accurate observations of a comet may often be made here. This was the fact with regard to each of the two comets found during the year. The observer was Mr. Wendell. The positions he obtained were extensively employed in the computation of orbits by various astronomers, and preceded by twenty-four hours those obtained elsewhere.

Variable Stars. — The large telescope has been frequently used in observing variable stars, too faint to be visible with the west equatorial, the work of which is mentioned below. The collection of data for charts of the immediate neighborhood of many variable stars has also been undertaken with the large telescope.

Spectra and Color of Stars. — This subject has long been under consideration, in the hope that good means may soon be found of determining the position of the lines and the distribution of the light in different types of stellar spectra. As yet, the observations made have been experimental and preliminary. A large spectroscope was ordered from Hilger, of London, for the purpose of research in this direction, and was received early in 1882. Owing, however, to the difficulty which the maker of the instrument found in understanding, from mere correspondence, the essential objects to be aimed at in its construction, this spectroscope has not proved satisfactory for the proposed work. A more promising form of apparatus is now under consideration.

Meridian Circle.

After an interruption of about a year, the fundamental observations with the meridian circle were resumed on February 8, 1883. Before the commencement of the work the level and azimuth of the instrument were brought nearly to zero, by scrapping the segmental bearings of the pivots. Between February 8 and November 1, 2383 observations were made of the fundamental stars, 136 of Polaris, and 121 of the Sun, making in all 2640. As heretofore, all the instrumental constants are referred to Polaris alone. An average of 10.5 independent determinations

of the declination of this star have been made at each transit. The actual number of declinations observed is 1420.

The level of the instrument as determined with the Clark reversible level has remained remarkably constant during the year, the entire range of variation having been only $0^s.17$. The individual measures differ on the average by about $0^s.02$ from the adopted mean values.

As formerly, the reversible level has been used to determine the variation of the index error of the circle for the interval during successive transits of the Pole Star. The long collimator has been used for the same purpose during observations near the equinoxes and the solstitial points; but on account of the variation of the fixed point of the collimator during the daytime, and especially on account of the frequent unsteadiness of the image, the results obtained are far less satisfactory than those derived from the level.

A series of observations undertaken for the determination of the longitude of McGill College Observatory, Montreal, extended from June 2 to June 23 inclusive. The plan of the campaign involved an exchange of observers in addition to the determination of personal equation by the usual methods. Accordingly, after three nights at Cambridge and Montreal, Professor Rogers exchanged stations with Professor McLeod. The results for personal equation at each station are as follows: —

Montreal.	Cambridge.
Rogers, *minus* McLeod.	Rogers, *minus* McLeod.
June 13 + .096^s.	June 28 + .143^s.
June 14 + .111.	June 30 + .144.

The observations at Cambridge were made with the Russian Transit, but simultaneous observations were made with the meridian circle by Mr. Brown. The whole number of observations made during the series is about 400, and their reduction is nearly completed.

After the completion of the reduction to 1875.0 of the zone observations made between 1870 and 1879, it was found by direct comparison with the Durchmusterung positions brought forward to the same epoch, that observations were wanting for 306 stars. It was also found that in many cases, especially between 19 hours and 1 hour of right ascension, the wrong star was observed when there were two or more stars in the same field. During the obser-

vations no less than 528 stars were for various causes marked in the observing records as not seen. Since, in a large number of cases, the failure to observe was due to the faintness of the stars under a bright field illumination, it was thought desirable to improve the character of the illumination before commencing the work of re-observation. Accordingly, Mr. George B. Clark, after a series of experiments with a dark field illumination for lines ruled upon glass, arranged a set of mirrors in the tail-piece of the telescope, by which the light, passing from a lamp swinging upon gimbals, through an opening in the tube, is thrown down upon the glass plate in two planes at an angle of about 45°.

When a red-shade glass is interposed, an even illumination is obtained, under which stars of the tenth or eleventh magnitude can be easily seen.

The re-observation of the scattering zone stars was begun October 9, and between this date and November 1, 512 observations were made. The gap which has been filled extends from 19^h 40^m, to 1^h.

The reduction to 1875.0 of the zone observations made between 1870 and 1879 has been completed. The reduction of the observations of fundamental stars for the same interval is also completed. The entire computing force assigned to the Meridian Circle is now engaged in the preparation of copy for publication. This work will be completed in about five weeks.

It will be seen therefore that practically two volumes are ready for publication. The third volume, containing the observations of secondary stars brighter than the magnitude 6.5 and the final catalogues, is in a forward state of preparation.

The final catalogues will depend upon about 46,600 observations, distributed as follows: observations of zone stars, 24,300; of primary stars, 15,200; of secondary stars brighter than the magnitude 6.5, 7,100.

Meridian Photometer.

The position of this instrument, as a permanent portion of the Observatory work, may now be regarded as fairly established. After a thorough trial it has shown itself capable of determining absolute stellar magnitudes more satisfactorily than any other photometer that we have tried. It bears the same relation to photometry that a transit circle does to measurements of position,

and will be regarded as the standard to which all our other measures will be reduced. As it is capable of measuring any star of the ninth magnitude or brighter, its field of work is, for the present, indefinitely great. We are now able to determine satisfactorily the brightness of any such stars when they are desired for any special purpose by other astronomers.

One hundred and thirty-three series, including about twenty thousand sittings, have been made since November 1, 1882, by Mr. Wendell and myself. The most important investigation in progress has been the revision of the Durchmusterung magnitudes. The results of the estimates of brightness made at thirteen observatories, and extending over a period of ten or fifteen years, will thus be brought together and reduced to a single system.

The stars adopted as standards in the Uranometria Argentina, have been observed with this instrument on at least three nights each. The scale has thus been determined in terms of the photometric scale. A comparison has also been made with the measures of the brighter of these stars, obtained with the small meridian photometer. The results with the new instrument are on the average 0.04 magnitudes fainter. This difference is so small that its existence may be regarded as doubtful. No systematic difference depending upon the right ascension is perceptible. On the other hand, a comparison with the Durchmusterung, Uranometria Argentina, and a series of eye estimates, indicate well-marked systematic errors in these catalogues, due to the presence of the Milky Way. In seven and eighteen hours of right ascension the stars are estimated two or three tenths of a magnitude too faint in each of these catalogues. The proximity of bright stars, especially those in Orion, appears a reasonable explanation of this source of error.

Some miscellaneous observations have also been taken, of which the most interesting are those of Neptune, of the comparison stars of *S Cancri*, and of other variables. Observations of Neptune on eleven nights give its light as 7.71 ± .02.

MISCELLANEOUS.

Variable Stars. — The study of the variable stars by Mr. Chandler has been an important part of the Observatory work. The bibliography is nearly completed so far as the first extraction of references is concerned. Notes have been prepared to exhibit

the evidence of variability which has been published with regard to about twelve hundred stars. This list excludes many cases in which the evidence is entirely inadequate. A table, giving all the published maxima and minima of each of the variables of long period, is now in process of construction. The preparation of this table has led to the important result that an interval of several years occurs in which no observations appear to have been made of about thirty of these objects. About one hundred and forty stars belong to this class, and since last April all of them have been observed by Mr. Chandler, with the six-inch Clacey Equatorial mounted in the west dome. According to the present plan of work, he observes each of these stars at least twice a month, and more frequently during its brightest phases. Charts of the vicinity of these variables have been prepared, and some progress made towards their completion. Similar charts have been made for about seventy telescopic stars suspected of variability, and nearly two hundred observations of these stars have been obtained. The color of the variable stars is also estimated, about three hundred observations of this class having already been made.

A circular was distributed asking the aid of amateurs and others in the observation of stars known or suspected to be variable. It is believed that by co-operation much valuable material may be collected and much time saved. Numerous replies have been received and important results have been obtained, especially by Mr. H. M. Parkhurst, of New York, and by the Rev. J. Hagen, S. J., of Prairie du Chien, Wisconsin. The great difficulty encountered by most of the observers was that of identifying with certainty the fainter stars, although this is one of the first things that should be learned by any person desiring to do useful astronomical work.

Astronomical Photography. — With the assistance of Mr. W. H. Pickering, Instructor in Physics at the Massachusetts Institute of Technology, an investigation was undertaken in astronomical photography. Two objects were kept in view: first, the determination of the light and color of the brighter stars, and, secondly, the construction of a photographic map of the whole heavens. After numerous preliminary observations, a method was employed by which a photograph of the brighter stars included in about one-twelfth of the entire heavens could be obtained on a single plate.

Maps were also obtained containing a region of about 15° square, containing stars as faint as the eighth magnitude. The color exercised a marked influence on the intensity of the photographic images, in some cases producing a difference equivalent to four magnitudes. It is thought that photography may offer the most delicate test we yet have of the color of a star, — differences too small to be perceptible by the eye becoming distinctly visible in the photographic images.

A circular has been issued to request the donation of specimens of astronomical photography, to be added to those already belonging to this Observatory. Many of the early efforts to render this mode of research available were made by Professors W. C. and G. P. Bond, and the numerous results of their work now preserved here have great historical interest. The photographs of the solar eclipses of 1869 and 1870, taken under the direction of Professor Winlock, form another important portion of the collection. It is hoped that the additional specimens received in response to the circular will permit the formation here of a series fully illustrating the origin and progress of astronomical photography. Many fine examples of the art have already been presented to the Observatory. It is intended hereafter to issue a catalogue of the collection, in which the proper acknowledgment will be made of the liberality of each contributor to it. It will be desirable at the same time to indicate the existence of other important astronomical photographs, and the places where they are preserved. Information of this description will accordingly be gratefully received. Original negatives, or prints on glass from such originals are especially welcome, since they are free from many of the defects of paper prints. Much inconvenience and delay in the reception of these specimens, and of any gifts intended for the Observatory which cannot be sent by mail, may be avoided by forwarding them by a vessel bound for Boston, instead of by one destined for New York.

Time Signals. — The attention of the public has been directed to this department of our work by the introduction of the new system of standard time. The policy of this Observatory, with regard to the change, has been to avoid taking any action tending to force it upon the public against their wishes, but to point out, and so far as possible to avert the difficulties that would arise should two systems of public time remain in general use. An

ırly decision was required, since the railroads centring in oston made their assent to the change conditional on the doption of the new time in dropping the Boston time-ball. .ction was delayed, however, until the official assent of the city ? Boston had been given to the adoption of the new time for strik- ıg the hour-bells and for the public clocks. The unanimity and ıse with which the community adopted the new time was largely ıe to the activity of Mr. Edmands, Assistant in Charge of the ime Service.

It is probable that the use of our signals will soon be extended y the introduction of new devices. The Rhode Island Electric ompany and the New England Telephone and Telegraph Com- any are now receiving our signals without charge while they onduct experiments with this view.

The Boston time-ball was dropped on 361 days, at noon, — on 21 days by telegraph, and on 40 days by hand. On four days it ıiled to drop at noon, but was dropped five minutes later, — on ıree days by telegraph and on one day by hand, — according to ıe usual arrangement. On December 6, 1882, the ball was ropped at 8 h. 30 m. A.M., and at 4 h. 0 m., P.M., as well as at oon, for the benefit of those desirous of making exact observa- ons of the transit of Venus.

Telegraphic Announcements. — The system of announcing stronomical discoveries employed here for some years past has eceived an important extension during the last year. An asso- iation of over fifty European observatories has been formed, with :s headquarters at Kiel, for the purpose of expediting the an- ouncement of astronomical discoveries. The Smithsonian Insti- ıtion, which had for many years rendered an important service ɔ astronomy by transmitting astronomical telegrams between 'urope and America, courteously signified its readiness to trans- ər this function to the Observatory of Harvard College, upon əarning that this Observatory was prepared to undertake it. 'he change was announced by a circular issued by the Smith- onian Institution on January 10, 1883, and since that time the 'bservatory has distributed in this country the astronomical ıtelligence received from the European association, and has ɔrwarded to Kiel information of American discoveries.

The Observatory is fortunate in having secured the assistance f Mr. John Ritchie to take charge of this department of its work.

Both Messrs. Chandler and Ritchie, by whom the present system of telegraphing astronomical announcements was devised and carried into effect, are therefore now included in the corps of the Observatory.

The early transmission by telegraph of observations and computed elements of comets between this observatory and that of Lord Crawford at Dun Echt is continued as in former years. The similar interchange of information with Dr. Oppenheim of Berlin is likewise maintained. The intelligence thus transmitted is distributed in Europe and here by means of special circulars.

Transit of Venus, 1882. — For reasons stated in the last Report, no elaborate preparations were made at this Observatory for observing the transit of Venus on the sixth of last December. But as the weather was favorable during most of the day, a considerable number of observations was obtained. Six telescopes were employed in observing the contacts. With the large equatorial, photometric and spectroscopic observations were also undertaken. The photometric measures furnished the result that the relative light of the Sun and Venus was in the ratio of 100 to 1.6, while the light of the sky surrounding the Sun would be expressed on the same scale by 7.5. This comparative darkness of Venus with respect to the sky was confirmed by direct observation. The spectroscope furnished no evidence of an atmosphere surrounding Venus; but the dispersion employed was not large.

The diameter of Venus was carefully determined by Professor Rogers and Mr. Chandler, with the results 16″.09 and 16″.35. The method employed was that of transits over inclined lines. An account of all these observations appears in the Proceedings of the American Academy of Arts and Sciences, XVIII. 15.

Almucantar. — An interesting series of observations has been made by Mr. Chandler to test the merits of an instrument devised by him some years ago. It consists essentially of a telescope floating upon mercury, and is designed for the observation of transits over given parallels of altitude. Its name has been derived from this circumstance. This instrument, although having an objective of only 1¾ inches aperture, has given results of a high degree of accuracy, the probable error of a single determination of zenith distance being 0.″5 to 0.″6. Mr. Chandler is now directing the construction of a large instrument of the same kind with a telescope five inches in aperture, and intends to apply it

certain important problems in practical astronomy, for the lution of which it possesses advantages over other astronomical struments.

While passing last summer in Europe, I visited the observatories Greenwich, Oxford, Cambridge, Paris, Brussels, Bonn, Strasırg, Berne, Geneva, Milan, Vienna, Berlin, and Potsdam, and e private observatories of Dr. Huggins, of Mr. Common, and of r. Ranyard. I was also enabled to present some of the results work in progress here at meetings of the Royal Astronomical ciety at London, the Astronomische Gesellschaft at Vienna, and e Liverpool Astronomical Society. The most important result my journey was the acquisition of copies of some valuable unblished manuscripts. By the courtesy of Lady Herschel and of eutenant-Colonel John Herschel, I was permitted to examine e original manuscripts of Sir William Herschel's observations of e light of the stars. This led to the discovery of two unpublished talogues, of which I was allowed to make a copy. The reducn of the four published catalogues by means of our photometric easures has already been mentioned in my last Report. The two ditional catalogues furnish means for completing this work, so at we have now a most valuable determination of the light of the stars in Flamsteed's Catalogue at a time when no other timates of their light are known to exist. By the kindness of ofessor Schönfeld, Director of the Bonn Observatory, I was rmitted to make a copy of the unpublished observations of varile stars made by the late Professor Argelander after the publition of Volume VII. of the Bonn Annals. These manuscripts ll be very valuable in connection with the bibliography to which ference has already been made.

Publications.

The printing of Volume XIV. of the Observatory Annals has ne on steadily during the year, the printers having constantly en supplied with copy.

The publications named below have appeared since the similar t in the last Report was drawn up, either as official communicans from the Observatory, or as papers prepared by its officers dividually.

Thirty-seventh Annual Report.

Statement of work done at the Harvard College Observatory

during the years 1877–82. By Edward C. Pickering. Cambridge, 1882.

First Circular of Instructions for Observers of Variable Stars, acting in co-operation with the Harvard College Observatory, dated January 27, 1883.

Circulars relative to the Collection and Distribution of Astronomical Intelligence. Cambridge, 1883.

On the Telegraphic Transmission of Astronomical Data. Part ii. The Phrase Code.

Circular on Astronomical Photography, dated February 21, 1883.

Address by Professor Pickering at the June Meeting of the Royal Astronomical Society. The Observatory, vi. 199. Astronomical Register, xxi. 150.

Address by Professor Pickering at the Second Annual General Meeting of the Liverpool Astronomical Society. Abstract of Proceedings, 2. Astronomical Register, xxi. 278.

The Wedge Photometer. By Edward C. Pickering. Proc. Am. Acad. of Arts and Sciences, xvii. 231.

Observations of the Transit of Venus, December 5 and 6, 1882, made at the Harvard College Observatory. Edward C. Pickering, Director. Id. xviii. 15.

On a Method of determining the Index Error of a Meridian Circle at any Instant, depending upon the observed Polar Distance of Polaris. By W. A. Rogers. Id. xviii. 284.

Studies in Metrology. By W. A. Rogers. Id. xviii. 287.

On the Reduction of Different Star Catalogues to a Common System. By W. A. Rogers. Id. xviii. 399; Astronomische Nachrichten, cvi. 257.

A Study of the Centimeter marked "A," prepared by the U. S. Bureau of Weights and Measures for the Committee on Micrometry. By W. A. Rogers. Proc. Am. Soc. of Microscopists, 1883, p. 184.

A Critical Study of the Action of a Diamond in ruling Lines upon Glass and Metals. By W. A. Rogers. Id. p. 149.

Note on Probable Errors. By W. A. Rogers. Id.

Vice-President's Address on the German Survey of the Northern Heavens. By W. A. Rogers. Proc. Am. Assoc. for the Advancement of Science, 1883, p. 53.

Results of Tests with the Almucantar in Time and Latitude. By S. C. Chandler, Jr. Id. (Also Sidereal Messenger, ii. 269.)

Mountain Observatories. By Edward C. Pickering. Appalachia, iii. 99. The Observatory, vi. 287.

On the Orbit of the Great Comet. By S. C. Chandler, Jr. Astronomische Nachrichten, ciii. 347.

On the Orbit of the Great Comet. By S. C. Chandler, Jr. Id. ciii. 381.

Elements of Comet 1882, I. By O. C. Wendell. Id. civ. 287.

Elements and Observations of the Comet 1883, Brooks-Swift. By S. C. Chandler, Jr. Id. cv. 127.

Ephemeris of Comet 1883, Brooks-Swift. By John Ritchie, Jr. Id. cv. 143.

On the Variability of 36 (Uran. Argentina) Ceti. By S. C. Chandler, Jr. Id. cv. 333.

New Planetary Nebulae. By Edward C. Pickering. Id. cv. 335.

Erratum in the Position of a Variable Star. By Edward C. Pickering. Id. cv. 351.

Observations of Comet 1883, I. (Brooks-Swift) at Harvard College Observatory. By S. C. Chandler, Jr. Id. cvi. 9.

A convenient Method of finding or identifying an Asteroid. By O. C. Wendell. Id. cvi. 205.

Date of Discovery by Brooks of Pons' Comet. By John Ritchie, Jr. Id. cvii. 43.

On the Outburst in the Light of the Comet Pons-Brooks, September 21–23. By S. C. Chandler, Jr. Id. cvii. 131.

It is evident that the present rapid accumulation of results will soon involve an important question regarding their publication. The funds especially provided for this purpose will prove entirely inadequate, and the application to this purpose of the unrestricted funds of the Observatory must be considered. This subject will probably be more fully treated in my next Annual Report.

EDWARD C. PICKERING, *Director.*

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X.

SIR WILLIAM HERSCHEL'S OBSERVATIONS OF VARIABLE STARS.

By Edward C. Pickering.

Presented January 9, 1884.

The discovery last summer of two additional catalogues of the Light of the Stars by Sir William Herschel has been announced elsewhere. At the same time a Journal was found, which gives the dates of observation for the individual comparisons contained in the six catalogues. The suggestion was made by Mr. Chandler, that the observations of variable stars contained in these catalogues would thus be rendered of value. The observations contained in Table I. were kindly forwarded by Lieut. Col. Herschel, who has taken much trouble in furnishing me with all the material available for the following discussion. The successive columns of Table I. give a current number, the number of the catalogue of Herschel in which the star is contained, the usual designation of the star, and its Flamsteed number. The next columns give the date, the observation, and the resulting magnitude. The latter is derived from the photometric observations made in 1879–82 at Harvard College Observatory with the meridian photometer, and will appear in the Annals of the Observatory, Volume XIV. The values assumed for the intervals employed by Herschel are obtained from a discussion of all his catalogues, and will appear in the same volume; they are 0.1 for a period, 0.2 for a comma, and 0.4 for a dash.

The scale employed may be defined as that in which a magnitude corresponds to the ratio whose logarithm is four tenths, and which coincides with the scale of Argelander for the magnitude 5. It is best illustrated by the statement that the magnitudes 3, 4, 5, and 6 on the scales of the Uranometria Nova, Heis, and the Durchmusterung would be expressed by 3.1, 4.2, 5.0, and 5.8 on the photometric scale.

TABLE I.

No.	Cat.	Name.	[illegible]	Date.	Comparison.	Magn.
1	I.	η Aquilæ	55	1795, July 19	65, 30, 55. 60	3.8
2		"		" "	60. 55	4.1
3		"		" "	55 — 41 — 38. 44	3.9
4		"		1795, July 25	41, 55	4.5
[illegible]	I.	g Herculis	30	1795, May 23	1, 30	4.7
6		"		1795, May 25	30 — 25	5.1
7		"		" "	11. 35 — 6, 30	4.9
8		"		" "	6, 52, 30	5.2
9		"		" "	52, 42. 30, 34	5.6
10		"		1795, Sept. 20	30. 1	4.4
11		"		" "	30 —, 25	4.9
12	I.	u Herculis	68	1795, May 22	71. 68, 72	5.2
13		"		" "	68, 90, 72	5.1
14		"		1795, Aug. 18	68, 59, 61	5.1
15		"		" "	69 — 68	5.8
16	II.	o Ceti	68	1779, Nov. 30	$\beta > o > \alpha$	2.4
17		"		1798, Feb. 18	68. 82	4.0
18	II.	η Geminorum	7	1795, Mar. 20	$\mu - \eta - \xi''$	3.3
19		"		" "	$\mu - \epsilon - \eta$	3.6
20		"		1795, Nov. 7	27; 13, 7 — 81	3.2
21		"		1796, Feb. 1	13; 7	3.3
22		"		" "	13. 7 —, 31	3.1
23		"		1796, Nov. 30	13 —, 123 Tauri, 7	3.2
24	II	ζ Geminorum	48	1795, Nov. 7	55. 77, 34 — 48	4.1
25	III.	δ Cephei	27	1796, Nov. 5	32, 27	3.8
26	IV.	β Lyræ	10	1782, May 12		—
27		"		1795, May 5		—
28		"		1795, Sept. 15	14 —, 10	3.8
29	IV.	R Lyræ	13	1796, Aug. 28	12. 13	4.5
30		"		" "	21. 20, 13	4.7
31	IV.	ρ Persei	25	1795, Aug. 21	23, 25 — 41, 46	3.4
32		"		1796, Sept. 7	45 — — 39 — 25	3.6
33	IV.	λ Tauri	35	1796, Jan. 1	1 — 2, 35. 38	4.0
34		"		1796, Nov. 30	123 —, 35	3.6
35	V.	X Sagittarii	3	1795, Sept. 15	2, 3	—
36	VI.	δ Libræ		1795, May 11	37, 31. 35, 44. 10. 43. 45. 6	5.2
37		"		1795, May 18	37, 31, 19	5.4
38		"		" "	7, 19	5.6
39		"		1797, May 22	19. 18	5.9

The following remarks appear in the original record: —

No. 16. "*o Ceti* is less than β and larger than α. See p. 26."

No. 22. "Seems to be larger than it has been."

No. 26. "To the n-eye γ much larger than β. (Mem. This star is changeable, and was then at its minimum.)"

No. 27. "*β Lyræ* much less than γ, $10^h\ 45^m$ com. time."

No. 28. "10 seems to be at its minimum."

The variations of several of these stars are irregular, or at least the law governing them is not yet known. It is evident that these obser-

vations will hereafter form a most valuable test of the correctness of any assumed law, since in many cases they precede by more than half a century any other observations of these stars of the same degree of precision. This is shown in Table II., which gives in successive columns the name of the star, the number of observations contained in Table I., the year in which the variability was discovered, and the number of years by which this followed the observations of Herschel. A dash is inserted when the discovery preceded the observations. In these cases the observations of Herschel have less value, since we have contemporaneous or antecedent observations which serve to determine the nature of the changes. The last three columns give the period in days, and the magnitude at maximum and minimum, according to the catalogue of Professor Schönfeld.*

TABLE II.

Name of Star.	No. Obs.	Discov.	Years.	Period.	Variation. Schönfeld.	
η Aquilæ	4	1784	—	7.2	3.5	4.7
g Herculis	8	1857	62	irr.	5	6.2
u Herculis	5	1869	74	irr.	4.6	5.4
o Ceti	3	1596	—	331.3	1.7–5.0	8–9
η Geminorum	9	1865	70	229.1	3.2	3.7–4.2
ζ Geminorum	1	1844	49	10.2	3.7	4.5
δ Cephei	1	1784	—	5.3	3.7	4.9
β Lyræ	3	1784	—	12.8	3.4	4.5
R Lyræ	2	1856	60	46.0	4.3	4.6
ρ Persei	3	1854	59	irr.	3.4	4.2
λ Tauri	3	1848	52	3.9	3.4	4.2
X Sagittarii	1	1866	71	7.0	4	6
δ Libræ	5	1859	64	2.3	4.9	6.1

The individual stars will now be considered in turn, so far as the material exists for a more complete reduction than is given in Table I.

η Aquilæ. — The variation in light of this star has been discussed by Argelander.† A light curve is given by which the brightness at any time may be expressed in terms of an arbitrary scale of grades. This scale is defined by expressing in grades the light of the comparison stars used in determining the changes in brightness of the variable. Points have been constructed for each of these stars, with grades as abscissas, and the photometric magnitudes as ordinates. A straight

* Zweiter Catalog von veränderlichen Sternen. Mannheim, 1875.

† Astron. Nach., xix. 399.

line drawn nearly through these points serves to convert the scale of grades into photometric magnitudes, or into actual light ratios. From this we may conclude that the maximum and minimum light of the variable expressed in photometric magnitudes is 3.7 and 4.5. The range is accordingly less than that ordinarily given, but this is partly due to the difference in the scales. The observations of Herschel expressed in grades equal 9.6, 6.1, 8.5, and 1.0. The first three of these correspond to periods of $1^d\ 19^h$, $1^d\ 4^h$, and $1^d\ 14^h$, after a minimum, if the light of the star was increasing, or to $3^d\ 4^h$, $5^d\ 6^h$, and $3^d\ 14^h$, if the light was diminishing. The other observation, No. 4, which was made six days later, on July 25, serves to decide between these two hypotheses. The light was then sensibly that of a minimum at 1.2 grades. As the period of the star is about $7^d\ 4^h$, a minimum about a day preceding the observations 1, 2, and 3 would also be indicated by observation 4. This would agree with the hypothesis that the light was increasing on July 19, but would controvert the view that it was diminishing.

The time of the observations is defined only by the limits of twilight, which in the latitude of Slough would extend to within about two hours of midnight in July. The star would culminate near midnight, and could be easily observed as long as darkness lasted. The times of observation may therefore be written, 1795, July $19^d\ 12^h \pm 2^h$, and 1795, July $25^d\ 12^h \pm 2^h$. The mean of the three results on July 19 would give an interval from the minimum of $1^d\ 12^h$, or would place the preceding minimum at 1795, July $18^d\ 0^h$, with an uncertainty of several hours, since a small error in the light corresponds to a large deviation in the time of minimum. The elements of Argelander indicate a minimum 1795, July $18^d\ 19^h\ 42^m$ Paris M. T.

g Herculis. — The variations in light of this star are irregular, or the law governing them has not yet been discovered. Should this law ever be determined, these observations may have great value, since they anticipate by sixty-two years the first observations of equal accuracy previously known. Probably the variations are so slow that the hour at which the observations were made will not be needed.

u Herculis. — The same remark applies to this star as to the preceding. The observations are accordant, and anticipate other similar observations by seventy-four years.

o Ceti. — Some other observations by Sir William Herschel are given by Argelander.* As this star had been observed for many years previously, these observations are not of especial importance.

* Bonn Beob., vii. 329.

η *Geminorum.* — These observations precede by seventy years those taken elsewhere. They will therefore have great value in determining the period when the nature of the variations is more accurately established. The small change in light, however, makes the result derived from any small number of observations somewhat doubtful.

ζ *Geminorum.* — A comparison of the results obtained by Argelander * with the photometric measures gives the variation in light of this star from 3.6 to 4.2. It was therefore apparently observed by Herschel near its minimum. The light curve indicates that the observation preceded or followed a minimum by about nineteen hours, but the change in brightness during this time is much less than the uncertainty of the observation. The ephemeris of Schönfeld indicates a minimum for Ep. —2434 at 1795, Nov. 8^d $5^h.6$, which does not differ from the time of observation by as much as the uncertainty of the comparison.

δ *Cephei.* — According to the curve of Argelander,† this star has the magnitudes of 3.5 and 4.3 at maximum and minimum. The observation of Herschel would correspond to 8.0 grades. This indicates a minimum preceding it by 1^d 2^h or 2^d 20^h, according as the light was increasing or decreasing. The star is above the horizon nearly all night, hence the time of observation is fixed only by the limits of twilight. We may therefore call the time of observation, 1796, Nov. 5^d $12^h \pm 6^h$. The elements of Schönfield give for Ep. —2987 a minimum at 1796, Nov. 4^d 7^h 24^m, which agrees as well as could be desired with the observation. As in the case of η *Aquilæ*, the observations of contemporaneous observers fix the period of this star so accurately that a correction based upon a small number of observations does not seem justifiable.

β *Lyræ.* — The variations of this star have been so thoroughly determined by other observers that these observations cannot add much to our knowledge of the subject. Only one observation, No. 28 of Table I., is sufficiently precise to be of value, and the interval here employed —, is too large to be estimated with accuracy. This observation has therefore not been reduced.

R *Lyræ.* — The variations of this star are so small that it is doubtful if the observations of Herschel can be utilized.

ρ *Persei.* — The same remark may be applied to this star as to *g Herculis.*

λ *Tauri.* — This star belongs to the Algol class. The maximum

* Bonn Beob., vii. 389.

† Astron. Nach., xix. 395.

brightness as given in the photometric catalogue is 3.6. The agreement of the observation No. 34 is probably accidental, since the large interval —, cannot be estimated with accuracy. As far as it goes, however, it indicates that the star was at its full brightness. The other observation, No. 33, indicates a diminution of light, or that the star was near a minimum. The law of variation of light is not known, but probably the change in magnitude amounts to about 0.8. The star retains its full brightness except for about two hours before and after each minimum. We may accordingly assume that a minimum preceded or followed the observation No. 33 by about one hour. On this day the sun set at about $3^h\ 47^m$ and λ *Tauri* set at $16^h\ 10^m$. Allowing for twilight, we may accordingly assume 1796, Jan. $1^d\ 10^h \pm 5^h$ for the time of observation. For the other date we obtain, in like manner, 1796, Nov. $30^d\ 11^h \pm 6^h$. The ephemeris of Schönfeld, applying the equation of light, gives 1795, Dec. $31^d\ 22^h.6$, for Ep. —6500. A correction to the ephemeris of $-11^h.5$ is thus indicated. This exceeds the possible error in the time, added to the probable error in magnitude. In other words, if the star was really below its full brightness, the minimum must have occurred several hours after the computed time. In like manner, we obtain 1796, Dec. $1^d\ 22^h.3$ for Ep. —6485, or the nearest minimum does not occur until 35 hours after the observation No. 34. Accordingly, as the observation indicated, the star should have had its full brightness. The first minimum previously known of this star occurred on Dec. 6, 1848. If it were possible to determine the hour of Herschel's observation, the mean period of this star would be determined with great precision. An uncertainty of one hour would correspond to about half a second in a single period.

X *Sagittarii.* — This star varies in light from about 4.5 to 5.3 in a period of a little over seven days. The only comparison made by Herschel places this star a little fainter than 2 *Sagittarii.* The latter star is commonly placed in Ophiuchus, — in fact, it is nearly in line with 52 and 58 *Ophiuchi*, and between them. Its magnitude, according to the Uranometria Argentina is 6.8, which corresponds to 6.6 on the photometric scale. This would make the variable much too faint, even if at its minimum. It is also strange that Herschel should have employed a star at so great a distance (about 8°), when he might have taken others about equally faint and nearer. The hypothesis that 2 *Sagittarii* was much brighter then than now, is negatived by the fact that Herschel compared it with 52 *Ophiuchi*, and found it only slightly brighter. The magnitude of this star in the Uranometria

Argentina is 6.5, corresponding to a photometric magnitude of 6.3. This value, although reducing the discrepancy, would still make the variable 6.5, which is 0.7 fainter than its light at minimum. This comparison is not given in the catalogue of Herschel, and accordingly is not checked by appearing under both 2 and 3 *Sagittarii*.

The southern declination of the star restricts the time of observation within narrow limits. The star sets at 9^h, and twilight would not be over until about 7^h. Accordingly, the time of observation would be 1795, Sept. $15^d\ 8^h \pm 1^h$. The elements of Schönfeld * give a minimum at 1795, Sept. $15^d\ 16^h$, for Ep. —3902. The period of Schmidt,† on the other hand, gives 1795, Sept. $13^d\ 22^h$. If, then, the star was really at its minimum when observed by Herschel, this observation determines a correction to the period with great certainty.

δ *Libræ*. — The observations of this star are so important that they require a more detailed discussion. The law of variation of the light has been determined by Professor Schönfeld.‡ Table III. gives the names of the various comparison stars, the letters by which they are designated by Professor Schönfeld, and the brightness in grades that he assigned to them. The next columns give the light according to the photometric measures made at this Observatory, and according to the Uranometria Argentina. The grades are reduced to magnitudes, as described on page 271, according to each of the scales. The results found by subtracting the magnitudes derived from the grades from that given in the two catalogues are given in the last two columns.

TABLE III.

Names.	Desig.	Grades.	H. P.	U. A.	H. P.	U. A.
37 Libræ	*f*	15.2	4.9	5.5	—2	—2
ϵ Libræ	*e*	12.5	5.2	5.5	—2	0
15 Hev.	*d*	8.5	5.0	5.6	+8	+3
178 Bode	*c*	3.5	5.6	6.2	0	+1
B. A. C. 4945	*b*	0.0	—	6.6	—	0

The light of the variable at maximum and minimum equals 13.0 and 2.0 grades respectively. This corresponds to a variation from 4.9 to 5.8 on the photometric scale, and from 5.5 to 6.4 on the scale of the Uranometria Argentina.

The observations of Sir William Herschel are compared in Table IV.

* Zweiter Catalog.

† Astron. Nach., lxxxvii. 109.

‡ Astron. Nach., lxxiv. 342.

The successive columns give the Flamsteed numbers, the photometric magnitudes, and the magnitudes according to Sir William Herschel. These have been derived from a discussion of all six catalogues of Herschel. They have a special value for the present purpose, since they indicate the brightness of these stars at the time the observations now under consideration were made. The fourth column gives the magnitude according to the Uranometria Argentina. The magnitude of 19, δ *Libræ*, according to each of these scales is inserted in brackets.

TABLE IV.

Fl.	H. P.	W. H.	U. A.	Fl.	H. P.	W. H.	U. A.
37	4.9	5.1	5.5	37	4.9	5.1	5.5
31	5.2	5.0	5.5	31	5.2	5.0	5.5
35	5.4	5.9	5.8	19	[5.4]	[5.2]	[5.7]
44	5.5	5.4	5.9	7	5.4	5.0	5.7
19	[5.2]	[5.1]	[5.7]	19	[5.6]	[5.2]	[5.9]
43	5.0	4.8	5.5	19	—	[5.9]	[6.2]
45	5.0	5.2	5.5	18	—	6.0	6.8

The scale of the Uranometria Argentina differs two or three tenths of a magnitude from that of the photometer for stars of the fifth and sixth magnitude. For the stars used in this comparison the difference amounts to four tenths of a magnitude. Applying this correction, the results become more readily comparable without affecting the conclusions derived from them. In Table V. the brightness of δ at the time of the various observations, and at maximum and minimum, as stated in the first column, is compared. The following columns give the magnitude derived in Table IV. from the three authorities, after applying a correction of four tenths of a magnitude to the Uranometria Argentina. The last column gives a mean value which may be employed, since all the observations have been reduced to the same scale.

TABLE V.

	H. P.	W. H.	U. A.	Mean.
1795, May 11	5.2	5.1	5.3	5.2
1795, May 18	5.4	5.2	5.3	5.3
" "	5.6	5.2	5.5	5.4
1797, May 22	—	5.9	5.8	5.8
Maximum	4.9	—	5.1	5.0
Minimum	5.8	—	6.0	5.9

A minimum is clearly indicated in 1797, May 22, and the star seems to have been below its maximum brightness on the other nights also. Converting the magnitudes into grades, and comparing with the light curve of Professor Schönfeld, we may infer that on 1795, May 11, the observation was made $4^h.7$ before, or $5^h.2$ after, a minimum. The observations of 1795, May 18, in like manner, indicate that a minimum would follow in $3^h.4$, or had passed $3^h.8$. The observation of 1797, May 22, indicates a brightness that does not differ sensibly from that at minimum. The star changes in brightness by about a tenth of a magnitude within an hour of minimum.

The hours within which the observations must have been made are limited by the twilight, which would, for observations of such faint stars, be appreciable within two hours of midnight. The comparison stars 43 and 44 *Libræ* would be above the horizon from $8^h.0$ to $16^h.4$. Their altitudes would exceed 10° from $9^h.2$ to $15^h.2$.

The period of δ *Libræ* is $2^d\ 7^h\ 51^m\ 20^s = 2^d.3273148$. Accordingly, if a minimum occurred near midnight on 1797, May 22, others would have occurred on the afternoons of 1795, May 11, and 1795, May 18. We may therefore assume that the observations of 1795 were made after, and not before minima. Subtracting from the time 1795, May $11^d\ 12^h$, the interval $5^h.2$, adding $0^h.2$ for the difference in longitude of Slough and Paris, and adding $0^h.1$ for the equation of light, we obtain 1795, May $11^d\ 7^h.1$, for the Paris heliocentric time of minimum. The times of minima indicated for the other observations are given with this in the first column of Table VI. The second column gives the ephemeris time for the epoch given in the third column. The last column gives the observed minus the computed times of minima.

TABLE VI.

Observed.	Computed.	Epoch.	O. — C.
d. h.	d. h.		h.
1795, May 11 7.1	May 11 11.9	—10980	—4.8
1795, May 18 8.5	May 18 11.5	—10977	—3.0
1797, May 22 12.3	May 22 21.8	—10661	—9.5

The mean of these results indicates a correction of six hours to the ephemeris, or of seven hours, if we assign somewhat greater weight to the last observation. A most fortunate coincidence brought all the observations so near minimum that the star had in each case less than

its normal light. Observations of the maximum light would have had comparatively little value.

The times of minima on May 11 and May 18, 1795, are somewhat uncertain, since the assumed light of the comparison stars may be in error. All the stars with which δ *Libræ* was compared have accordingly been arranged in sequences by Mr. Chandler, since the above reduction has been made. The details of these observations will be published elsewhere, but they give a correction for the minima of May 11 and May 18 of —2.3 and —2.5, instead of —4.8 and —3.0 hours.

XII.

RECENT OBSERVATIONS OF VARIABLE STARS.

BY EDWARD C. PICKERING.

Presented March 12, 1884.

THE work of observing variable stars is a branch of astronomical research which can be successfully prosecuted at observatories not provided with the means for undertaking large pieces of routine work. Hence, where these means exist, the observation of variable stars is usually neglected, not from any doubt of its interest or importance, but because it is assumed that attention will be paid to it at institutions less fully equipped, and especially by the numerous amateur observers to whose resources it appears so well adapted.

But in order to obtain the best results in this line of research, some systematic division of the labor has become important. At present, for want of system, some variable stars are observed with unnecessary frequency, while others of no less interest are completely neglected. A bibliography of the variable stars, which is in course of preparation by Mr. S. C. Chandler, Jr., will exhibit large gaps in the observations of many important objects in the list. In such cases, the value of the earlier observations is often impaired by the difficulty of connecting them with those recently made.

In the hope of promoting a more systematic observation of the variable stars, a pamphlet upon the subject, and a subsequent circular, have been issued during the past year by the Harvard College Observatory. In response to the recommendations of the pamphlet, a number of observers signified their inclination to undertake the proposed work, some of whom have already reported many valuable observations. It is to be anticipated that their example will be followed by others, so that the frequent renewal of the special lists of stars required by each participant in the work will become inconvenient. Under these circumstances it seems desirable to make a published statement of the present condition of this branch of scientific inquiry, so that each observer may judge for himself what part of the work can be most profitably undertaken with the means at his disposal. This advantage would

obviously be lost by waiting for the reduction and publication of the observations. It is intended to publish another circular early in 1885, giving, so far as practicable, the results obtained during 1884 by all observers of variable stars. The value of this circular will depend upon the amount of assistance which the various astronomers interested in the subject may be inclined to afford. Those who have already undertaken to communicate their observations to this Observatory will, no doubt, continue their co-operation. If the greater part of the results obtained by independent observation elsewhere are also communicated in a form so far condensed that they can be furnished with little trouble to the observers themselves, the proposed circular will exhibit a statement of the course of observation during the year sufficiently complete to form a highly useful guide for subsequent work. It is therefore hoped that observers of variable stars, whether professional astronomers or amateurs, will be generally disposed to furnish the information necessary to the completeness of the circular. This information relates to the following subjects: —

1. Method of observation. If photometric, some account of the instrument and the manner of using it. If not photometric, whether the observations are made by Argelander's method, by the division into tenths of the interval in brightness between two comparison stars, one slightly brighter and the other slightly fainter than the star observed, or by direct estimation of magnitude.

2. Stars observed during 1884, and the number of nights on which each was observed. In naming the stars, it may be convenient to use the numbers given in the first column of Table I. below.

3. The time and form of publication of the observations now contemplated by the observer.

4. Plans for 1885, with regard to the stars selected for observation, and the number of nights on which it is proposed that each shall be observed.

Further information, although not directly required for the purpose of the circular, will be gratefully received. If the observations are not to be published by the observer, a copy of them would be most acceptable. If they are, any results already reached, as, for example, the times and magnitudes of the maxima and minima of the stars, the dates of the separate observations, or the number of nights in each month of the year upon which a given star was observed, would be of much service.

Table I. exhibits the results of observation of variable stars for 1883, so far as they are at hand, in order to show the nature of the

TABLE I. — VARIABLE STARS.

No.	H.P.	Name.	R. A. 1875.			Dec. 1875.		Max.	Min.	Per.
			h	m	s.	°	′	m.	m.	d.
0a	—	Ceti	0	15	26	-20	45.1	5.2	7.0	—
1	51	T Cassiopeiæ		16	29	+55	5.9	6.5 — 7.0	11 — 11.2	436
2	54	R Andromedæ		17	28	+37	53.0	5.6 — 8.6	<12.8	404.7
3	—	S Ceti		17	42	—10	14.5	7.0 — 8.0	<10.7	323.6
4	—	B Cassiopeiæ		17	52	+63	27.2	>1	?	—
5	—	T Piscium		25	31	+13	54.6	9.5 — 10.2	10.5 — 11.0	Irr.
6	94	α Cassiopeiæ		33	25	+55	51.1	2.2	2.8	Irr.
6a	—	U Cephei		51	18	+81	12.1	7.0	9.5	2.5
7	—	S Cassiopeiæ	1	10	30	+71	57.2	6.7 — 8.5	<13	615
8	—	S Piscium		11	2	+ 8	16.3	8.8 — 9.3	<13	406.6
8a	—	Piscium		16	22	+14*	12.7	10	14	—
8b	—	Ceti		19	31	— 4	36.6	6.5	7.8	—
8c	—	R Sculptoris		21	13	—33	11.5	6¼	7¾	207
9	—	R Piscium		24	12	+ 2	14.1	7.4 — 8.3	<12.5	345
10	—	S Arietis		57	55	+11	53.5	9.1 — 9.8	<13	288.8
11	—	R Arietis	2	9	1	+24	28.4	7.6 — 8.5	11.9 — 12.7	186.2
12	370	o Ceti		13	1	— 3	43.9	1.7 — 5.0	8 — 9	331.8
13	—	S Persei		13	54	+58	0.8	8.5 ?	<9.7	—
14	—	R Ceti		19	39	— 0	55.7	7.0 — 8.7	<12.8	167.1
15	—	T Arietis		41	22	+16	59.3	7.9 — 8.2	9.4 — 9.7	324
16	480	ρ Persei		57	10	+38	21.3	3.4	4.2	Irr.
17	496	β Persei	3	0	2	+40	28.4	2.2	3.7	2.9
18	—	R Persei		22	6	+35	14.3	8.1 — 9.2	12.5	206.6
19	657	λ Tauri		53	45	+12	8.2	3.4	4.2	4.0
20	—	T Tauri	4	14	43	+19	14.8	9.2 — 11.5	12.8 — <	Irr
21	—	R Tauri		21	27	+ 9	52.9	7.4 — 9.0	<13	325.6
22	—	S Tauri		22	22	+ 9	40.1	9.9	<13	378
22a	—	Doradus		35	19	—62	19.4	5½	6¾	—
23	—	V Tauri		44	48	+17	19.6	8.3 — 9.0	<12.8	168.6
24	—	R Orionis		52	13	+ 7	56.8	8.7 — 8.9	<13	378.8
25	877	ε Aurigæ		53	0	+43	38.2	3.0	4.5	Irr.
26	880	R Leporis		53	55	—15	3.7	6 — 7	8.5 ?	437.8
27	—	R Aurigæ	5	7	12	+53	26.6	6.5 — 7.4	12.5 — 12.7	465
27a	—	S Aurigæ		18	52	+34	2.8	9.4	<13	—
28	—	S Orionis		22	50	— 4	49.9	8.3 ?	<12.3	—
29	1005	δ Orionis		25	37	— 0	25.6	2.2 ?	2.7	Irr.
29a	—	Orionis		29	42	— 5	33.5	10	13	—
30	1091	α Orionis		48	24	+ 7	23.3	1	1.4	Irr.
31	1160	η Geminorum	6	7	20	+22	22.4	3.2	3.7 — 4.2	229.1
31a	—	Monocerotis		16	26	— 2	8.1	7	<10	—
32	1205	T Monocerotis		18	29	+ 7	9.1	6.2	7.6	26.8
33	—	R Monocerotis		32	21	+ 8	50.7	9.5	11.5	Irr.
34	1256	S Monocerotis		34	6	+10	0.5	4.9	5.4	3.4
35	—	R Lyncis		50	59	+55	30.2	9 ?	<12.8	—
36	1334	ζ Geminorum		56	41	+20	45.1	3.7	4.5	10.2
37	—	R Geminorum		59	49	+22	53.8	6.6 — 7.8	<12.3	371.0
38	—	R Canis min.	7	1	50	+10	13.1	7.2 — 7.9	9.5 — 10.0	335.0
38a	—	Puppis		9	43	—44	26.2	3½	<6	136
38b	—	V Geminorum		16	10	+13	21.8	8.5	13 — 13½	276
38c	1417	U Monocerotis		24	50	9	31.0	6.0	7.2	46.0
39	—	S Canis min.		25	56	+ 8	35.0	7.2 — 8.0	<11	332.2
40	—	T Canis min.		27	3	+12	0.6	9.1 — 9.7	<13	335.2
40a	—	Canis min.		34	34	+ 8	40.2	8½	13.5	406

* Declination +12° 12′.7, according to Mr. Parkhurst.

TABLE I. — VARIABLE STARS.

No.	Class.	Discoverer.	Date.	Obs. 1882.	Obs. 1880–82
0a	—	Chandler	1881	2 C. 36 S.	—
1	II.	Kruger	1870	14 C.	W.
2	II	Argelander	1858	12 C.	Sm. W.
3	II.	Borelly	1872	6 C.	Sf. W.
4	I.	Tycho Brahe	1572	—	—
5	II.	Luther	1855	6 C.	—
6	III.	Birt	1831	—	Sm. W.
6a	V.	Ceraski	1880	—	Sm. W.
7	II.	Argelander	1861	15 C. 26 P. 8 S.	Sf. Sm. W.
8	II.	Hind	1851	7 C.	Sm.
8a	—	Peters	1880	—	—
8b	—	Gould	1874?	—	—
8c	II.	Gould	1872?	—	—
9	II.	Hind	1850	9 C.	Sm. W.
10	II.	Peters	1865	6 C.	—
11	II.	Argelander	1857	7 C. 19 H. 1 Z.	Sm. W.
12	II.	Fabricius	1596	5 C.	Sm.
13	II.	Kruger	1873	13 C. 17 H.	Sf.
14	II.	Argelander	1866	4 C.	—
15	II.	Auwers	1870	10 C. 24 H. 7 Z.	—
16	II.?	Schmidt	1864	35 S.	Sm.
17	V.	Montanari	1669	—	Müller, Sm. W
18	II.	Schonfeld	1861	7 C. 6 H.	H. Sm. W.
19	V.	Baxendell	1848	—	—
20	—	Hind	1861	7 C.	—
21	II.	Hind	1840	10 C.	Sf. Sm. W.
22	II.	Oudemans	1855	10 C.	Sf. W.
22a	—	Gould	1874?	7 L. 9 U.	—
23	II.	Auwers	1871	13 C. 6 P.	H. Sf. W.
24	II.	Hind	1848	11 C.	H. Sf. W.
25	III.	Fritsch	1821	10 S.	Sm.
26	II.	Schmidt	1855	7 C. 22 S.	Sm.
27	II.	At Bonn	1862	9 C. 26 H.	Sf. Sm.
27a	II.	Dunér	1881	10 C. 8 P.	D. Sm.
28	II.	Webb	1870	10 C	Sf.
29	III.	J. Herschel	1834	8 S.	—
29a	—	Bond	1863	6 C.	Müller
30	III.	J. Herschel	1836	—	Sm.
31	II.?	Schmidt	1866	—	Sm.
31a	—	Schönfeld	1883	2 C.	—
32	IV.	Gould	1871	61 S.	W.
33	II.	Schmidt	1861	11 C.	Sm.
34	IV.	Winnecke	1867	8 S.	—
35	II.	Kruger	1874	12 C. 14 P.	H.
36	IV.	Schmidt	1844	—	Sm.
37	II.	Hind	1848	14 C. 19 P 26 S.	W.
38	II	At Bonn	1854	16 C 3 H. 3 Z.	Sf. W.
38a	II.	Gould	1872	10 L. 12 U.	—
38b	II.	Baxendell	1880	17 C.	Baxendell
38c	II.?	Gould	1873	36 S.	—
39	II.	Hind	1856	14 C.	Sf. W.
40	II.	Schönfeld	1865	9 C.	—
40a	II.	Baxendell	1879	9 C.	Sm.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h m s	° ′	m.	m.	d.
41	—	S Geminorum	7 35 32	+28 44.6	8.2 — 8.7	<13	294.2
42	—	T Geminorum	41 48	+24 2.7	8.1 — 8.7	<13	288.1
42a	—	S Puppis	43 6	—47 8.8	7½	9	—
43	—	U Geminorum	47 41	+22 19.7	8.9 — 9.7	13.1	Irr.
43a	—	Puppis	55 0	—12 32	8½	<14	310
44	—	R Cancri	8 9 40	+12 6.5	6.2 — 8.3	<11.7	354.4
45	—	V Cancri	14 36	+17 40.9	6.8 — 7.2	<12	272
46	—	U Cancri	28 37	+19 19.5	8.2 — 10.4	<13	305.7
47	—	S Cancri	36 48	+19 29.0	8.2	9.8	9.5
48	—	S Hydræ	47 3	+ 3 32.4	7.5 — 8.5	<12.2	256.4
49	—	T Cancri	49 32	+20 19.7	8.2 — 8.5	9.8 — 10.5	484.2
50	—	T Hydræ	49 35	— 8 31.0	7.0 — 8.1	<12.5	289.4
50a	—	R Carinæ	9 29 6	—62 14.2	4.4	9.8	313
51	—	R Leonis min.	38 4	+35 5.2	6.1 — 7.5	<11.0	374.7
52	1752	R Leonis	40 50	+12 0.5	5.2 — 6.4	9.4 — 10.0	312.6
52a	—	l Carinæ	41 40	—61 55.9	3.7	5.2	31.2
52b	—	Leonis	53 3	+21 51.6	8½	8.6<13	280!
52c	—	Antliæ	10 4 22	—37 7.1	6¼	<8	—
52d	—	Carinæ	6 23	—60 50.3	6¼	9	—
52e	—	U Leonis	17 21	+14 38.1	9⅓	Inv.	—
52f	1869	Hydræ	31 22	—12 44.1	4½	6	—
53	1880	R Ursæ maj.	35 47	+69 25.9	6.0 — 8.1	13	303.4
54	—	η Argus	40 13	—58 40.2	>1	6.3	Irr.
54a	—	T Carinæ	50 18	—59 51.2	6.2	6.9	—
55	—	R Crateris	54 25	—17 20.4	>8	<9	—
56	—	S Leonis	11 4 23	+ 6 8.5	9.0 — 9.7	<13	187.6
57	—	T Leonis	32 2	+ 4 3.9	10?	<13	—
58	—	X Virginis	55 27	+ 9 46.1	7.8?	<10	—
59	—	R Comæ	57 51	+19 28.8	7.4 — 8.0	<13	363
60	—	T Virginis	12 8 12	— 5 7.2	8.0 — 8.8	<13	337
61	—	R Corvi	13 10	—18 20.3	6.8 — 7.3	<11.5	318.6
61a	—	— Virginis	27 26	— 3 43.8	8	14	210±
62	—	T Ursæ maj.	30 42	+60 10.6	7.0 — 8.3	12.2	255.6
63	2147	R Virginis	32 10	+ 7 40.6	6.5 — 7.5	10.0 — 10.9	145.7
63a	—	R Muscæ	34 28	—68 43.3	6.6	7.3	0.9
64	—	S Ursæ maj	38 28	+61 46.7	7.7 — 8.2	10.2 — 11.1	224.8
65	—	U Virginis	44 46	+ 6 14.0	7.7 — 8.1	12.2 — 12.8	207.4
66	—	W Virginis	13 19 35	— 2 31.2	8.7 — 9.2	9.8 — 10.4	17.3
67	—	V Virginis	21 21	— 2 19.0	8.0 — 9.0	<13	251
68	2275	R Hydræ	22 53	—22 25.6	4.0 — 5.5	10?	469.3
69	2289	S Virginis	26 29	— 6 20.6	5.7 — 7.6	12.5	374.0
69a	—	Virginis	14 3 37	—12 42.7	9	14	—
69b	—	R Centauri	7 35	—59 19.8	6	10	—
70	—	T Bootis	8 14	+19 39.1	9.7?	<13	—
71	—	S Bootis	18 41	+54 22.7	8.1 — 8.5	13.2	272.4
72	—	R Camelopardi	27 8	+84 23.8	7.9 — 8.6	12?	266.2
73	2445	R Bootis	31 41	+27 16.9	5.9 — 7.5	11.3 — 12.2	223.0
73a	2459	Bootis	37 50	+27 3.6	5.2	6.1	370?
73b	—	Bootis	48 33	+18 12.1	9.1	12.0 — 13.6	173.8
74	2506	δ Libræ	54 18	— 7 51.6	4.9	6.1	2.3
74a	—	Libræ	15 3 37	—19 33.9	10	<13.5	700±
74b	—	R Triang. Austr.	8 37	—66 2.1	6.6	8.0	3.4
75	—	U Coronæ	13 6	+32 6.4	7.6	8.8	3.5
76	—	S Libræ	14 13	—19 47.3	8.0	12.5?	—
77	—	S Serpentis	15 48	+14 45.9	7.6 — 8.6	12.5?	361.0

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Obs. 1883.	Obs. 1880–82.
41	II.	Hind	1848	6 C.	Sf. W.
42	II.	Hind	1848	7 C.	Sf. W.
42*a*	—	Gould	1874?	—	—
43	II.?	Hind	1855	24 C.	H. Sf.
43*a*	II.	Pickering	1881	12 C.	—
44	II.	Schmidt	1829	13 C.	Sm. W.
45	II.	Auwers	1870	15 C. 13 P. 21 S.	Sm. W.
46	II.	Chacornac	1853	11 C.	Sm.
47	V.	Hind	1848	—	Sm.
48	II.	Hind	1848	12 C.	D. Sf.
49	II.	Hind	1850	14 C. 14 P.	—
50	II.	Hind	1851	11 C.	Sf. W.
50*a*	II.	Gould	1871	—	—
51	II.	Schönfeld	1863	10 C. 22 S.	Sm.
52	II.	Koch	1782	12 C. 14 S.	D. Sf. Sm.
52*a*	—	Gould	1871	16 L. 17 U.	—
52*b*	II.	Becker	1882	8 C.	Becker
52*c*	—	Gould	1872	—	—
52*d*	—	Gould	1871	—	—
52*e*	—	Peters	1876	7 C.	—
52*f*	—	Gould	1871	—	—
53	II.	Pogson	1853	14 C. 31 H. 23 S. 7 Z.	Sm. W.
54	II.?	Burchell	1827	—	—
54*a*	—	Thome	1872	10 L. 11 U.	—
55	II.	Winnecke	1861	10 C.	Sf.
56	II.	Chacornac	1856	13 C.	—
57	II.	Peters	1865	4 C.	—
58	II.	Peters	1871	9 C.	—
59	II.	Schönfeld	1856	8 C. 30 P.	—
60	II.	Boguslawski	1849	5 C.	—
61	II.	Karlinski	1867	9 C.	Sf.
61*a*	II.	Henry	—	11 C.	—
62	II.	Hencke	1856	17 C. 30 S.	H. Sf. Sm. W.
63	II.	Harding	1809	17 C. 26 S.	H. Sm. W.
63*a*	IV.	Gould	1871	10 L. 16 U.	—
64	II.	Pogson	1853	17 C. 35 S.	D. Sf. Sm. W.
65	II.	Harding	1831	14 C.	D. Sm. W.
66	II.?	Schönfeld	1866	12 C.	D.
67	II.	Goldschmidt	1857	10 C. 22 P.	W.
68	II.	Miraldi	1704	2 C. 41 S.	D. Sm. T.
69	II.	Hind	1852	12 C.	—
69*a*	II.	Palisa	1880	7 C.	—
69*b*	—	Gould	1871	9 L. 11 U.	—
70	I.?	Baxendell	1860	—	—
71	II.	At Bonn	1860	15 C.	D. Sm.
72	II.	Hencke	1858	7 C.	Sf. Sm.
73	II.	At Bonn	1858	15 C. 32 S.	Sm. W.
73*a*	—	Schmidt	1867	—	—
73*b*	II.	Baxendell	1880	11 C.	Baxendell
74	V.	Schmidt	1859	—	Sm.
74*a*	II.	Palisa	1878	5 C.	Palisa, Weis
74*b*	IV.?	Gould	1871	2 L. 6 U.	—
75	V.	Winnecke	1869	—	Sm.
76	II.	Borelly	1872	6 C.	Sf.
77	II.	Harding	1828	6 C.	D.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875. h.	m.	s.	Dec. 1875. °	′	Max. m.	Min. m.	Per. d.
78	2553	S Coronæ	15	16	18	+31	49.1	6.1 — 7.8	11.9 — 12.5	361.0
78a	—	Libræ		34	46	—20	46.5	9	<14	—
79	2639	R Coronæ		43	25	+28	32.5	5.8	13.0	Irr.
80	2647	R Serpentis		44	56	+15	30.8	5.6 — 7.6	<11	357.6
80a	—	V Coronæ		45	4	+39	57.0	7.7	12	360.0
81	—	R Libræ		46	32	—15	44.5	9.2 — 10.0	<13	723
82	2678	T Coronæ		54	16	+26	16.5	2.0	9.5	—
83	—	R Herculis	16	0	37	+18	42.5	8.0 — 9.0	<13	319.0
83a	—	W Scorpii		4	28	—19	48.6	10	<13	224.3
84	—	T Scorpii		9	36	—22	33.5	7	<10	—
85	—	R Scorpii		10	12	—22	31.8	9 ? — 10.5	<12.5	223
86	—	S Scorpii		10	13	—22	28.8	9.1 — 10.5	<12.5	176.9
86a	—	Ophiuchi		14	40	— 7	24.0	9.0	<13.5	326
87	—	U Scorpii		15	16	—17	29.3	9 ?	<12	—
87a	—	Ophiuchi		19	46	—12	8.5	7.5	10.5	365
88	—	U Herculis		20	16	+19	10.8	6.6 — 7.7	11.4 — 11.6	408.3
89	2772	g Herculis		24	32	+42	9.6	5	6.2	Irr.
90	—	T Ophiuchi		26	35	—15	46.6	10	<12.5	—
91	—	S Ophiuchi		27	4	—16	48.5	8.3 — 9.0	<12.6	233.8
91a	—	W Herculis		30	48	+37	35.6	8.0	<14.5	289
91b	—	Urs. Min.		31	40	+72	31.9	8.6	10.5	180 ?
91c	—	R Draconis		32	22	+67	0.7	7.2	13<	245.9
92	2828	S Herculis		40	13	+15	9.2	5.9 — 6.8	11.5 — 12.2	303
93	2830	Ophiuchi		52	30	—12	38.0	5.5	12.5	—
93a	—	V Herculis		53	41	+35	15.5	9.0	11.7	—
94	—	R Ophiuchi	17	0	36	—15	51.9	7.0 — 8.1	<12	302.4
95	2879	α Herculis		8	57	+14	32.1	3.1	3.9	Irr.
95a	2883	U Ophiuchi		10	12	+ 1	21.0	6.1	6.8	0.9
96	2890	u Herculis		12	42	+33	14.1	4.6	5.4	38.5
97	—	Serpentarii		23	9	—21	20.0	>1	?	—
[illegible]	2972	X Sagittarii		39	41	—27	45.6	4	6	7.0
99	3035	W Sagittarii		57	2	—29	34.7	5	6.5	7.6
100	—	T Herculis	18	4	22	+31	0.1	7.2 — 8.3	11.4 — 12.1	165.1
101	—	T Serpentis		22	43	+ 6	13.1	9.1 — 10.0	<12.8	342.8
102	—	V Sagittarii		24	4	—18	22.1	7.5 ?	9.5 ?	—
103	—	U Sagittarii		24	32	—19	13.9	7.0	8.3	6.7
104	—	T Aquilæ		39	45	+ 8	36.9	8.8	9.5	Irr.
105	3176	R Scuti		40	40	— 5	52.6	4.7 — 5.7	6.0 — 8.5	71.1
105a	—	κ Pavonis		44	3	—67	23.2	4.0	5.5	9.1
106	3193	β Lyræ		45	28	+33	13.0	3.4	4.5	12.9
107	3224	R Lyræ		51	32	+43	47.1	4.3	4.6	[illegible]
108	—	S Coron. Austr.		52	43	—37	10.0	9.8	11.5 ?	6.1
109	—	R Coron. Austr.		53	29	—37	10.4	10.5 — 11.5	<12.5	31
110	—	R Aquilæ	19	0	21	+ 8	2.6	6.4 — 7.4	10.9 — 11.2	345.1
111	—	T Sagittarii		9	1	—17	15.2	7.6 — 8.1	<11	381
112	—	R Sagittarii		9	21	—19	35.5	7.0 — 7.2	<12	270.0
113	—	S Sagittarii		12	7	—19	19.1	9.7 — 10.4	<12.7	230
114	3395	R Cygni		33	28	+49	55.1	5.9 — 8.0	13	425.3
115	—	11 Vulpeculæ		42	26	+27	0.5	3	?	—
116	—	S Vulpeculæ		43	16	+26	58.7	8.4 — 8.9	9.0 — 9.5	67.5
117	3434	χ Cygni		45	46	+32	36.0	4.0 — 6.0	12.8	406.5
118	3436	η Aquilæ		46	6	+ 0	41.2	3.5	4.7	7.2
119	—	S Cygni	20	2	53	+57	37.6	8.8 — 9.5	<13	322.8
120	—	R Capricorni		4	17	—14	45.0	8.8 — 9.7	<13	347
121	—	S Aquilæ		5	52	+15	14.9	8.9 — 9.9	10.7 — 11.6	147.2

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Obs. 1883.	Obs. 1880-82.
78	II.	Hencke	1860	13 C. 43 S.	H. Sm. W.
78a	—	Peters	1878	—	—
79	II.?	Pigott	1795	14 C. 56 S.	Sm. W.
80	II.	Harding	1826	10 C.	Sm.
80a	II.	Dunér	1878	13 C.	H. Sf. Sm. W.
81	II.	Pogson	1858	2 C.	—
82	I.	Birmingham	1866	—	Sm.
83	II.	At Bonn	1855	12 C. 4 H.	H. Sm.
83a	II.	J. Palisa	1877	3 C.	Sm.
84	I.	Auwers	1860	2 C.	—
85	II.	Chacornac	1853	7 C.	Sm.
86	II.	Chacornac	1854	6 C.	Sm.
86a	II.	Schönfeld	1881	8 C.	Sm.
87	I.?	Pogson	1863	—	—
87a	—	Dunér	1881	3 C.	Dreyer, D.
[redacted]	II.	Hencke	1860	14 C. 34 S.	Sm.
89	III.	Baxendell	1857	09 S.	Sm.
90	II	Pogson	1860	4 C.	—
91	II.	Pogson	1854	4 C.	—
91a	—	Dunér	1880	—	D. W.
91b	II.	Pickering	1881	20 C.	Pickering
91c	II.	Geelmuyden	1876	18 C. 21 S.	H. Sm. W.
92	II.	At Bonn	1856	10 C. 25 H.	H. Sm. W.
93	I	Hind	1848	—	—
93a	II	Baxendell	1880	11 C.	Baxendell
94	II	Pogson	1853	7 C.	D.
95	III.	W. Herschel	1795	—	Sm.
95a	V.	Sawyer	1881	12 S.	—
96	III	Schmidt	1866?	—	Sm.
97	I.	Fabricius	1604	—	—
98	IV	Schmidt	1866	16 S.	Sm.
99	IV.	Schmidt	1866	44 S	Sm.
100	II	At Bonn	1857	18 C. 13 H.	H. Sm. W.
101	II.	Baxendell	1860	10 C.	—
102	II.	Quirling	1865	6 C.	—
103	IV.	Schmidt	1866	5 C.	Sm.
104	II.	Winnecke	1860	9 C.	—
105	II.	Pigott	1795	8 C. 79 S.	Sm. W.
105a	IV.	Thome	1872	1 U.	—
106	IV.	Goodricke	1784	—	Müller, Sm. W.
107	II.?	Baxendell	1856	17 S.	—
108	IV.?	Schmidt	1866	1 C.	Sm.
109	II ?	Schmidt	1866	1 C.	Sm.
110	II	At Bonn	1856	1 C.	—
111	II	Pogson	1863	6 C.	Sf.
112	II	Pogson	1858	7 C.	D.
113	II	Pogson	1860	7 C.	—
114	II.	Pogson	1852	14 C. 22 P. 17 H. 6 Z.	Sm. W.
115	I	Anthelm	1670	—	—
116	II.	Hind	1861	12 C. 26 H. 4 Z.	W.
117	II.	Kirch	1686	11 C. 55 P. 26 H. 67 S. 4 Z.	Sm. W.
118	IV	Pigott	1784	16 S.	Sm. W.
119	II.	At Bonn	1860	9 C. 15 P.	Sf.
120	II.	Hind	1848	6 C.	D.
121	II.	Baxendell	1863	10 C.	—

TABLE I. — *Continued.*

No.	H P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
122	—	R Sagittæ	20 8 22	+16 21.0	8.5 — 8.7	9.8 — 10.4	70.4
123	—	R Delphini	8 53	+ 8 42.7	7.0 — 8.5	12.8	284.0
124	3547	P Cygni	13 11	+37 38.7	3 — 5	<6	—
125	—	U Cygni	15 44	+47 30.1	7.8 ?	9.8 ?	—
126	3557	R Cephei	34 20	+88 45.2	5 ?	10 ?	—
126a	—	— Cygni	87 17	+47 41.8	8	12	423.
127	—	S Delphini	37 19	+16 38.4	8.4 — 8.6	10.4 — 11.1	275.6
128	—	T Delphini	80 34	+15 56.7	8.2 — 8.9	<13	331.4
129	—	U Capricorni	41 11	—15 23.2	10.2 — 10.8	<13	203.5
130	3654	T Cygni	42 12	+33 55.0	5.5 ?	6 ?	—
131	—	T Aquarii	43 20	— 5 45.3	6.7 — 7.0	12.4 — 12.7	203.2
132	—	R Vulpeculæ	58 49	+23 19.5	7.5 — 8.5	12.5 — 13.0	137.5
132a	—	Capricorni	21 0 19	—24 25.5	9½	14	—
132b	—	T Cephei	7 52	+67 58.9	5.6	9.5	382
133	—	T Capricorni	15 6	—15 51.4	8.9 — 9.7	<13	269.4
134	—	S Cephei	86 45	+78 8.6	7.4 — 8.5	11.5	485
134a	—	Nova Cygni	87 2	+42 18.2			—
135	3845	μ Cephei	3[illegible] 41	+58 12.4	4 ?	5 ?	Irr.
136	—	T Pegasi	22 2 48	+11 55.7	8.8 — 9.3	<12.5	367.5
137	3981	δ Cephei	24 32	+57 46.6	3.7	4.9	5.4
137a	—	Lacertæ	37 43	+41 43.0	8.6	<13.5	315.
138	—	S Aquarii	50 25	—21 13.4	7.7 — 9.1	<11.5	279.4
139	4078	β Pegasi	57 45	+27 24.2	2.2	2.7	Irr
140	—	R Pegasi	23 0 22	+ 9 52.1	6.9 — 7.7	12 ?	382.0
141	—	S Pegasi	14 14	+ 8 14.2	7.6	<12.2	—
142	4103	R Aquarii	37 21	—16 11.9	5.8 — 8.5	11 ?	388.0
143	4234	R Cassiopeiæ	52 4	+50 41.5	4.8 — 6.8	<12	425.9

information desired. It is thought best not to delay the publication of the present circular in order to obtain additional information, much of which must be procured from Europe; but astronomers will confer a favor upon this Observatory by sending material which may be used next year to make the table for 1883 more complete. It is highly desirable that this information, as well as that respecting the observations of 1884, should reach this Observatory as early as February 1, 1885, in order that the proposed circular may be issued as early in the year as possible. Information received later, however, may be made serviceable in any circular which may afterwards be prepared.

The bibliography undertaken by Mr. Chandler, as above mentioned, will eventually furnish the means of preparing a catalogue of all the stars now known to be variable. The list in Table I. is from his provisional catalogue of known variables, which consists of Schönfeld's Second Catalogue,* with forty-eight additional stars whose variability seems certain.

* Zweiter Catalog von veränderlichen Sternen, Mannheim, 1875.

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Obs. 1883.	Obs. 1880-82.
122	II.?	Baxendell	1859	10 C.	Sm.
123	II.	Hencke	1859	7 C. 24 H.	W.
124	I.	Janson	1600	—	—
125	II.	Knott	1871	8 C. 10 P.	Bm. Sf. W.
126	II.?	Pogson	1856	3 C.	Sf.
127	II.	Birmingham	1881	8 C.	Bm. K. Sf. Sm.
127*a*	II.	Baxendell	1860	8 C.	Sm.
128	II.	Baxendell	1863	7 C.	D. Sm.
129	II.	Pogson	1858	8 C.	—
130	—	Schmidt	1864	—	—
131	II.	Goldschmidt	1861	8 C.	Sm. W.
132	II.	At Bonn	1858	15 C.	W.
132*a*	—	Peters	1867	—	—
132*b*	II.?	Ceraski	1878	6 C.	H. Knott, Sm.
133	II.	Hind	1851	11 C.	—
134	II.	Hencke	1858	7 C.	Sf. W.
134*a*	I.	Schmidt	1876	—	—
135	III.?	Hind	1848	—	D.
136	II.	Hind	1863	6 C. 18 P.	Sm.
137	IV.	Goodricke	1784	—	Sm. W.
137*a*	—	Deichmüller	1883	11 C.	—
138	II.	Argelander	1853	5 C.	—
139	III.	Schmidt	1847	—	Sm.
140	II.	Hind	1848	6 C.	Sm.
141	II.	Marth	1864?	6 C.	—
142	II.	Harding	1811	8 C.	Sm.
143	II.	Pogson	1853	11 C. 26 P.	Sm.

The first column of the left-hand page of Table I. gives a provisional number for designating the star. This number is taken from Schönfeld's Catalogue when the star occurs there; in other cases, a letter is added to the number. Other letters may be employed in effecting additional interpolations. The second column contains numbers from the catalogue of stars observed with the meridian photometer at the Harvard College Observatory, to be printed in Volume XIV. of the Annals of that institution. The letters H. P. (Harvard Photometry), prefixed to a number, will denote a reference to this catalogue. The following columns contain the right ascension and declination of the star for 1875, its magnitude at maximum and minimum, and its period in days.

The first column of the right-hand page repeats the number to be used for the provisional designation of the star. The second gives the class to which the star belongs, upon the system of classification employed in the Proceedings of the American Academy of Arts and Sciences, XVI. 257. Upon this system, Class I. includes tem-

porary stars; Class II., stars undergoing large variations in periods of several months; Class III., irregularly variable stars undergoing but slight changes in brightness; Class IV., variable stars of short period, like *β Lyræ* or *δ Cephei;* Class V., Algol stars, or those which at regular intervals undergo sudden diminutions of light, lasting for but a few hours. The third column gives the name of the discoverer, and the fourth column the date. The fifth column gives the number of nights on which each star was observed in 1883 by the observers, whose initials are appended to the figures. These initials are placed in alphabetical order, and are explained below. The last column contains the names of other astronomers who are known to have observed the corresponding stars since 1880. Some of these names have been abbreviated as follows: Dunér, D.; Hartwig, H.; Safarik, Sf.; Schmidt, Sm.; Wilsing, W.

The initials in the last column but one of Table I. refer to the following series of observations: —

C. This series is carried on by Mr. S. C. Chandler, Jr., at the Harvard College Observatory. The telescope employed was made by Mr. Clacey. Its aperture is 6¼ inches, and the magnifying power employed is generally 45; sometimes, 125 or 200. The observations were begun in March, 1883, but their number has been greatly increased since October. The present plan contemplates two or three observations of each variable belonging to Class II. during every month, whenever it is sufficiently bright to be visible. The observations are made by Argelander's method. Estimates of magnitude are also made independently.

H. These observations are made by the Rev. J. Hagen, S. J., at Prairie du Chien, Wisconsin. After the middle of November, 1883, the observations were independently repeated by Mr. Zwack. The instrument is a telescope by Merz, three inches in aperture. The observations are made by the division into tenths of the interval between two comparison stars. A copy of all the observations has been received at the Harvard College Observatory, and is available for the discussion of the variations of any of the stars observed.

L. These observations were made by Mr. H. A. Lawrence, and will be mentioned below, under the heading U.

P. These observations are made by Mr. H. M. Parkhurst, at Brooklyn, N. Y., with a telescope made by Fitz, nine inches in aperture. The magnifying powers employed are 56 and 150. Many of the observations are made by Argelander's method, and the remainder with photometric apparatus devised by Mr. Parkhurst, in

order to effect an optical diminution of the aperture, without diminishing the brightness of the field, until the disappearance of the star. The telescope being fixed, the pencil of rays is gradually intercepted and slightly deflected by a prism, the proportion of the light being determined by the time which elapses after the passage of the transit wire by the star. Both accuracy and facility of reduction are much increased by placing over the object-glass caps bounded by logarithmic curves. A copy of the observations has been received at the Harvard College Observatory, and is available for the discussion of the variations of any of the stars observed.

S. These observations are made by Mr. E. F. Sawyer, at Cambridgeport, Mass., by means of an opera-glass for the brighter stars, and of a field-glass for the others. It is to be noticed that the observations of the star 95 *b*, made by Mr. Sawyer on twelve nights, consist in the observation of twelve minima, each series usually including a large number of comparisons. Mr. Sawyer has sent the results of all his observations to the *Astronomische Nachrichten*.

U. These observations were made by Professor Winslow Upton, of Brown University, during an expedition to observe the total eclipse of the Sun which occurred on May 6, 1863. The observations were chiefly made on board the U. S. S. Hartford, but partly on Caroline Island, in the Pacific Ocean. No instruments except a field-glass were employed in the comparisons, which were made by the division into tenths of the interval between two comparison stars. Most of the observations were independently repeated by Mr. H. A. Lawrence. The stars observed were all south of —30° declination; no account has been received of any other observations of these southern stars during 1883.

Z. These observations were made by Mr. Zwack, and have already been mentioned under the heading H.

Although it is of course impossible to prepare a complete list of stars suspected of variability, as distinguished both from known variables and from stars about the magnitude of which observers have slightly differed, the attempt has been made in Table II. to provide a list of stars for the variability of which there is evidence enough to make them interesting objects. When the variability of any of these stars has been fully established, it will be very desirable to determine their maxima, minima, periods, and light curves. In observing these objects, the comparisons should be made either by Argelander's method or by some other of sufficient precision to decide the question of variability. The mere estimation of magnitudes cannot suffice for this purpose.

TABLE II. — SUSPECTED VARIABLES.

No.	H. P.	R. A. 1875.			Dec. 1875.		Max.	Min.	Authority.
		h.	m.	s.	°	′			
1	23	0	6	49	+14	29.7	2.5	3.1	Schwab
5	—		17	27	—10	9.1	10	—	Borelly
9	—		37	51	+ 6	36.9	9	12	Hind
25	—	1	15	0	+ 9	1.6	10	Inv.	Tempel
47	—		47	45	+ 8	9.9	6.7	8	Argelander
49	—		49	25	—68	33.6	6.6	7.5	Gould
59	—	2	10	24	+58	22.3	8.5	9.5	Safarik
73	—	3	37	37	+ 9	0.2	9½	Inv.	Palisa
75	—		38	1	+23	41	—	—	Wolff
81	—		46	29	+ 7	24.1	6¾	8	Gould
87	—		57	46	+23	38.4	9 5	12	Kreutz
93	—	4	14	32	+19	31.1	9.2	10.0	Baxendell
97	—		32	28	+13	28.5	9.5	Inv.	Palisa
109	867		49	42	—16	37.2	5.4	6.0	Gould
111	881		53	57	+ 3	25.8	6	6¼	Gould
113	883		54	7	—12	43.4	4.8	5.7	Gould
139	—	5	23	49	— 1	7.7	9	—	Argelander
143	—		27	23	+21	51.5	8½	11½	Schmidt
145	1018		28	18	+10	9.6	5.7	6.7	Gould
147	1021		28	56	— 6	5.5	5	6	Falb & Gould
161	—	6	11	14	— 1	31.6	8	9½	Copeland
167	—		59	22	+23	5.9	9.0	12	Safarik
189	—	7	36	2	—31	22.3	6.5	7.4	Gould
195	—		43	52	—40	20.6	6.5	7.2	Gould
201	—		59	44	+23	8.7	9.5	—	Palisa
205	—	8	2	26	+19	48.3	9.7	14	Peters
209	—		21	3	+ 9	0	9½	—	Palisa
227	1684	9	13	13	—23	57	6	8½	Schönfeld
239	—		27	25	—56	29.0	3½	4½	Gould
251	—		44	32	+30	41.7	9	Inv.	Schmidt
259	—	10	1	27	—51	34.8	6½	7½	Gould
269	—		12	55	+ 7	49.8	9½	—	Palisa
271	—		12	55	—60	42.5	3.3	4.5	Gould
277	—		21	55	—73	23.7	4.2	5.1	Gould
293	—		45	33	—20	35.2	6	8	Gould
294	—		47	2	+14	22.9	9	—	Peters
311	—	12	7	31	+ 0	16.8	8	8.7	Harrington
337	—		32	41	+17	11.8	8.8	10.0	Weiss
339	—		32	54	+17	10.7	—	—	Weiss
345	2160		37	2	—13	10.4	5	7	Schönfeld
347	2164		39	15	+46	•7.4	6	—	Schmidt
365	2293	13	28	2	—12	35.1	5.7	6.3	Schmidt
373	2343		43	27	+16	25.1	—	—	Schmidt
375	—		47	47	+11	41.1	8½	—	Hind
381	—		56	25	— 1	46.6	8	9	Copeland
383	—		58	15	— 8	35.9	11	—	Peters
405	—	14	39	59	—56	8.3	6	7	Gould
407	2475		42	41	+ 6	28.9	6	8	Hussey
411	—		43	42	—76	9.0	5.5	6.2	Gould
421	—		58	9	—68	14.2	7.0	7.4	Gould
433	—	15	26	53	—48	55 8	7	9½	Gould
437	—		28	59	—20	45.0	—	—	Peters
441	—		30	47	—15	45.5	—	—	Peters

TABLE II. — *Continued.*

No.	H. P.	R. A. 1875.			Dec. 1875.		Max.	Min.	Authority.
		h.	m.	s.	°	′			
447	—	15	36	28	—10	31.1	7.0	8.8	Weiss
449	—		38	46	—34	17.3	5½	6½	Gould
451	—		39	14	—20	44.3	11	Inv.	Peters
453	—		43	23	+28	40.0	11	12½	Schmidt
459	—	16	1	11	—21	11.4	11	< 13	Peters
471	—		22	22	—19	14	—	Inv.	Peters
475	—		30	47	+ 7	22.1	7	8	Chandler
483	—		44	45	— 5	57.6	8	11	Birmingham
509	3048	18	2	43	+28	44	4½	—	Schwab
511	—		9	19	—34	8.8	6.2	7.4	Gould
513	—		9	49	+71	3.1	9	—	Schmidt
517	—		27	58	+36	53.9	7½	9	Birmingham
531	—		53	32	—37	8.3	10	—	Schmidt
547	3362	19	25	40	+27	41.9	8.3	3.9	Klein
549	—		27	9	+17	28.5	6½	9½	Nature
555	—		35	19	+12	52.9	6½	9½	Argelander
557	—		37	57	+35	55.2	8½	10	Argelander
567	—	20	7	8	—22	21.4	11	— ?	Peters
601	—	21	1	24	—21	51.3	11½	Inv.	Peters
609	—		12	42	—50	27.6	6.1	7.3	Gould
615	—		56	30	—17	13.7	11	14 ?	Peters
625	3977	22	23	57	—26	42.7	5½	6.7	Schmidt
635	—	23	14	51	+55	25.8	8.2	8.8	Argelander
651	—		51	30	— 9	30.4	9.7	14 ?	Peters

The list given in Table II. is extracted from Mr. Chandler's unpublished catalogue of suspected variables, and comprises the objects in that catalogue the variability of which is suspected on reasonably good evidence. The first column contains the provisional numbers by which the corresponding stars are designated in Mr. Chandler's catalogue. The second column, as in Table I., gives the H. P. number. The remaining columns contain the right ascension and declination for 1875, the supposed magnitude at maximum and at minimum, and the authority for the suspicion of variability.

Accurate observations of these stars, or of other similar objects, are much to be desired. Information respecting the dates of observation, and results derived from the comparisons, which may be communicated by observers, will be published in the circular for 1885, whether they relate to 1884 or to previous years.

HARVARD COLLEGE OBSERVATORY,
Cambridge, Mass.

[From the American Journal of Science, Vol. XXVIII, July, 1884.]

Art. II.—*Light of Comparison Stars for Vesta*; by Edward C. Pickering.

In Professor Harrington's important "Study of Vesta," which appeared in this Journal, III, xxvi, 461, the light of the planet was determined from comparisons with the two stars DM. +22° 2163 and 2164. The observations were made with the wedge photometer, and were accordingly differential, so that the resulting magnitudes of Vesta depend upon the assumed magnitudes of the stars, which were taken from the Durchmusterung. It therefore appeared desirable that the stars should be observed with the large meridian photometer of the Harvard College Observatory, with the object of providing means for the reduction of Professor Harrington's results to absolute measures. The meridian photometer has been described in the Monthly Notices of the R. Astron. Society, xlii, 365.

The following table exhibits the results respectively obtained for the two comparison stars. The first column contains the numbers of the series to which the observations belong, the second the dates, and the third the initials of the observers, E. C. Pickering and O. C. Wendell. The fourth and fifth columns contain residuals expressed in tenths of a magnitude. The mean results, from which these residuals are derived, when corrected for atmospheric absorption, are 9·06 for DM.+22° 2163 and 5·48 for DM.+22° 2164. The fifth observation of DM.+22° 2163 was rejected because it appeared that an error of 30° in reading the graduated circle of the photometer had probably occurred in one of the four comparisons which constitute a complete observation with the meridian photometer. The residual corresponding to the rejected observation is placed in brack-

ets. If the presumed error of 30° is left without correction, this residual would become −0·9 instead of +0·2. The separate reduction of the four comparisons gives the residuals —2·4, 0·0, −0·5, +0·1. Correcting the first reading by 30°, its residual is reduced to —0·3.

No. of Series.	Date, 1884.	Obs.	Residuals 2163	2164
249	March 16	P.	—0·1	0·0
251	March 18	W.	0·0	0·0
252	March 22	P.	0·0	−0·1
254	March 25	W.	+0·1	+0·1
255	March 31	P.	[−0·2]	−0·2
261	April 14	P.	+0·1	+0·2

The corrections to be applied to the DM. magnitudes of the stars appear from these observations to be +·28 for DM.+ 22° 2163 and+ ·18 for DM. +22° 2164. From these corrections may be derived the formula $M-m=·023m+·058$, in which M denotes the photometric magnitude of Vesta corresponding to the magnitude m given by Professor Harrington.

In the following table the first column is repeated from Professor Harrington's table in the article above mentioned. The second column contains the corresponding magnitudes of Vesta computed for mean opposition, after correction by the formula just obtained. By mean opposition is understood, as usual, the situation in which a planet is in exact opposition to the Sun, while both the planet and the Earth are at their mean distances from the Sun. The third column contains the residuals from the mean, 6·64, of the corrected magnitudes thus found. The last column contains the residuals showing the accordance of Professor Harrington's observations of the two comparison stars. Taking the differences between the two columns of his table headed 2164 and 2163, we have a series of quantities expressed in seconds of time, the mean of which is 20·6; it corresponds to the photometric difference in magnitude resulting from the observations made here with the meridian photometer. This photometric difference is 9·06−5·48 = 3·58. These data show that in Professor Harrington's observations one second of time may be expressed in terms of magnitude by ·174. The final column of the table here given accordingly contains the products by ·174 of the differences between Professor Harrington's columns 2164 and 2163, diminished by the photometric difference 3·58. If reduced to the equator, the quantity ·174 becomes ·16, which furnishes a determination of the constant of reduction required by the particular wedge employed. The last line of the table contains the numerical means of the quantities in the last three columns. It may be observed that in the first and third lines of the table the large residuals in the third column are accompanied by large residuals in the final

column and are therefore partly attributable to errors of observation. In the seventh line from the end of Professor Harrington's table, 5·84 is assumed to be a misprint for 6·84.

Sidereal time of Observation. April, 1883. d.	h.	m.	Magn. of Vesta.	Residuals. Vesta.	Stars.
9	XII	17	7·21	+·57	−·24
13	XII	31	6·59	−·05	+·05
	XIV	57	6·17	−·47	−·26
	XV	24	6·39	−·25	−·19
15	XII	38	6·55	−·09	+·02
	XIII	19	6·43	−·21	+·07
	XV	43	6·32	−·32	−·14
16	X	3	6·73	+·09	·00
	XI	55	6·85	+·21	+·03
	XII	26	6·43	−·19	+·14
	XII	55	6·78	+·14	+·11
	XIII	24	6·69	+·05	−·14
	XIII	49	6·79	+·15	−·12
	XIV	18	6·79	+·15	+·05
17	IX	39	6·52	−·12	+·09
	X	11	6·47	−·17	+·12
	X	40	6·48	−·16	+·11
	XI	5	6·75	+·11	−·03
	XI	29	6·51	−·13	+·12
	XI	50	6·52	−·12	+·14
	XII	11	6·61	−·03	+·02
	XII	31	6·62	−·02	−·22
19	XII	41	6·67	+·03	−·02
23	XII	8	6·82	+·18	+·07
	XII	58	6·85	+·21	+·17
26	XI	55	6·91	+·27	−·02
28	XII	27	6·84	+·20	+·03
			6·64	±·17	±·10

The mean result for the magnitude of Vesta, 6·64, may be compared with the results formerly obtained at this Observatory and published in the Astronomische Nachrichten, cii, 151. The value obtained from observations on 12 nights in 1880 was 6·49, and from observations on 10 nights in 1881–2 was 6·45. The differences between these values and that derived from Professor Harrington's observations do not seem large, considering the fact that the two methods of observation were very dissimilar. In measuring large intervals of brightness with the wedge photometer systematic errors may perhaps result from irregularities in the tint of the glass and other causes. On the other hand, the small meridian photometer used in the observations of Vesta was not designed for measuring the light of objects fainter than the sixth magnitude, and even the brightest asteroids were seen in the instrument with some little difficulty.

The magnitude 6·51 found for Vesta in vol. xi of the Annals of this Observatory, page 294, was obtained by an indirect process, and its close agreement with the later results just mentioned is probably accidental.

Harvard College Observatory, Cambridge, Mass., May 19, 1884.

[Transactions American Society Mechanical Engineers, Vol. V.]

CXLVI.

ON A PRACTICAL SOLUTION OF THE PERFECT SCREW PROBLEM.

BY WILLIAM A. ROGERS, CAMBRIDGE, MASS.

AT the outset of a discussion of the problem indicated by the title of this paper, it is clearly essential that the term "perfect screw" shall be defined in the most explicit way. Perfect is a relative term. For certain purposes a piece of mechanism may be perfect, while it might fail to meet the most simple requirements of another problem. In another paper the writer has used the illustration furnished by the carpenter who was called to level up his comparator, but it will bear repeating in this connection. He had been furnished with an astronomical level, but in a short time he returned in great disgust, saying that "the level was good for nothing—that it bobbed all about." "But," said he, "I have a level at home which will settle at the same spot every time," and he insisted that he should be allowed to go home and get the level that would "settle at the same spot every time." He was allowed to do the work in his own way, and shortly afterward he triumphantly pointed out the evidence that the bed of the comparator was perfectly level. It need not be said that the most elementary test showed that the bed was *not* level, notwithstanding the evidence pointed out by our good friend the carpenter.

A piece of mechanism of the class which the French would call mechanism of precision may be termed perfect *when it meets all the requirements of the purpose for which it was constructed.* Let us apply this definition to any mechanism which involves the use of a good screw.

The cross-head of a planer receives its vertical movement through two screws. If one screw has a pitch differing from the other, it is evident that new adjustments will be required for every elevation. But if, after the proper adjustment has been made at one elevation, it is found that the working parts of the planer remain constant at whatever height the cross-head may be raised, the screws may in this case properly be called perfect. Yet, if these two screws were removed from their connection with other working parts, it

would without question be found that measurable errors of pitch could be detected and measured.

A short screw made at the works of the Waltham Watch Company will be presently described. If reliance could be placed upon the severe tests of direct measurement which have been applied to graduations produced by this screw, it might be fairly called perfect. On this bar of speculum metal 5,000 lines are ruled within a space of half an inch, producing what is called a diffraction grating. When this grating is subjected to examination under the spectroscope, there are certain optical tests of the accuracy of the spacing for short intervals which will at once detect errors which must always elude the most careful tests by direct measurement. To the naked vision there would not seem to be much difference between this grating and those produced by Rutherford, and especially the magnificent gratings from the machine of Professor Rowland, but tried by optical tests the difference is really so great that if all the errors could be charged to the screw itself the claim of perfection could not hold for a moment.

It has been intimated that the errors shown by optical tests in diffraction gratings may not after all be entirely chargeable to the screw. The flatness of the surface ruled, any unequal friction between the nut and the screw, the character of the groove cut by the ruling diamond—these and many other considerations determine the character of the grating. It is now well known that, severe as the optical test is, in the detection of periodic errors depending on single revolutions of the screw, it fails in the detection of errors separated at wide intervals. Indeed, even in the most perfect of all machines, Professor Rowland's, he is obliged to employ a "corrector" to eliminate the errors which are beyond the limits of direct measurement. The writer is well aware that he should speak with a good degree of moderation in this connection, since he has to a certain extent failed where Professor Rowland has succeeded; but Professor Rowland has a supreme knowledge of the problem both as a physicist and as a mathematician, and his success has been achieved by the power of keen analysis, aided by his little "corrector" and a precise knowledge how to use it, having as the basis of his work a most excellent but not a perfect screw.

Let us take another illustration. A cathetometer is an instrument for the measurement of vertical distances by means of one or more telescopes attached to a vertical standard upon which there is a graduated scale, usually one meter in length with subdivisions to

millimeters. There are several well-known manufacturers of physical apparatus in Europe, who advertise that these graduations are without sensible error. An investigation of the errors of several of these graduated scales during the past three years has shown that in every case they were nearly within the requirements of the optical power of the telescopes employed, but it needed only the most superficial examination under the microscopes of the comparator to place them instantly far below the lowest limit required in an exact standard of length.

Illustrations almost without limit might be multiplied to show the necessity of defining the limit of accuracy with which one ought to be content in mechanical construction. It goes without saying that real progress begins when the mechanician recognizes that there is such a limit. A short time since, the writer asked Mr. Sharpe, of the firm of Darling, Brown & Sharpe, if he would undertake to grind a perfect cylinder. His reply was very suggestive. He said: "We are not making perfect mechanism of any kind any longer in this establishment. A few years ago we felt competent to undertake perfect work of any and every kind, but we have grown wiser since then." Need it be said that the work done by this company is in many respects of a higher grade than it was ten years ago?

Five or six years ago, the writer was ruling lines 120,000 to the inch more or less, and he thought nothing of obtaining for the probable error of a set of measures of graduations, figures low down in the millionths of an inch. It has since been learned by some not very pleasant experience that figures do not always tell the truth, especially figures which represent what are known as "probable errors"—that while straining at very small gnats, several very large camels walked by unperceived.

Let us now endeavor to answer the question — *What ought we to expect of a perfect screw?* Those of you who are accustomed to make screws will at once say that the answer depends to a large extent upon the length of the screw. And so it does under the ordinary methods of construction, but in the Rogers-Ballou process, which will presently be described, it is claimed that a screw 6 feet in length can be cut with nearly the same accuracy as a screw 6 inches long.

At this point it is important that the errors to which screws are subject should be defined with the utmost clearness. They are of three kinds:

(*a*) An error in the total length. Supposing the pitch to be uniform at every point between the terminal threads, the whole length may either exceed or fall short of the unit of length adopted, *e. g.*, the yard at the standard temperature, 62 Fahr.

(*b*) Even if the whole length is correct, the pitch of the screw for even revolutions may not be uniform. In a perfect screw the distance from face to face of every thread in a line parallel with the axis of the screw will be the same. That is, the inclined planes formed by the threads are everywhere parallel and equidistant.

(*c*) Even if conditions (*a*) and (*b*) are fulfilled there may yet remain a very troublesome class of errors, which are a function of single revolutions of the screw. If I rule 11 lines corresponding to even tenths of a revolution of the screw, I may find, from an examination of the spaces formed, that there is a gradual but very small increase in the length of each successive space up to a certain point, when a maximum value is reached. After this a diminution takes place which goes on until the amount of decrease is equal to the amount of the previous increase. Errors of this class are usually designated "periodic errors," since they are a function of a complete revolution of the screw. Expressed in mathematical language, every measured space gives an expression of the form

$$\Delta = m + a \textit{ sin. } x + b \textit{ cos. } x + a' \textit{ sin. } 2x + b' \textit{ cos. } 2x, \textit{ etc.},$$

in which:

$\Delta =$ the required error.

$m =$ a constant.

$x =$ the angle of revolution.

$a, b, a', b',$ *etc.* $=$ unknown coefficients to be determined from a series of equations by the process of Least Squares.

It is important that we shall ascertain what efforts have been made to overcome these errors in the construction of screws.

It is well known that the earliest systematic efforts to place the screw problem upon a substantial and scientific basis were made by Whitworth, but he profited by the labors of still earlier investigators. The following account of the early efforts in this direction, communicated to me by Mr. H. J. Chaney, Warden of the Imperial Standards of Great Britain, is such a clear and concise statement of what was accomplished by the early investigators in this field that it is quoted entire, although it was not communicated for the purpose of publication:

"In the rapid development of steam machinery there was felt a necessity for accuracy and interchangeability in parts, which in the screw took practical form nearly half a century since; first in the production of a standard guide screw, and subsequently in the demand for a uniform system of screw threads.

"In this country it is perhaps to the eminent engineering firm of Messrs. Maudslay & Co. that we are indebted for the first attempt to construct a perfect system of screws. For his dividing engine, however, Mr. Bryan Donkin had constructed in the year 1828 a standard screw fitted with a compensating bar, by means of which the errors of different parts of the screw were allowed for. Many screws were cut by this machine, some of which were given to various scientific friends. Sir Joseph Whitworth among others had one of these screws in the year 1843.

"Messrs. Maudslay had the advantage of the assistance of a workman whose name is now identified with all that is systematic and accurate in screw work—Whitworth, and who subsequently left them to take part under Mr. Clements, of Lambeth, in the construction, as I understand, of Babbage's Difference Engine, and there produced with Clements the first standard guide screws.

"In a paper communicated to the Institution of Civil Engineers in 1841, Whitworth discussed the question of the want of uniformity of screw threads, and put forward a series of sizes adapted to the use of engineers. These sizes differed from Maudslays', and appear to have been a compromise between sizes then generally in use. For iron piping, Whitworth took, as is well known, some sizes which had been adopted by Messrs. James Russell & Son, pipe manufacturers.

"For engineering purposes the Whitworth thread appears now to be generally adopted. For many other purposes the want of a common standard gauge for screws is much felt. A committee of the British Association appointed in 1881 for the purpose of determining a gauge for the manufacture of small screws used in electrical apparatus and clock-work, adopted a pitch similar to the Whitworth pitch for all sizes down to a ¼ inch, and also adopted the Whitworth thread above or below ¼ inch. This committee have made no definite report, and there appears to be much difference of opinion on the questions as to the inch or millimeter units, the angle of the threads, descriptive number of each size, etc."

At the outset of a discussion of the screw problem, and especially as a preliminary to any attempt to improve upon existing methods

of construction, it seemed important to ascertain just what degree of accuracy had been attained thus far in the manufacture of precision screws. Accordingly, in 1879 the writer visited Baltimore, Philadelphia, Schenectady, New York and Providence, and obtained transfers from screws by Perreaux, Bianchi, Clement, Brown & Sharpe, and Rutherford. As far as could be learned, these were the only screws at that time in this country possessing any claim to more than ordinary accuracy. In London, a yard with subdivisions into inches was obtained from the dividing engine used by Troughton & Sims in ordinary work. In Paris a meter with subdivisions to decimeters was obtained from the dividing engine of Desmoulins-Froment. Access could not be obtained to the dividing engine of Brunner Frères, but a standard centimeter subdivided to tenths of millimeters was obtained from this firm.

Application was made to Sir Joseph Whitworth & Co. for a screw one meter in length, but the reply was returned that the company was not prepared to do work of this class with the degree of precision required. Froment, of Paris, however, accepted the order, but it was not until after two years that the screw was delivered.

It does not seem necessary to include in this paper a full account of the investigation of the errors of these screws. The results can be stated in a few words.

(*a*) In only two cases was the total length found to be substantially correct,—viz., in a yard and meter made by Brown & Sharpe and in a meter by Froment. But in both of these cases the total length was varied to correspond with the unit of length adopted by means of a "corrector." Brown & Sharpe have always exercised their undoubted right of declining to allow a personal inspection of their processes, but I cannot be far from right in saying that a corrector was employed not only in the correction of the total length, but also in the correction of errors due to the irregularities of the screw. In Paris, Froment accorded the rare privilege of a personal inspection of his dividing engine. It was estimated that the corrector eliminated errors amounting to about one-tenth of a millimeter, or about one-two hundred and fiftieth of an inch. In the remaining cases the error in the total length was in no case less than one two-hundred-and-fiftieth of an inch, and in one case it reached one-tenth of an inch in one yard.

(*b*) In every case in which a corrector was not employed the errors depending on single revolutions of the screw were very

large, while the variation in the pitch at different points along the screw varied between $\frac{1}{100}$ inch and $\frac{1}{2000}$ inch.

If one can judge of the screws made by German manufacturers by the graduations of German cathetometers, they would appear to be at least of no higher grade than those of French or American manufacture.

It appears safe to conclude that with the exception of the Rutherford screw, of a few micrometer screws by Alvan Clark & Sons, and perhaps of a small number of screws of the same class by Hilger, of London, by Brunner Frères, of Paris, and by Repsold, of Hamburg, there was not in the year 1880 a single screw in existence which could be shown by a published discussion of its errors to be sufficiently uniform in pitch to entitle it to the rank of a precision screw.

One does not need to go very far in assigning a cause for the failure to make any important advance in the construction of screws. According to the existing methods of manufacture, the maker of a screw has absolutely no precise knowledge of the form and dimensions of the thread which he cuts till the screw is completed. It is well known that several devices have been employed to test the accuracy of the screw during its construction, but they nearly all involve *the errors of a combination of threads, instead of the errors of single threads.*

It has been the custom to assume that the residual errors of a screw can be worked out by grinding with a lead nut. The ordinary methods of grinding are wholly inadequate, especially in the elimination of the errors depending on one revolution of the screw, when combined with uniformly increasing or decreasing variations in pitch for successive threads.

The action of a grinding nut may be likened to that of a harrow upon a ploughed field. The harrow will easily smooth down the furrows freshly turned by the plough, but it would be making a too serious demand upon it to require that it should level down hills or even hillocks. The whole difficulty in grinding consists, first, in the fact that the action between the nut and the screw is to a certain extent mutual, and, second, that the threads of a screw are ground *in combination, and not each by itself.* A grinding nut will easily work out short and irregular variations in pitch, but it will not eliminate long sweeps of errors except by accident. I shall presently recur to this matter in connection with the discussion of two typical screws.

(c) *The apparent irregularities in screws are due, first, to the errors in the pitch of the screw itself, and, second, to the unequal friction between the nut, the screw, and the ways upon which the carriage driven by the nut moves.* Abundant experience has proved this statement to be true. In the first dividing engine constructed for the writer by Buff & Berger, of Boston, the nut was at first connected rigidly with the carriage. After an experience of two years it was found impossible to get the same system of errors for the screw in successive trials if the slightest change was made in the relation of the working parts of the machine. By touching a screw here or there it was found possible even to reverse the sign of the correction depending on one revolution of the screw. About 1878 a *free nut* was first employed, *i.e.*, the nut now travels freely upon the screw, without the slightest binding, pushing the carriage before it. From that time to this, the system of corrections required for the screw has remained unchanged, and I now use precisely the same values as were computed five years ago. The constancy of the corrections was greatly aided by the use of finely powdered graphite as a lubricant. A similar experience with some micrometer screws made for the meridian circle of Harvard College Observatory by Mr. Geo. Clark, of the firm of Alvan Clark & Sons, was very instructive. One of the screws was mounted and dismounted nine times, and in every case different systems of errors were obtained, the extreme difference being about $\frac{1}{5000}$ inch.

There are a few fundamental requirements which must be absolutely met in the successful construction of a screw. Let us try to state these requirements in the most simple and positive terms:

(1) The shaft to be threaded must maintain a true cylindrical form during every part of a revolution and during every successive revolution, *i. e.*, the axis of motion must be a straight line.

(2) The cutting tool must travel in a line exactly parallel with the axis of the screw to be cut. This requirement demands first that the ways upon which the carriage of the screw-cutting machine travels shall be straight, allowing the carriage to move in a horizontal plane. Especial attention must therefore be paid to the elimination of the flexure of the bed-plate upon which the ways are cut. Second, the ways must be free from horizontal curvature, allowing the carriage to move in a true vertical plane. Expressed in general terms, the conditions to be fulfilled require that every movement of the cutting tool with respect to the screw to be cut *shall be referred to an invariable reference plane.*

(3) The cutting tool must give to each single thread approximately its proper form and pitch during each successive operation of cutting, independently of every other thread.

If these conditions can be fulfilled in the construction of a screw, there is no reason why the residual errors may not be reduced far below the limit reached in our present practice.

Let us now venture to define the limit which ought to be reached. First, a screw ought to be capable of measuring as closely as a skillful mechanic can calliper. In ordinary practice that limit may be placed at about $\frac{1}{20000}$ inch, but in the hands of a person in which the sense of feeling has been cultivated even but slightly, a good calliper will detect variations in the diameter of a small cylinder amounting to about $\frac{1}{40000}$ inch.

Second, experience has shown that the limit of *certainty* in measuring short spaces, *e. g.*, two or three inches, is about $\frac{1}{50000}$ inch, while for longer intervals, in which flexure comes into play, the limit should be placed at between $\frac{1}{20000}$ and $\frac{1}{30000}$ inch.

It would appear, therefore, that we ought to demand of a precision screw that it shall have no error much exceeding the lowest limit named. Of course, the average error of adjacent threads would in this case be far less. If it is possible, therefore, to construct a screw whose error shall not rise above this limit, it may fairly be termed a screw of precision, or, if you choose, a "perfect screw."

In the fall of 1882, Mr. Geo. F. Ballou, who is now superintendent of the Ballou Manufacturing Co. of Hartford, Conn., which has undertaken the manufacture of improved lathe and precision screws by what has been designated the Rogers-Ballou process, joined the writer in an attempt to give a practical application to the principles of construction which have been outlined in this paper. For about four years previous to this time, Mr. Ballou had been engaged in making a dividing engine which Mr. Chas. Van Woerd, at that time mechanical superintendent of the Waltham Watch Company, undertook to construct upon my order.

A high limit of precision was soon reached in the construction of a short screw, having a working length of about 4 inches, although the hope of obtaining from it an improvement in diffraction gratings was not realized. But in every attempt to make a screw having a length of half a meter it was found impossible to go beyond a certain limit by the ordinary methods of construction and correction. After experimenting in various ways for nearly two years,

Mr. Van Woerd decided to adopt the form of a sectional screw. Threads were cut upon ferrules $1\frac{3}{4}$ inches in length, each ferrule being cut from the same part of the leading screw. These ferrules were then placed upon a cylindrical shaft, and adjusted in such a way that the threads of the adjacent ferrules would match.

The method itself is not new; Whitworth tried it and abandoned it many years ago. Mr. Van Woerd, however, by the method described in patent No. 293,930, claimed that the difficulties encountered by Whitworth were entirely overcome.

Mr. Ballou had done all of the actual work of construction up to the point of the application of the new method of making the ferrules.

About this time an order was received from Professor Wm. A. Anthony for the construction of a dividing engine for the Physical Department of Cornell University. Upon accepting the order, a shop was fitted up in Boston with tools of the best quality, chiefly from the establishment of Pratt & Whitney, and Mr. Ballou undertook the construction of the engine, mainly from his own designs, and of the screw which is its essential part. The completed machine was shipped in just 35 weeks after the actual commencement of the work; and the screw, which will be presently discussed, was cut and ground in 27 hours from the time the first tracing of a thread was made. It was at that time practically perfect for about 20 inches, and nearly as perfect as it afterward became by the process of grinding adopted. Notwithstanding the fact that the work was done upon a common lathe in which the errors of the leading screw were enormously large, the result showed that the method employed was based upon correct mechanical principles, and was entirely feasible.

This method can be described in a very few words.

Let the reader hold clearly in mind the following:

There are:

(*a*) An ordinary lathe, the ways of which have been made as nearly straight as possible.

(*b*) A shaft between dead centers which maintains a cylindrical form during every revolution and every part of a revolution.

(*c*) A microscope provided with Tolles' opaque illuminator for viewing opaque objects, attached to the carriage moved by the leading screw of the lathe.

(*d*) A graduated bar mounted independently of the carriage with subdivisions which are multiples of single threads of the leading screw.

(*e*) A slide moving parallel with the leading screw, by means of a very short and firmly mounted micrometer screw of comparatively large diameter and attached firmly to the carriage. The tool-post is secured firmly to this secondary slide.

(*f*) A mechanical means of determining when the leading screw has made a complete revolution.

The method of proceeding was as follows:

(1) The graduated bar having been leveled up and set parallel to the axis of the screw to be cut, the micrometer of the microscope was set upon the initial line. The lathe was then started with the leading screw "in feed." After the screw had made, for example, nearly ten revolutions, the lathe was stopped and the remainder of the even revolution was completed by hand manipulation. The deviation of the micrometer line from the corresponding graduation upon the bar was then measured in terms of the screw-head of the secondary micrometer screw. In this way the errors of the leading screw with respect to the graduations of the standard bar were determined and written down upon a strip of paper pasted to the vertical face of the bar.

(2) The carriage was then started again with the cutting tool in operation, and by means of a rough pointer, the micrometer screw working the secondary slide was fed either forward or backward, in accordance with the corrections before determined. Hence, when any even revolution was completed, it would be found that the line of the bar would be nearly under the cross wire of the microscope. This operation was kept up until the screw was finished.

At the completion of the operation of cutting, it was found—

First, That the total length of the screw corresponded nearly with the length of the line standard from which it was cut.

Second, That there were at many points minute irregularities of pitch, due to the fact that the application of the corrections intermediate between the main divisions had not been exactly made.

Third, That the crucial test of the removal of these irregularities by grinding with a brass nut was a complete success. As had been predicted, they were for the most part removed after an hour's grinding.

The method of testing was as follows: Two half nuts with projecting arms resting upon the ∧ shaped way were first placed at a fixed interval apart. A microscope was mounted upon one nut and coincidence was made between the micrometer wire of

the microscope and a line drawn upon the upper surface of the other nut. It is obvious that if the relation between the different threads of the screw remained constant, the line under the microscope would remain constant. This constancy under a half-inch objective was maintained for about twenty inches. Then the nuts began to separate, and the separation continued until the maximum deviation amounted to about $\frac{1}{5000}$ of an inch; but near the end the nuts came back to their first relation.

In order to eliminate these residual errors, together with the remaining errors which were a function of one revolution of the screw, the following method was employed. The grinding nut was made in two halves, in such a manner that a constant relation was maintained between the two halves, both in their normal and in reversed positions. By grinding the screw first with the two halves of the nut in their normal relation and then in reversed relations, the tendency was to continually work out the periodic errors of the screw, with the exception of minute errors which were transferred from the screw to the nut during the operation of grinding.

In order that the nut might grind without disturbing the general relation between the threads, a cast-iron cylinder with a centre at the bottom was filled with the best sperm oil, and the screw was mounted vertically upon this centre. At first two broad fans were attached to the nut in the hope that the resistance of the oil, which in this case would be symmetrical with respect to the axis of the screw, would be sufficient to drive the nut upon the screw. As this movement was found to be too slow, a guiding rod was used.

The grinding process was continued for three weeks, and the results obtained confirmed previous experience. At the end of the first week the maximum error of $\frac{1}{5000}$ of an inch had been reduced about one-half, but it was found that small errors had been introduced in the mean time at other points, through a slight transfer of the errors of the screw to the nut itself and from thence back to the screw. Near the end of the second week the screw was clearly less perfect as a whole than at the commencement of the operation of grinding. Mr. Ballou then recut the nut, making its diameter a little less than than that of the screw. Within a few hours thereafter a decided improvement was observed. A new nut was made at the end of the second week having its diameter still a little less than before. During the third week the gain consisted for the most part in eliminating the errors

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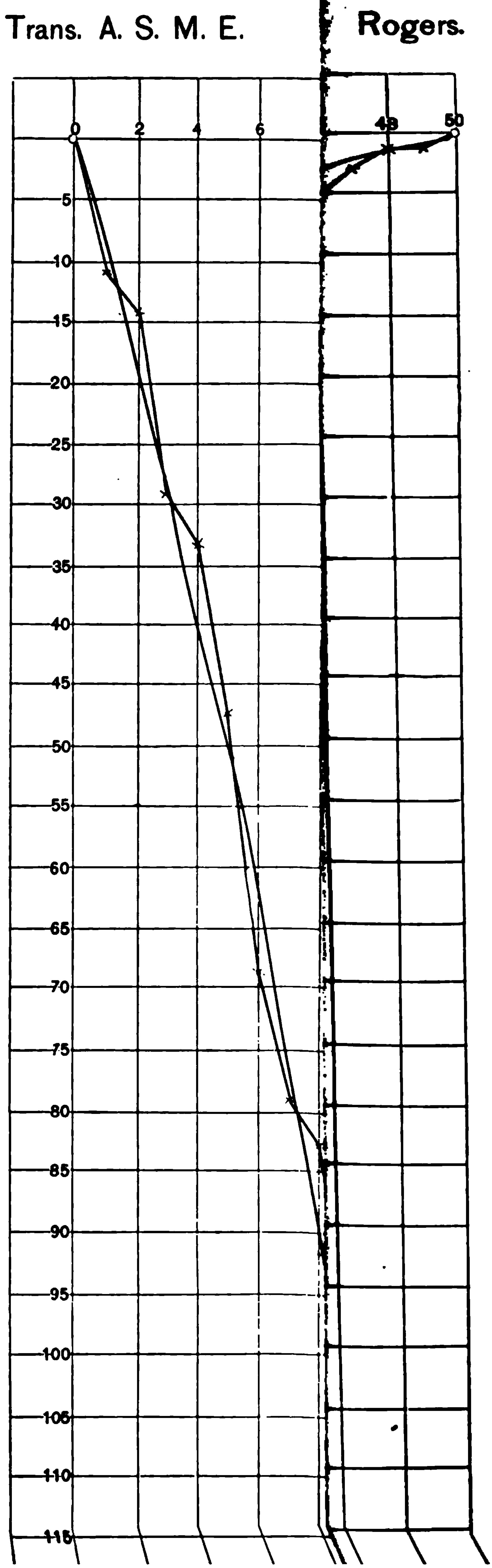


which had been introduced during the second week. Throughout the entire operation of grinding, reversals were made every hour both of the two halves of the nut and of the screw upon its centres.

For a comparison of the old with the new method of cutting screws the sectional screw made by Mr. Van Woerd has been chosen; *first*, because its errors are less than those for either of the long screws which preceded it, and, *second*, because it seemed important to ascertain whether a sectional screw can be made which possesses decided advantages over the ordinary form. It is pretty certain that this particular screw is the best of its class ever made. The workmanship upon it could hardly be better. If it is found that errors of considerable magnitude remain, we may conclude that they should be charged to the method itself.

The method of obtaining transfers from this screw was as follows: It had been found by a direct comparison both with a half meter and a half yard, standard at 62° Fahr., that

The half meter = 400.4210 revolutions of the Waltham screw.

The half yard = 366.1382 revolutions of the Waltham screw.

Since the screw was cut 8 threads to the centimeter, it is therefore $\frac{421}{800}$ of a millimeter too long, or nearly a millimeter in a meter.

1 Div. = .0000050 inch.

VAN WOERD SECTIONAL SCREW.

Comparison of the computed with the observed readings for:

Subdivisions of the half meter.

Spaces.	Observed Reading of Index.		Computed Reading of Index.		Δ Expressed in hundred thousandths of an inch.
	Rev.	Div.	Div.	Δ Div.	
1	8	63	84	− 21	− 10
2	16	140	168	− 28	− 14
3	24	196	253	− 57	− 28
4	32	271	337	− 66	− 33
5	40	327	421	− 94	− 47
6	48	369	505	− 136	− 68
7	56	431	589	− 158	− 79
8	64	459	624	− 165	− 83
9	72	533	758	− 225	− 113
10	80	606	842	− 236	− 118
11	88	696	926	− 230	− 115
12	96	782	1010	− 228	− 114
13	104	864	1095	− 231	− 115
14	112	973	1179	− 206	− 103
15	120	1065	1263	− 198	− 99
16	128	1161	1347	− 186	− 93
17	136	1249	1431	− 182	− 91
18	144	1325	1516	− 191	− 95
19	152	1436	1600	− 164	− 82
20	160	1525	1684	− 159	− 60
21	168	1605	1768	− 163	− 82
22	176	1692	1852	− 160	− 80
23	184	1811	1937	− 126	− 63
24	192	1914	2021	− 107	− 53
25	200	2021	2105	− 84	− 41
26	208	2118	2189	− 71	− 35
27	216	2201	2273	− 72	− 36
28	224	2290	2358	− 68	− 34
29	232	2355	2442	− 87	− 43
30	240	2435	2526	− 91	− 46
31	248	2522	2610	− 88	− 44
32	256	2604	2694	− 90	− 45
33	264	2717	2779	− 62	− 31
34	272	2819	2863	− 44	− 22
35	280	2903	2947	− 44	− 22
36	288	2990	3031	− 41	− 20
37	296	3073	3115	− 42	− 21
38	304	3154	3200	− 46	− 23
39	312	3241	3284	− 43	− 22
40	320	3331	3368	− 37	− 18
41	328	3409	3452	− 43	− 21
42	336	3512	3536	− 24	− 12
43	344	3605	3621	− 16	− 8
44	352	3693	3705	− 12	− 6
45	360	3785	3789	− 4	− 2
46	368	3862	3873	− 11	− 5
47	376	3951	3957	− 6	− 3
48	384	4040	4042	− 2	− 1
49	392	4124	4126	− 2	− 1
50	400	4210	4210	+ 0	+ 0

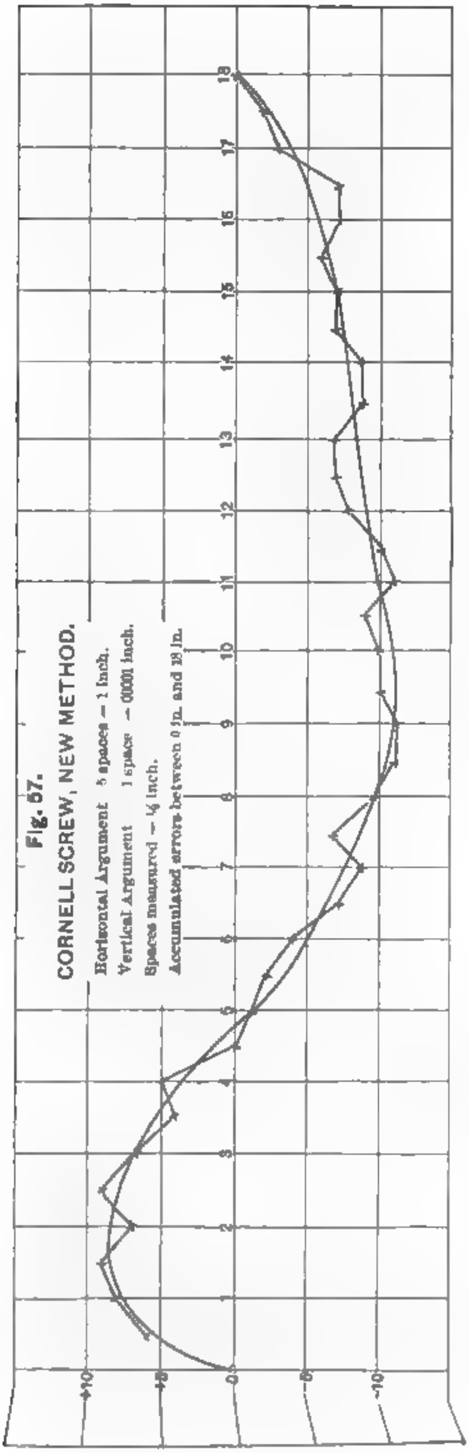


Fig. 57.
CORNELL SCREW, NEW METHOD.
Horizontal Argument 5 spaces = 1 inch.
Vertical Argument 1 space = 00001 inch.
Spaces measured = ⅛ inch.
Accumulated errors between 0 in. and 18 in.

Comparison of the computed with the observed readings for:

Subdivisions of the half yard.

Spaces.	Observed Reading of Index.		Computed Reading of Index.		Δ Expressed in hundred thousandths of an inch.
	Rev.	Div.	Div.	Δ Div.	
1	10	1677	1705	− 28	− 14
2	20	3368	3410	− 42	− 21
3	30	5055	5115	− 60	− 30
4	40	6735	6820	− 85	− 43
5	50	8375	8525	− 150	− 75
6	61	77	230	− 153	− 76
7	71	1708	1935	− 227	− 114
8	81	3416	3640	− 224	− 112
9	91	5129	5345	− 216	− 108
10	101	6845	7051	− 206	− 103
11	111	8559	8756	− 197	− 98
12	122	288	461	− 173	− 86
13	132	1982	2166	− 184	− 92
14	142	3690	3871	− 181	− 91
15	152	5415	5576	− 161	− 80
16	162	7131	7281	− 150	− 75
17	172	8824	8986	− 162	− 81
18	183	580	697	− 117	− 58
19	193	2303	2396	− 93	− 46
20	203	4039	4101	− 62	− 31
21	213	5733	5806	− 73	− 37
22	223	7445	7511	− 66	− 33
23	233	9131	9216	− 85	− 42
24	244	831	921	− 90	− 45
25	254	2534	2626	− 92	− 46
26	264	4282	4331	− 49	− 25
27	274	6013	6036	− 23	− 12
28	284	7697	7741	− 44	− 22
29	294	9390	9446	− 56	− 28
30	305	1102	1151	− 49	− 24
31	315	2849	2856	− 7	− 3
32	325	4530	4562	− 32	− 16
33	335	6246	6267	− 21	− 10
34	345	7960	7972	− 12	− 6
35	355	9776	9677	− 1	− 1
36	366	1882	1882	+ 0	+ 0

The transfer from the Cornell screw was kindly made for me by Professor Anthony after the engine had been mounted upon the firm foundation prepared for it in the new Physical Laboratory of Cornell University. The transfer consists of forty half-inch spaces, but only thirty-six of these have been fully investigated.

It is necessary at this point to define clearly a class of errors which will be designated *accumulated errors.* They will be indicated by the symbol Σ, which is the usual symbol for a summed series.

If a given space is subdivided into any number of approximately equal parts, the error of any space with respect to the mean of all the spaces is called a relative error. The accumulated error is the algebraic sum of the relative errors reckoned from the initial line. Thus, if the second space of a given series of subdivisions is one unit longer than the first, the third one unit longer than the second, the fourth one unit longer than the third, etc., the error of the fifth line expressed in aliquot parts of the whole space subdivided will be three units. But in this summation of the relative errors, the accidental errors of observation are carried along with every subsequent summation. Hence the error, for example, of the middle point may not be found to be the same as would be obtained from a direct comparison of the two halves. Indeed, it ought not to be expected that in practice the summation of the errors of fifty spaces should give the same accumulated error for spaces 10, 20, 30 and 40 as would be obtained by a direct comparison of these spaces. If a substantial agreement is obtained, however, whatever the number of sub-divisions, it may be assumed that the errors of the separate spaces have been correctly determined.

Let us now apply this test to the errors of the Cornell screw. The measures given below were obtained by a comparison of each space with a constant distance between the two stops of the comparator. If the distance between the stops differs from the mean of all the spaces, a constant must be taken from each in order to reduce it to the mean of all the readings. This constant has been subtracted in every series given below except in the first. Here, instead of obtaining the constant from the whole series for the reduction of the remaining series, it will be necessary to obtain it for the space measured in that series. The errors at intermediate points are found by adding to the relative errors for each space the uniformly distributed errors of the limiting lines of that space, as will be shown below. The uniformly distributed errors are printed in smaller type than the errors of the main subdivisions. The values given are expressed in terms of the micrometer screw of the microscope employed, in which

1 div. = .000020 inch.

INVESTIGATION OF HALF-INCH SPACES FROM THE CORNELL SCREW.

OBSERVED DATA.

SPACES =	½ In.	2/2 In. Σ	3/2 In. Σ	4/2 In. Σ	6/2 In. Σ	9/2 In. Σ	18/2 In. Σ
	div.	*div.*	*div.*	*div.*	*div.*	*div.*	*div.*
0	+ 0.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0
1	+ 2.5	+ 1.9	+ 1.5	+ 0.8	+ 0.6	− 0.1	− 0.3
2	+ 0.5	+ 3.9	+ 3.0	+ 1.7	+ 1.3	− 0.2	− 0.6
3	− 0.1	+ 2.9	+ 4.5	+ 2.6	+ 2.0	− 0.4	− 0.9
4	− 0.2	+ 2.0	+ 3.9	+ 3.4	+ 2.7	− 0.5	− 1.2
5	+ 0.4	+ 1.6	+ 3.3	+ 3.3	+ 3.4	− 0.6	− 1.5
6	− 1.4	+ 1.2	+ 2.8	+ 3.2	+ 4.0	− 0.8	− 1.8
7	− 1.8	+ 0.6	+ 1.7	+ 3.1	+ 3.1	− 0.9	− 2.1
8	− 0.3	− 0.1	+ 0.6	+ 3.0	+ 2.2	− 1.0	− 2.4
9	− 3.1	− 1.6	− 0.6	+ 1.7	+ 1.2	− 1.2	− 2.8
10	− 1.9	− 3.0	− 0.9	+ 0.4	+ 0.3	− 1.8	− 3.1
11	− 2.1	− 3.5	− 1.1	− 0.9	− 0.6	− 2.4	− 3.4
12	− 2.9	− 4.0	− 1.4	− 2.3	− 1.6	− 3.0	− 3.7
13	− 1.3	− 4.2	− 2.0	− 2.9	− 2.0	− 3.6	− 4.0
14	− 1.7	− 4.4	− 2.6	− 3.5	− 2.4	− 4.2	− 4.3
15	+ 0.5	− 4.8	− 3.3	− 4.1	− 2.8	− 4.8	− 4.6
16	− 2.6	− 5.2	− 4.2	− 4.6	− 3.1	− 5.4	− 4.9
17	− 1.6	− 6.0	− 5.1	− 4.7	− 3.5	− 6.0	− 5.2
18	− 0.6	− 6.5	− 6.1	− 4.8	− 3.9	− 6.6	− 5.5
19	+ 0.0	− 6.1	− 5.5	− 4.9	− 3.8	− 6.4	− 5.2
20	− 1.3	− 5.2	− 4.9	− 5.0	− 3.6	− 6.2	− 4.9
21	+ 0.4	− 5.0	− 4.2	− 5.0	− 3.5	− 6.1	− 4.6
22	− 1.5	− 4.8	− 4.3	− 5.1	− 3.4	− 5.9	− 4.3
23	+ 0.3	− 4.3	− 4.4	− 5.1	− 3.2	− 5.7	− 4.1
24	+ 0.6	− 3.8	− 4.5	− 5.2	− 3.0	− 5.5	− 3.8
25	+ 0.2	− 3.8	− 4.5	− 5.2	− 2.8	− 5.4	− 3.5
26	− 1.1	− 3.9	− 4.5	− 5.2	− 2.6	− 5.2	− 3.2
27	− 1.8	− 4.0	− 4.5	− 5.1	− 2.4	− 5.0	− 2.8
28	− 0.5	− 4.0	− 3.3	− 5.1	− 2.2	− 4.5	− 2.5
29	+ 0.0	− 3.5	− 2.1	− 4.6	− 2.0	− 3.9	− 2.2
30	− 0.4	− 3.0	− 1.4	− 4.5	− 1.8	− 3.4	− 1.9
31	− 0.8	− 3.0	− 1.3	− 4.2	− 1.5	− 2.8	− 1.6
32	− 0.8	− 2.9	− 1.2	− 3.8	− 1.2	− 2.3	− 1.3
33	+ 0.0	− 2.4	− 1.0	− 2.9	− 0.9	− 1.7	− 1.0
34	+ 1.5	− 1.9	− 0.7	− 1.9	− 0.6	− 1.2	− 0.7
35	+ 0.5	− 0.9	− 0.3	− 0.9	− 0.3	− 0.6	− 0.3
36	+ 1.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0	+ 0.0

RESULTS.

ARGUMENT—HALF-INCH SPACES.

SPACES.	(1) Σ	(2) Σ	(3) Σ	(4) Σ	(6) Σ	(9) Σ	(18) Σ	*MEAN Σ	MEAN IN HUNDRED THOUSANDTHS OF AN INCH.	Δ
0	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+ 0	+6
1	+3.1	+2.9	+3.0	+2.9	+3.0	+3.0	+3.2	+3.0	+ 6	+2
2	+4.2	+3.9	+4.0	+3.9	+4.1	+4.0	+4.5	+4.1	+ 8	+1
3	+4.7	+3.5	+4.5	+4.3	+4.6	+4.3	+5.1	+4.5	+ 9	−2
4	+4.1	+2.0	+3.4	+3.4	+4.0	+3.6	+4.7	+3.5	+ 7	+2
5	+5.1	+2.5	+3.9	+4.5	+5.0	+4.5	+5.8	+4.4	+ 9	−2
6	+4.3	+1.2	+2.8	+3.8	+4.0	+3.5	+5.2	+3.4	+ 7	−3
7	+3.2	−0.2	+1.6	+2.7	+3.3	+1.0	+4.1	+2.1	+ 4	+1
8	+3.5	−0.1	+1.9	+3.0	+4.1	+1.2	+4.6	+2.4	+ 5	−5
9	+1.0	−2.2	−0.6	+1.3	+2.0	−1.2	+2.1	+0.2	+ 0	−1
10	−0.3	−3.0	−0.5	+0.4	+1.1	−2.1	+1.0	−0.5	− 1	−2
11	−1.7	−3.1	−0.9	−0.5	+2.0	−3.0	−0.3	−1.0	− 2	−2
12	−4.0	−4.0	−1.4	−2.3	−0.6	−5.1	−2.5	−2.6	− 4	−3
13	−4.7	−4.0	−2.5	−2.9	−2.1	−5.4	−3.0	−3.3	− 7	−2
14	−5.8	−4.4	−4.0	−3.6	−3.0	−6.2	−4.0	−4.2	− 9	+2
15	−4.7	−3.3	−3.3	−2.7	−3.9	−4.7	−2.7	−3.5	− 7	−3
16	−6.7	−5.2	−5.2	−4.6	−3.4	−6.8	−4.6	−4.9	−10	−1
17	−7.7	−6.2	−6.1	−5.4	−4.0	−6.0	−5.4	−5.5	−11	+0
18	−7.6	−6.5	−6.1	−5.2	−3.9	−6.6	−5.5	−5.6	−11	+1
19	−7.0	−5.5	−5.2	−4.5	−3.6	−5.9	−5.0	−4.9	−10	+0
20	−7.7	−5.2	−5.6	−5.0	−4.4	−6.5	−5.8	−5.4	−10	+1
21	−6.7	−4.1	−4.2	−4.6	−3.7	−5.6	−4.8	−4.5	− 9	−2
22	−7.6	−4.8	−5.6	−6.1	−4.8	−6.4	−5.8	−5.6	−11	+1
23	−6.7	−4.4	−5.2	−5.7	−4.1	−5.4	−5.1	−5.0	−10	+2
24	−5.5	−3.8	−4.5	−5.2	−3.0	−4.2	−3.9	−4.1	− 8	+1
25	−4.6	−3.2	−3.4	−4.2	−2.0	−3.4	−3.6	−3.3	− 7	+0
26	−5.2	−3.9	−3.5	−4.5	−2.3	−3.8	−4.2	−3.7	− 7	−2
27	−6.4	−4.3	−4.5	−5.4	−3.3	−5.0	−5.3	−4.6	− 9	+0
28	−6.3	−4.0	−3.5	−5.1	−3.0	−5.0	−5.3	−4.8	− 9	+2
29	−5.7	−3.0	−2.1	−4.3	−2.2	−4.5	−4.8	−3.5	− 7	+0
30	−5.5	−3.0	−1.4	−3.9	−1.8	−4.3	−4.6	−3.3	− 7	+1
31	−5.6	−3.0	−1.6	−3.9	−1.5	−4.5	−4.9	−3.2	− 6	−1
32	−5.8	−2.9	−1.8	−3.8	−3.2	−4.7	−5.2	−3.6	− 7	+0
33	−5.0	−3.1	−1.0	−2.9	−3.1	−4.2	−4.6	−3.5	− 7	+4
34	−2.9	−1.9	−0.2	−1.4	−1.6	−2.1	−2.6	−1.6	− 3	+1
35	−1.8	−1.1	−0.8	−0.8	−0.9	−1.4	−1.5	−0.8	− 2	+2
36	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+0.0	+ 0	

The method of deriving the quantities given in the above table from the data given in the previous table will be obvious from the following illustrations:

* Excluding (1).

FOR THE INCH SPACES.		FOR THE 8 INCH SPACES.	
From measures of ½ inch spaces.	From measures of inch spaces.	From measures of ½ inch spaces.	From measures of inch spaces.
Δ Σ	° sum.	Δ Σ	sum
+ 2.5 + 1.0 + 1.0	+ 1.9 + 2.9	+ 2.5 + 2.4 + 2.4	+ 0.6 + 3.0
+ 0.5 − 1.0 + .0	+ 3.9 + 3.9	+ 0.5 + 0.4 + 2.8	+ 1.3 + 4.1
+ 1.5		− 0.1 − 0.2 + 2.6	+ 2.0 + 4.6
		− 1.2 − 1.3 + 1.3	+ 2.7 + 4.0
		+ 0.4 + .2 + 1.5	+ 3.4 + 4.9
		− 1.4 − 1.5 + 0.0	+ 4.0 + 4.0
		+ 0.12	

Since the greater part of the error of the screw is at the end from which the accumulated errors have been reckoned, the relative errors have been determined of the first ten revolutions of the screw, assuming the length of the first half inch to be correct. The results are given in hundred-thousandths of an inch.

Spaces.	Σ
1	+ 14
2	+ 10
3	+ 28
4	+ 6
5	+ 3
6	− 14
7	− 12
8	− 20
9	− 14
10	+ 0

The errors depending on single revolutions of the Waltham screw were found by measuring ten ruled spaces corresponding to even tenths of a revolution. The results for the Cornell screw were kindly communicated by Professor Anthony. The figures represent millionths of an inch.

Spaces.	Cornell Screw.	Waltham Screw.
	Σ	Σ
1	− 2	+ 10
2	− 2	+ 14
3	− 8	+ 10
4	− 9	− 2
5	− 8	− 17
6	− 12	− 33
7	− 14	− 34
8	− 11	− 33
9	− 2	− 21
10	+ 0	+ 0

The nature and magnitude of the errors of these screws will be more clearly seen from the curves upon the following plate, which have been described from the data given.

It will be noticed that there is a dotted curve in Fig. 55. The difference between the two curves represents the effect of a demand of my landlord for an increase of rent, in consequence of which the dividing engine was removed into new and, it may be added, more commodious quarters. After it was remounted, it was found that the old system of errors no longer held. The present discussion refers to data derived from observations made since the removal.

It will be seen that there is a pretty close correspondence between the curves for the English and the metric subdivisions. The former of course includes errors to a certain extent depending on single revolutions of the screw.

One more fact should be stated in order to complete the argument in favor of the new method, and it is a fact of supreme importance. At the time the line bar was graduated from which the Cornell screw was cut, the writer had not succeeded in completely eliminating the accumulated errors. It would be too much to expect that there should be a complete coincidence between the errors of the bar and the errors of the screw, but the coincidence is nevertheless so close that it would make but little difference whether the accumulated error of the bar at the middle point was derived from the screw, or that of the screw from the bar.

It has already been stated that the method has only been tried under great disadvantages, but the Ballou Manufacturing Company have now completed a screw machine having a direct capacity for screws 6 feet in length and an indirect capacity for screws having a length not much exceeding 18 feet. A smaller machine for screws less than 6 feet, and especially adapted for cutting all kinds of micrometer screws, is in process of construction. This paper is already by far too long to admit of a description of these machines, but Mr. Bishop, the president of the company, will gladly offer every facility to any member of the society who desires to make a personal examination either of the method or of its results. You will receive, I am sure, a hearty welcome and courteous attention at the office in Hartford.

DISCUSSION.

Mr. Oberlin Smith.—I want to ask a question of Professor Rogers. I understand from him that one of the great practical difficulties that would seem natural in all this work is the deflection by gravity of horizontal bars used for the guides, whether they be cylindrical or whether they be prismatic. My question is, whether experiments in such work have been made having the screws and slides all vertical, so that deflection would not come in as a factor at all, and having the nut and other moving parts attached to it balanced about the axis of the screw, so that as it traveled up and down there would be vertical stress only.

Prof. Rogers.—Experiments have been made in this direction, but not, as far as I am aware, in this country. Professor Wild, of the Central Physical Observatory of St. Petersburg, has investigated the problem. He found that the flexure—that is not quite the proper term in this connection—he found that the change which in some way takes place when a bar is supported vertically is nearly or quite as troublesome as when it is placed in a horizontal position. Simple flexure can always be neutralized by proper supports.

The required degree of precision must be secured under the ordinary conditions in which screws are used. When a stiff screw three feet in length is supported at the middle point the effect of flexure will be eliminated in ordinary practice.

But in the new screw machine embodying the methods which have been described, a very neat device has been adopted by which the flexure can be eliminated at any desired number of points, so that the element of length does not now enter as a disturbing cause.

Mr. Towne.—Mr. President, without doubt it may seem to some persons present that this discussion has reference to what may be termed mathematical theory rather than to ordinary machine shop practice, but a matter in my own experience shows that this is not so. In cutting some screws recently for use in "Emery" testing machines, it became a matter of importance to produce two screws of identical pitch, as nearly as the same could be reasonably done. The screws have a diameter of three and one-half inches and a pitch of two threads to the inch. A lathe for cutting such screws was made especially for us by the Pratt & Whitney Co. (who gave great care to the work throughout), having unusual solidity and stiffness,

and having as a lead screw a copy of a Whitworth screw which Pratt & Whitney then had, and which had never previously been used, and of which they made as careful a copy as they could. After cutting the first pair of screws in this lathe, and chasing a pair of bronze nuts for use on them, and assembling the machine, an error in the screws was shown very apparently in this way: The two screws stand vertically in the machine, and carry a horizontal cross-head. On each screw is a pair of bronze nuts, one above and the other below this cross-head, the separation between the nuts on each screw being, approximately, nine inches. The work is all very carefully fitted, and, in the position when they were first assembled, the nuts, while in contact with the cross-head on each side, were still free, and there was no binding anywhere. On turning the screws, however, so as to raise the cross-head slightly, thereby bringing the nuts into a new longitudinal position on the screws, the nuts in one case were found to be separated—so much so that there was considerable freedom between the nut on the top of the cross-head and the cross-head itself; and in another position the distance was decreased, so that motion of the nuts was stopped. The total error, if I remember correctly, was something like six or seven thousandths of an inch—quite enough, with close work, to make binding. So that, in what may be termed ordinary machine shop practice, this matter of precision of screws comes in very frequently, and it is interesting to all of us to see how great precision is going to be obtained, not only in fine instruments, such as we see here, but I hope also in tools of more frequent use.

I wish to ask Professor Rogers one question, due, perhaps, to my not following him more closely. If I understand his process correctly, it differs from what has preceded it chiefly in the reproduction in the screw of the divisions of the plate or bar. The correction of each thread of a finished screw is not novel, as I understand, but it is the reproduction on the screw in cutting it of the divisions of the bar. Am I correct in that?

Prof. Rogers.—You are partly correct and partly not. The novelty of the process—I will not say novelty, the process itself—consists in a method of cutting single threads in such a way that we know just what is being done during every step of the operation.

If my screw has twenty threads to the inch, and my bar is subdivided to twentieths of a revolution of the screw, then for every entire thread cut I must maintain a coincidence between the line

upon the bar and the fixed line of the microscope. If this coincidence is not maintained, the screw which is being cut will have an error of just that amount at that point. In the ordinary process, you really do not know whether you are cutting a single thread correctly until the entire operation is completed. We deal with single threads, and in such a way that no one thread can ever have an error greater than can be ground out by means of a lead nut. The screw exhibited was cut from a bar having half-inch graduations, but the subdivisions of the bar now in use with the new screw machine are tenths of inches. As the leading screw has five threads to the inch, the errors can be obtained for every half revolution.

Attention is called to the fact that as long as the screw which is being cut has the same temperature as the leading screw it will have the same absolute length as the graduations of the bar from which it was cut. Since in the new machine the entire shaft is immersed in oil, the trouble with temperature is reduced to a minimum.

Mr. Towne.—If I understand correctly, then, in tracing the first line of the thread, we will say, that is, the first revolution of the screw, and having made one exact revolution, you then compare the progress of your line on the work with the divisions of your bar. Am I correct?

Prof. Rogers.—Yes, sir.

Mr. Towne.—If you find an error in that thread, how is that error corrected? Perhaps you touched on that point in the paper.

Prof. Rogers.—Yes, I thought I had done so. It must be remembered that the cutter has two motions; a primary motion and a secondary motion. The primary motion is the movement of the carriage by means of the leading screw; the secondary motion is the movement of the slide to which the cutter is attached through the short and firm micrometer screw.

There is a certain amount of preliminary work, however, which must be done before the screw is cut. We must first ascertain how far the lines upon the bar are away from the fixed micrometer line of the microscope, for even revolutions when the leading screw runs free, *i. e.*, without doing any work, and secondly, we must ascertain whether the errors are substantially the same during every comparison; for unless this constancy is maintained it will be impossible to eliminate the errors of the leading screw. The tabular corrections which have been derived from observation

cannot serve as a guide unless they are practically constant for every repetition of the observations.

I will repeat the various steps of the operation. The carriage is now at one end of the lathe bed, and the index of the leading screw is set at zero. Upon looking into the microscope, I notice that the first graduation upon the bar does not coincide with the fixed line of the eye-piece micrometer of the microscope. I slightly tap the bar at one end, and the coincidence between the two lines is perfect. A heavy bar can be moved into any required position by a series of light strokes with greater accuracy, and much more quickly than through the action of an abutting screw. I now allow the leading screw to make, *e. g.*, nearly five even revolutions, and by a hand movement I complete the fifth revolution. Again looking into the microscope, I observe that the line upon the bar is on one side of the micrometer line of the microscope, and I note how many divisions of the index of the secondary screw are passed over in bringing the secondary slide to which the microscope is attached into coincidence with the second line upon the bar. I record this number upon the slip of paper before me, which for convenience is pasted to the vertical face of the graduated bar. This operation is repeated for every five revolutions of the leading screw. I shall then have a system of corrections, some positive and some negative, which represent the errors of the leading screw for the conditions under which the observations were made. But these conditions are in many respects unlike those which hold while the cutting tool is doing its work. I therefore repeat the observations with the cutter at work upon a dummy shaft. In ordinary practice, however, it is not found necessary to take this precaution.

Having done this preliminary work we are now ready to cut the thread upon the cylinder which has been properly prepared in advance. The carriage is run back to the first position, and coincidence is made between the line upon the bar and the fixed line of the microscope after the cutting tool has been properly adjusted. The leading screw is thrown into feed, and during the movement from the first to the second line of the bar, the secondary micrometer screw is moved, by a continuous motion, the number of divisions of the index which corresponds with the previously determined error of the leading screw between these two points. To facilitate this movement the slip of paper upon the vertical face of the bar is divided into half-inch spaces, and a pointer is attached to the moving carriage. During the slow motion of the screw, the pointer

will serve as a guide to the eye in making a uniform distribution of the total error over the entire space. Stopping the lathe at five even revolutions, I find that the coincidence between the lines is now maintained. Starting again, I apply the correction required for the second five revolutions, and so proceed till the first tracing of the thread is completed. Every subsequent operation is a repetition of the first. It is to be noted that I am describing the actual process employed in cutting the first screw upon a Blaisdell lathe, but in the new screw machine the secondary motion of the cutter is under the complete control of the workman.

Having cut the screw, what do we find? First, that there are here and there slight irregularities in pitch, due to the fact that the errors of the leading screw have not been quite properly distributed between the points at which the errors have been determined. Just here is the vital part of the operation. By the ordinary process long sweeps of errors are introduced. You may grind till doomsday without removing errors of this class, but these single errors, these minute irregularities will disappear after a few moments' grinding with a lead nut charged with fine emery. Secondly, I find that the entire length of the screw corresponds with the distance between the terminal graduations of the bar. It is true that there has been a slight forcing of single threads, but there has been no such forcing of a combination of threads as takes place in ordinary practice, when the attempt is made to change the general pitch of a screw.

Mr. Oberlin Smith.—Then, sir, as I understand, the practical process consists in a secondary screw which corrects the errors of the first screw, thread by thread, by being moved in some definite relation to it, that relation varying with each individual thread, each of which has a personality of its own, and you are in the habit of doing that by hand. Of course, the lathe must move very slowly at present, but you are going to do it automatically at some future time.

Prof. Rogers.—Yes, sir.

Mr. Oberlin Smith.—What kind of gear do you use to connect the secondary screw with the primary so as to get that variation of motion?

Prof. Rogers.—In a general way the secondary motion is obtained either by means of a worm gear which moves the carriage itself upon the screw, or by a combination of circular gears working about different centers. There was a patent by the Putnam Machine

Company which prevented our using a secondary movement of the tool itself, even if we had designed to do so. To get around the patent—which it is always necessary to do [laughter]—we really found a much better way, by simply moving the carriage upon the screw itself. There is an adjustable sleeve—but really I cannot describe the apparatus without the drawings, and perhaps not, even if I had them here [laughter]. Mr. Ballou, my colleague in this investigation, is the mechanician.

Mr. Oberlin Smith.—That is what we call castigating a certain old gentleman around the remains of a tree. It is very common with mechanical engineers, and I believe it is very good engineering [laughter].

Prof. Webb.—Mr. Chairman, I think that is one of the great advantages of our patent system, that it stimulates every one, in endeavoring to evade an existing patent, to make something better than was made before; I know it has occurred in a great number of instances that something better and cheaper has been arrived at.

But it was another point that I wished to speak on: During the last two or three years, in looking around the machine shop of Cornell University, I have repeatedly met with a number of interesting mechanical appliances and ingenious mechanisms, and have inquired to whom they were due. I have always had as answer the name of a certain gentleman, an honored member of this society, who was formerly there. One of those things is a machine that will measure very accurately by ten-thousandths of an inch, from 12 inches down, and as that belongs to ordinary machine shop practice, I should like to hear from our president some remarks as to the applicability and importance of this new method of making screws in ordinary machine shop practice. I believe myself that it is a valuable thing for the machine shop. I have heard that certain firms have already expressed their intention, if this is a success, to turn out their old screws and put new ones into their lathes in order to have a systematic method of measuring. I cannot see that any further progress in accuracy of measurement is possible unless that is done, and I believe that the true method of measuring in a lathe is to have all the screws in that lathe graduated and to use them for the measurement of the work. I should like to hear upon this point from those acquainted with ordinary machine shop practice.

Mr. Hand.—Mr. President, if my memory serves me rightly, about six or eight years ago, you and I had something to do with a

pair of Whitworth screws on a measuring machine. They were very much out of truth. While you are telling about what Prof. Webb referred to, I would like to ask you to tell about the device you put on the machine, to eliminate the errors of those screws. At that time we knew nothing about this grinding process, and when we got stuck in the making of gauges, for the want of a perfect screw to measure with, you kindly came forward and helped us out by applying the corrective device referred to.

President Sweet.—I hardly think it within the scope of this discussion to branch off into the question of measuring machines, unless it be the will of the meeting that we deviate from the title of the paper. It seems to me there is not much to say when we confine ourselves to the perfect screw. It has been made more perfect in the screw described than anything ever made before. But when we get the perfect screw we have got to use it, and it is desirable that we shall maintain its accuracy: for that is an important thing. If our screw is made perfect, and we go to use it and it soon wears out, we are pretty nearly as badly off as if we had never had it.

Professor Rogers referred to his hobby. My hobby has been to get equal length of wearing surface wherever I thought it was practicable. I applied that to the measuring machine referred to by Prof. Webb by making the screw and the nut of the same length, believing that if we ground them together in that form, we would maintain better results than if we put a short nut on a long screw. I was not present to hear that part of the paper, but I believe it was stated that when they came to grind their screw they were all right until they got pretty nearly to the end; then they met with difficulties that caused them more trouble and more time to eliminate than all the work done up to that point. To give a fair illustration, let us imagine we have an edge which in the main is perfectly straight, but throughout its entire length there are slight undulations. Let us take a short piece of like character and try to grind those slight undulations out. The result will be just as it was with the screw. It will be all right through the center, but when we come to the ends we meet with the same difficulties—that they do not disappear, or you grind the end off too much or not enough. Now, let us take two straight edges of the same length with the same undulations, but with the undulations in one of different pitch from those in the other. If these be ground together, what will be the result? We will soon fetch down our two

straight edges, and they will be practically straight, as well on the ends as in the middle. That was the plan I adopted on the screw—to make the screw and the nut of the same length, and make them work back and forth. The screw and the nut were three inches long, and our measurement was only one inch. In our machine it made no difference to us whether we had sixteen threads to the inch or more or less than sixteen threads. We did not care whether it was 16 plus or 16 minus, because we had to read to a line, and that line could just as well be a spiral which would correct the error as to be a straight line parallel with the axis of the screw. We found our screw, if it continued 62 feet, would have one too many threads, so that by making the line a spiral that would make one complete turn in sixty-two feet, the error in the pitch was corrected.

The same principle was applied to the Richards machine, only in their case they have a short nut on a long screw. In that case, instead of making the line equivalent to a perfect spiral, the line had to be more or less curved, because those errors came in at the ends of the screw, when the short nut got up to the end, and the line had to be a curved line rather than a spiral.

Now, in applying this perfect screw to practical purposes, as I understand, they propose to make the lead screws of lathes.

In using a lathe for the ordinary purposes of a machine shop, you all know very well it will not be long before we have it worn down at the head-stock end, and not worn at all at the foot-stock end. We know we cannot make a screw and nut of the same length, and what shall we do? It is easy to do better than we do now, if we only had the courage. Cut away the threads at the foot-stock end—nine threads out of ten, eight out of nine, five out of six, etc., cutting out the most where the screw wears the least. This, of course, could not be determined accurately, because we do not know how much the screw is to be used on long and how much on short work, but it would help the matter very materially. None of us have the courage to cut away the threads of a nice screw. I am having a lathe made, and putting in all the notions I have thought of—a good many more notions than improvements, possibly—and though I have a good deal of courage in mechanics, I have not enough to make a lead screw as it ought to be made [laughter].

Mr. Bond.—Mr. Chairman, I would say in regard to the correction of that screw referred to by Mr. Towne, which I had the honor of superintending, we found errors in it that varied all the

way from minus .004 of an inch to plus .005. We determined to get these errors reduced as much as possible, and we ground three weeks on the screw in the attempt. It is 38 feet long and three inches in diameter, two threads to the inch, or one-half inch pitch, and a square thread. The sides being at right angles to the axis of the screw, made it very difficult to grind, but we managed it by taking two half nuts and lining them with a lead bushing. Having nuts about twelve or fourteen inches long, gave considerable length to the thread. We ran one nut back and forth the whole length of the screw to even up the thread. We then found there was very little difference in the tightness of the nut from one end of the screw to the other. Then we introduced a space between two nuts of about twelve inches, in order to separate them, and we found it went very easily in some places. We ran it along two or three days until we thought it was even, meanwhile measuring it. By introducing another length of two feet we still further separated these nuts to change the conditions, and then finally one of three feet was added, and we got it so that in all positions the nut apparently had the same resistance. We measured the screw, and found that the general errors had been reduced, as nearly as we could measure it, to about one-thousandth of an inch, either plus or minus. Of course, as the flexure of the screw came in, we measured it on all sides. In grinding this soft iron screw the sides of the thread were filled with emery. I think since the grinding there may have been a slight change in the pitch up to a certain time, but its use afterward always gave the same results. The screw that was cut by it and then measured had the same errors in it that the original screw had after grinding. We only use this long screw for screws that are nearly the length of the machine itself. We have shorter screws for doing short work, and we are in hopes of getting in time a perfect screw in this lathe. The use of this screw is only an expedient until that time comes.

Prof. Webb.—I suppose, to put it briefly, that Professor Rogers is introducing a new breed of screws. Now, we know that the agriculturists, when they have a fine animal, do not put him to hard work. They take care of him ; and yet they will have fine animals, and they pay the money for them, and I suppose that mechanical engineers, if they do not use a perfect screw where they need it, now that one is obtainable, will be worse off than the farmer is. An idea has occurred to me as to how they might be used. It is quite possible to have these screws so arranged that any part

of them can be used at will. You would use your lathe for accurate work only, and would keep account of the screws you cut, making the record show as nearly as possible how much the screw was worn by each job. In this way the wear could be very evenly distributed. There are various ways of fixing it so that all the screw can be used, and used regularly.

Mr. Reese.—Professor Webb has said that this is like the introduction of a new breed of chickens. It just occurred to me that if Professor Rogers proposes to introduce a new breed of chickens, some attention ought to be paid to the rooster [laughter].

I have been surprised that in this discussion and in this paper, which is a very good paper indeed, no attention has been paid to the molecular physics of the bar, which I believe will have something to do with the generation of high-bred screws. It seems to me that it would be impossible to get a uniform series of screws accurate unless you had a uniform structure of metal to start with. I presume it is expected to make those screws out of steel. Now, we all know that it is a very difficult matter to get a uniform bar of steel—a bar in which the molecular structure is uniform at all points. It is very difficult to get a bar of steel in which the carbon is uniformly distributed throughout the bar. Variation of temperature varies the molecular action. It varies the expansion and contraction—not only in its entirety, but in any point of the bar.

Some experiments that I made some years ago that are not yet complete may be referred to. I am sorry that I have not had the means to continue them and make them complete. Still they are complete enough to make a reference to them. They go to show that the shape of the molecule in the molecular structure of steel, and, in fact, of all metals, is spherical, and that, in the act of rolling, the spheres are drawn out along the line of their long axes, and that this is not their normal position in a bar of metal; it is abnormal. The spheroids ought to have their long axes across the bar. Now, I have taken a bar that was rolled in the ordinary way, with its molecules having their long axis running parallel with the length of the bar, and found its tensile strength to be 109,486 to the square inch. This bar when annealed stood 107,486 lbs. to the square inch. The bar exhibited an elongation of 10 per cent. in 8 inches before annealing, and 16 per cent. after annealing.

Another bar of steel rolled in the ordinary manner to 1.012 diameter, exhibited a tensile strength of 109,486 lbs. to the square

inch, and an elongation of 10 per cent. in 8 inches before annealing. This bar was passed through my machine, which caused the bar to rotate at high speed and under great pressure. The effect of this action was to twist the molecules so as to leave their long axes across the bar. This bar was rolled cold, its diameter was increased to 1.017, its tensile strength was reduced to 107,972, and its elongation increased to 20 per cent. in 8 inches. This bar was then annealed, but the annealing had no effect on its tensile strength, or its elongation before rupture. Annealing has no effect on a bar of steel in which the molecules are in their normal attitude. A perfect screw cannot be made from a bar of steel, in which the molecules do not exist in their normal attitude. The steel must possess a uniform physical structure, in which the molecules exist in their normal attitude, with their longer axes uniformly parallel to each other, before Professor Rogers can hope to secure the end he has in view.

Prof. Rogers.—Mr. President, if you will allow me—I do not like to take up so much of your time—but I have touched upon that problem. One of the most illustrious astronomers who ever lived, one who has made the largest contributions to our knowledge of the positions of the stars in the heavens, once said that "one is liable to fail of reaching practical results if the range of inquiry is extended too far." If we are compelled to determine the changes which take place in the molecular structure of steel before a perfect screw can be made, I fear it will be yet many years before such a screw can be made. This problem is an exceedingly interesting one, but fortunately it has but little bearing upon the ordinary behavior of steel under the changes of temperature, which ordinarily occur. But since this subject has been introduced I may say that I have during the past three years attempted to ascertain the effect of change of temperature upon the molecular structure of steel, copper and glass, by investigating the constancy of the coefficient of expansion of these metals.

I have two bars of glass having the dimensions $41 \times 1\frac{3}{4} \times 1\frac{3}{4}$ inches made for the Standards Department of the English Government by Chance & Sons in 1870. One of these bars was presented to me by Mr. Chaney, Warden of the Standards. I was allowed to bring with me from London a duplicate bar for the purpose of graduation. I have for two years studied the constancy of the coefficient of expansion of these bars. I cannot answer for the changes which may have taken place during the ten years in which they were allowed to assume a normal condition, but it is in my opinion

very certain that these bars of glass have at the present time a constant co-efficient of expansion.

The observations which have now been continued for nearly three years with bars of copper, brass and steel of various grades, all point to the constancy of their co-efficients.

The argument for that change of length in metals which is a function of their age is, as far as I can learn, based upon the following observations. First, that the zero point of a thermometer always rises during the first one or two years of its life; and, second, that certain standards which have been compared at wide intervals of time give evidence of an absolute change in length. With regard to the argument from the recognized changes in thermometers, it may be said that a rise in the zero point does not by any means indicate a change in the structure of the glass tube. In this case we have simply the magnified effect of a thin shell of glass upon a comparatively large mass of mercury. With regard to the observed changes in absolute length, it is safe to say that the observations are not in a single instance conclusive. Take for illustration the various comparisons of the U. S. Standard Yard "Bronze 11" which have been made with the Imperial Yard. In 1855, "Bronze 11" was assumed to be about one ten-thousandth of an inch *too long*. In 1879, it appeared from the observations of Mr. Chaney and Professor Hilgard to be 88 millionths of an inch too short. But according to the last report of the Standard Department, the observations of Professor C. S. Peirce, made during the summer of 1883, gave the relation, "Bronze 11 + .000022 inch=Imperial Yard. If we can trust the comparisons, we have had therefore in this instance both a decrease and an increase of length during the last thirty years. I am very confident that no change at all has taken place. Probably we must refer the apparent change to the different kinds of illumination under which the defining lines were observed in the different series of observations.

With regard to the experiment upon steel to which the last speaker has referred, it must be said that the seven seconds of time mentioned ought to have been at least seven hours. Any change of temperature in the entire mass of a given bar of metal is a function of that mass and of the time of exposure to a given temperature. If a brass bar with the dimensions $41 \times 1 \times 1$ inches and having the temperature, *e. g.*, 50° Fahr., is removed to a room in which a constant temperature of 70° is maintained, four or five hours must elapse before the bar will assume its normal condition

in the latter temperature. Contrary to the general impression, the effect of this change of temperature will be very slow for the first five or ten minutes.

The effect of the presence of the observer in a comparing room upon a bar of this kind will not be perceptible for ten or fifteen minutes. I can even handle the bar with impunity during the first five minutes. But the effect of a similar change of temperature upon a bar of small section will appear at once. Hence in my practice I have everything in readiness to make the first comparison of the bar having the least mass within one or two minutes after entering the comparing room. I then compare the bars in the order of their masses, allowing the limit of fifteen minutes for bars having the dimensions just named. When the comparisons are completed I leave the room quickly, and do not enter it again for about five hours. During this time the slight increase of temperature due to the presence of the observer will have been taken up by the bar. It would seem that in the experiment which the last speaker described, the mass of the bar was not sufficiently considered, and that the length of time of exposure to a given temperature or rather the time during which compression was taking place was not noted with sufficient accuracy.

I have not yet seen any decisive evidence of a molecular change in the structure of metals under moderately slow changes of temperature. Certainly this is the result of my experiments thus far. I venture the prediction that the result of the discussion which will assuredly take place during the next ten years will be, that with the exception of a few composite metals, *e. g.* zinc, all metals assume a normal condition soon after they pass from a molten to a solid condition, and that we may safely dismiss the consideration of all changes which in any way depend upon the age of the metal. Certainly, we need not take account of this source of disturbance in the production of screws.

Mr. Woodbury.—Mr. President, at the first meeting of the Society in New York a paper was read criticising the metric system, and at that time a statement was made that there was not a single leading screw in the world graduated on the metric system, and I should like to ask if that is true to-day?

Prof. Rogers.—No, sir, it is not true. As I said before, Mr. Van Woerd constructed a screw for me on the metric system. A metric screw by Froment of Paris is now in the office of the Coast Survey at Washington.

Mr. Woodbury.—Do any French tool makers use the metric system on their lathes?

Prof. Rogers.—Always.

Mr. Kent.—Mr. Reese just told us that he has branched into the domain of molecular physics. I think he has made a slight mistake in the name. It is molecular metaphysics. Physics can carry us only about as far as Professor Rogers carries us when he says, "A piece of mechanism of the class which the French would call mechanism of precision may be termed perfect when it meets all the requirements of the purpose for which it was constructed." That is as far as physics, I think, to-day can carry mechanical engineers; but philosophers and scientists may be carried further when they trench on the domain of molecular metaphysics, with which, I think, this Society has nothing to do.

Mr. Towne.—Mr. President, I wish to add just one word relating to the Whitworth screw made by Pratt & Whitney. There is no doubt but what extreme care was taken in the production of that screw, as has been said by Mr. Bond, and I believe we have in that screw as perfect a large screw as there is in this country. I appreciate the force of the suggestion which the president has made that later work done with that screw will probably be better than its first work.

Mr. Bond made reference passingly to another feature in this lathe which may be mentioned a little more fully, and I may be pardoned for doing it, as it was, I believe, at my suggestion that it was incorporated. That is, providing for a *second lead screw* in lathes of this kind, for doing short work. In our lathe the main screw is 28 feet long, I think, and is on the front of the bed. Behind the bed is a shorter screw, about 10 feet in length, cut in the same way, and corrected, I believe, in the same manner, the purpose of which is to cut nuts of short screws, and in that way obviate unequal wear on the main lead screw, and keep that for cutting long and fine screws, using the other screw for less important work and for short nuts. I think in any lathe to be used in ordinary machine shop work, particularly for cutting long threads, it is exceedingly desirable to have that secondary screw for cutting the nuts which are always required for such work, and for doing other short and unimportant work.

Mr. Bond.—Another lathe which might be referred to is made specially for cutting leading screws for lathes. It has two lead screws, and it is intended to use one to cut a certain length of

lathe screw, and the other for longer lengths, and in that way there is a tendency to make the wear uniform the whole length of both screws. It will take time to do this for all lengths of lathe screws, but that is what will be required.

Mr. S. W. Robinson.—Mr. President, I have two questions that I would like to have settled in my own mind. Possibly one of them has been settled already in the reading or in the discussion. If so, I did not notice it particularly. One is as to errors occurring in a single turn of the screw—a single thread—a single revolution. Professor Rogers speaks of using a microscope and micrometer screw to correct the position of the cutting tool, as I understand, for errors in the lead screw, as the work goes along. I heard reference once to divisions of a revolution into parts of a revolution—20ths, and so on. I did not quite catch whether a graduated standard bar was actually used or contemplated in making these screws, the graduation of which was fine enough, say, to divide the pitch of the screw being cut into 20 parts, and then stop at each graduation mark, or not to stop, either, but watch for the 20ths of the revolution and adjust the position of the tool at each mark. There might be a signal, for instance, giving notice of the approach of each 20th of a revolution. Every screw revolves so slowly in the cutting that there is time for a signal to be made and the observer to watch his opportunity and adjust the thread in each 20th of a revolution, if you please, or in any other number of divisions. It is evident that it can be carried to that extent, and I am not certain whether it was intended to be explained as if carried to that extent.

Now, this subject touches, perhaps, a point of interest in regard to the permanence of metal; that is, with respect to the bar, the permanence of the bar itself. If we could easily obtain a bar which could be divided into 20ths of a thread for its whole length and be perfect, it strikes me that this method of cutting a screw would result in cutting a perfect screw, that is, perfect within our present means of measuring, or present notion of precision in such matters, to say the least.

As to the permanence of metals, that question, if it is touched upon, seems to be an important one here, not only to the standard bar, but to the screw which is to serve as a standard. A screw that has been made with great care, of course, should be permanent as to its constitution and dimensions. Dimensions are the leading point. If there is a change in the physical constitution in any way,

such as in the relation of the molecules, there would be a change in the dimensions of the screw. I was once struck with a remark in a letter I received from a young man employed at the Elgin Watch Works. He said that there, in making mandrels that they desired to use in work of great precision, they were in the habit of getting out the blocks of steel nearly to size, tempering them, if they were to be tempered, then laying them away a year or two to season. Now, it seems to me that if there is anything in that, if it is necessary to season steel, it is quite necessary that these standard steel screws should be "seasoned," and also the standard graduated bars to be used in guiding the screw cutting, whether it be carried to the extent of divisions of the revolution or not. I would like to ascertain what has been done in regard to those two points.

Prof. Rogers.—The 6 foot screw in the new machine which has been referred to was cut in the way which Professor Robinson has described. It was not quite feasible to employ the principle of reversing the half nuts in a screw having this length. Since the provisional leading screw was cut five threads to the inch and the bar was graduated to tenths of inches, it is obvious that the periodic errors depending on half revolutions of the leading screw were eliminated, but this operation is rather tedious and requires a great deal of care. Hence, in the new machine, there is an automatic device for correcting the periodic errors which depend upon single revolutions of the screw. The larger share of the errors of this class are eliminated by means of a template. For the elimination of those which remain, dependence must be placed upon the principle of the reversal of a nut divided into sections. Professor Rowland has not yet published a description of his process of correcting a screw, but it is understood that he divides the nut into four sections. A short nut of this kind has recently been made for me by Mr. Ballou, and its action is very satisfactory. I suppose that Professor Rowland has anticipated me in the use of a nut having four sections. I may or may not have anticipated him in the use of one having two sections. When these sectional nuts are placed in position upon the screw, the faces of the threads of the screw and of the nut are everywhere nearly parallel. They may even touch at every point, but if there are periodic errors in the screw, the relation between the threads of the nut and of the screw will vary in every part of the revolution. By repeated reversals of the sectional nuts, therefore, the tendency will be to equalize the errors at every point.

Mr. Kent.—There has been a suggestion made about moving the leading screw along the lathe from end to end, and letting it project beyond the ends three or four feet, in order to have the nut move at all parts of the leading screw. I have a suggestion to make as to how that might be accomplished without moving the leading screw. Let the leading screw revolve in its own place all the time, and have a secondary bar or shaft above it, which may project beyond the end of the lathe, to which the tool post is attached. The nut of the leading screw is to be clamped to this shaft in any part of its length, and the position of the nut on the screw changed from time to time so as to wear the screw uniformly. The shaft slides between bearings so that it shall always pull in a straight line (illustrated on the blackboard).

Mr. Porter.—I would like to make one inquiry of Professor Rogers, and that is whether any attempt has ever been made to remove the periodic errors by a system of scraping instead of grinding. The reason I ask that question is this: It is obvious that when a perfect screw has been produced it is desirable that it be maintained as long as possible, that it shall have as long a lifetime and be capable of as much service in producing other perfect screws as possible. Now, any grinding material whatever finding its way into the metal makes it a permanent lap. If that can be avoided, it seems to me that a longer lifetime would be insured to the screw, and it has seemed to me, without having gone into the subject in my mind very carefully, that a system might be devised by which high points might be detected upon the face of the screw in the same manner as they are detected upon a plane surface and removed by scraping, which can be done in a manner exceedingly delicate. My inquiry is whether any attention has been given to that—whether any effort has been made in that direction.

Prof. Rogers.—Yes, the experiment was tried by Mr. Van Woerd, but without success. I do not think the plan would be possible. We do not deal with plane surfaces in cutting a screw. We deal with helices.

The objection to the use of emery in grinding is not a serious one in practice, especially if the steel is hard. We shall yet have hardened steel screws. Tempered steel behaves quite as well as annealed steel under variations of temperature. I regard the coefficient of my hardened steel standard as more securely determined than for any other metal except perhaps for copper.

Mr. Bond.—It may be hardened outside, and not in the center.

That is possibly the effect of a change of the co-efficient of expansion, because the main body of the bar is in a normal condition, and probably not very hard. We know that in standard cylindrical gauges that are made of tool steel, there are changes that come about that seriously affect the diameter of the ring which fits the gauge, but it is only in steel that is of very high carbon.

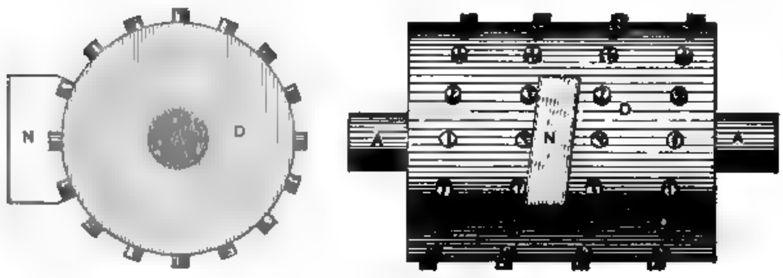
Prof. Rogers.—I shall still be obliged to say that I do not believe that the action which Mr. Bond describes is the result of a real change. All of my observations point to an absolute constancy in length soon after the steel is tempered. It cannot be possible that the effect of a change of two or three degrees in temperature at the surface of a steel plug two inches in diameter, will be to change the absolute diameter within a few seconds of time. Is it not more reasonable to suppose that the apparent increase of diameter may be due to the development of surface heat through surface friction in inserting the plug into the ring?

The principle of conservation of forces applies here as elsewhere. A change in the entire mass requires a certain amount of time, but there is some evidence that strictly surface changes may occur, and it is possible that the observations made by Mr. Bond may be explained in this way. The experience of Alvan Clark & Sons in applying the principle of local corrections in finishing an object-glass gives some color to this theory of local surface changes. Even in an object-glass of the largest diameter, two or three polishing strokes with the finger will change the curvature *at that point only*, by an amount which can be instantly detected by the eye in the optical tests employed. Are we to say that the slight change in temperature produced in the operation of polishing has affected the entire mass of glass? We might more reasonably go to the other extreme and say that in the operation of polishing the *direction* of the reflecting particles of glass which compose the surface is changed, and that the actual change in position is therefore enormously magnified by the change in the direction of the rays of light which reach the eye. It must, however, be clearly understood that this notion of surface changes, which are distinct from changes which occur in the whole mass, is only a tentative explanation of a very difficult problem. These slight changes are strictly local, and cannot in any sense be considered as affecting what we call the permanent "set" of metals. The observations of Mr. E. S. Wheeler show that in the case of zinc there may be a change of "set," but not in glass or steel. But in these experiments the change of tem-

perature was very violent. The bar of zinc after having been packed in melting ice was quickly removed and plunged into boiling water. After remaining in boiling water for two or three hours the bar was re-packed in ice. Under the changes of temperature which occur in ordinary experience, the zinc bar should assume its normal length in melting ice within 15 or 20 minutes, but Mr. Wheeler found that after the bar had remained in the ice bath for 24 hours, it was still much too long, and it was only after an immersion of three days that it assumed its normal length. These experiments were very carefully made, and probably a repetition of them would give nearly the same results which Mr. Wheeler found. A small part of the amount of change of "set" may possibly be due to the fact that the bars were not wholly immersed in the bath. It is my experience that the packing of ice should cover the bar to the depth of three or four inches, but the error which may arise from shallow packing cannot much exceed five or six mikrons, or about $\frac{1}{5000}$ inch.

As far as I can learn, the positive evidence of "set" in any published record, is limited to the paper by Mr. Wheeler, and here the evidence is limited to a composite metal, viz. zinc.

Mr. Oberlin Smith.—Mr. President, I want to suggest a possible method that has just occurred to me—perhaps, though, it will all be knocked into a cocked hat by the experience of some of the gentlemen—of making a prime leading screw from which to cut other screws. Suppose we had a large cast-iron drum, D, Fig. 65,—so



Proposed Master-drum for making "Perfect-Screw". Not drawn to Scale.

FIG. 65.

large that the flexure would not amount to anything. It could be made hollow for strength and to get less dead weight. Suppose it is a foot in diameter, and four or five feet long. Suppose that into that, in a spiral line as nearly as could be drawn, there were set a

great number of round pegs, forming the thread of the master screw, the nut consisting of a piece of metal, N, resting against their sides. You could obtain any degree of accuracy you wished, by making the diameter of the drum large and by making a great number of pegs. The nut running against quite a number of them would probably give an accurate enough surface contact. Any individual errors occurring would extend through a very small angular part of the whole revolution of the drum. Suppose each of these pegs was adjusted in the direction of the axis, which could be very easily done by making the pin part which runs into the drum slightly eccentric with the part which projects out. By revolving the peg, and sighting one side of it with the microscope of the comparator, by a very easy process, without any grinding or cutting, the whole thing could be adjusted and a true spiral could be obtained, irrespective too of the pegs having an exact uniform diameter. One side of all these pegs would be a "*perfect*" screw. Now, it seems to me that that could be revolved, and another perfect screw cut from it.

Prof. Rogers.—I should say that the method proposed is new, but I apprehend that in a practical application of the method the experience of Hoe would be repeated. In an attempt to produce an original graduation of a circle, Mr. Hoe fitted 360 pieces to the periphery of a wheel, with the expectation that by making the parts so nearly alike that they could be interchanged at will, he could obtain an exact division into 360 equal parts. I have not had the pleasure of seeing the apparatus, but I should not expect success with it, on account of the large number of surfaces with which one would be compelled to deal, viz., with 720.

Mr. Oberlin Smith.—You need have no extreme accuracy in making such a screw, only in adjusting it, and we machinists do not pretend to adjust those pegs; we turn that over to you. You have a microscope and a comparator, you can turn it round and look at every peg, and adjust it to the one-millionth of an inch.

Prof. Rogers.—Without doubt you might get a pretty good screw in that way, since there would be no repetition of errors. Unless one can control the errors of a leading screw, a complete independence of a leading screw is to be desired. The members of the Society will recall the account in one of the journals of the method by which a screw was cut by the natives of India, without the help of a leading screw. It has been said that this screw is quite the equal of the best lathe screws of the present day.

Mr. Durfee.—I think the suggestion of Mr. Smith was carried out some years ago by a mechanician by the name of Andrew Ross, of London, for the purpose of making a dividing engine for dividing instruments for measuring angles. Instead of using an ordinary worm, gearing into a large worm wheel, he had what was equivalent to a worm constructed something in this way: We will suppose a cylinder, on a horizontal axis, and around that cylinder one and a small fraction of a turn of a very deep threaded screw.

Through this thread was a series of pins having hardened cast steel hemispherical ends; these were adjustable in a direction parallel with the axis of a cylinder, and each one could be adjusted independent of the others. As the cylinder was turned each of the hemispherical ended pins came in contact successively with the hardened flat cast steel ends of a series of adjustable pins projecting through short arms securely fastened to the rim of the great wheel upon which the instrument to be divided was placed. These last named pins were each placed parallel to a tangent to the great wheel. A full description of the invention of Mr. Ross will be found, accompanied with illustrative engravings, in the 48th volume of the Transactions of the Society of Arts.

Mr. Oberlin Smith.—I would recommend this over that for its simplicity, as it is a common thing that any machinist could make in "three or four days" [laughter]. How much *longer* he would be at it I do not know, but I imagine that in Mr. Bond's workshop it would not be an extremely long job. The pegs being in duplicate, he could make them cheaply, and they need not be sized very accurately, as they could fit tight in the holes by being slightly tapered.

Mr. Porter.—I fancy, Mr. President, that each one of those pegs would need to be supplied with a worm wheel [laughter].

Mr. Oberlin Smith.—The eccentricity in those pegs need be so very slight—(I don't know how much, but probably a thousandth of an inch, or somewhere there, for the holes could be drilled within that limit easily enough)—the eccentricity would be so slight, and they would move so little, that they could be revolved with a long lever [laughter], and would answer the same purpose as Mr. Porter's worm gear exactly.

Prof. Webb.—Mr. Chairman, I should put a ball joint on the pins, so as to make them stand out [laughter].

Mr. Chas. E. Emery.—I must say that it seems to me impossible

to consider the proposition of spirally arranged eccentric pins seriously. Mr. Porter's satirical suggestion is apropos. It was suggested that the pin device could be made in an hour or two, but there are few shops in the country which could do it accurately in a day or two or a month or two, if at all, as it involves that the hole for each pin be exactly radial and at right angles to the cylinder, that the pin fit the hole precisely, and that the face of the eccentric on the pin be exactly parallel. Otherwise the incline forming the nut will touch different pins at different distances from the centre of the cylinder. The difficulty is to work within one two-hundred-thousandth of an inch mentioned by Professor Rogers, when those of us who have tried know how difficult it is to find mechanics that can work within one-thousandth of an inch. The proposition as made is impracticable, but it is not improbable that the sides of fixed teeth spirally arranged could be ground separately to exact distances determined by a comparator. The teeth could be made either by driving in pins or by cutting a rough thread on a long pinion. An accurate series of faces being once determined, the interpolations would be made by making an accurate incline to form a nut.

The suggestion of Mr. Porter that the surfaces could be scraped involves the unusual difficulty of operating upon a warped surface. A grinding wheel mounted so that it would always approach the thread at a fixed angle could be manipulated to touch any point, and gradually work it back to proper position by a comparator.

Mr. Brashear (*by invitation*).—In reference to the screw made by Professor Rowland, which Professor Rogers has referred to several times, I would say that I had the privilege of examining it while on a recent visit to the Johns Hopkins University, and I may say that Professor Rogers is correct in regard to the division of the correcting or grinding nut into four longitudinal parts, and I would also say that the correcting nut is the same length as the screw, *i. e.*, the same as used by your president when making the screw at Cornell. The grinding was done under water, emery being used as the grinding material, and the water was kept at the same temperature throughout the grinding process. Professor Rowland has published a preliminary paper on the work done by this magnificent screw, whose error, if I remember rightly, is not greater than one-half the mean length of a wave of light. He has also promised a paper on the method of its construction, which, added to that of Professor Rogers, will certainly

greatly enhance our knowledge of this difficult, yet important problem. Referring to the question of abrading metal surfaces by grinding, I think we have much to learn. We know that grinding with emery has been abandoned in nearly all mechanical work, and scraping has taken its place, but this plan would evidently be inadmissible in the correcting of errors in a screw. My own experience in using emery is, that no matter how carefully the work is done on ordinary steel or iron surfaces, the particles will imbed themselves in the surface, and many of them will remain there, the microscope always revealing them. I have made a goodly number of experiments in this line, and am convinced that we can find a grinding material for metal surfaces, which will possess all the good qualities of an abrasive powder without the *dangerous* qualities of emery, corundum, etc., which, becoming impacted in the wearing surfaces, must go on grinding as long as they are in use. In my experiments I have found elutriated or washed "Arkansas Oil Stone" powder to possess the same excellent qualities as emery without its dangerous qualities, save in a very limited degree. Superior to this is a material known as "Water of Ayr Stone" or "Scotch hone," which will cut glass very readily. To prepare these properly, they are crushed as fine as possible, then put into a glass jar with water enough to stand four or five inches above the powder. An ounce or two of mucilage is stirred in the water to assist in holding the finer particles in suspension. The powder and water are then thoroughly incorporated, the lighter matter that rises to the top skimmed off, and after the top water has stood undisturbed, say, for 30 minutes, it is drawn off with a syphon to within a half inch of the settled powder. The very fine particles still in suspension in the water that has been drawn off are allowed to settle, which for the finest powder will take a day or more. The clear water is now poured back, and again stirred, and the water drawn off after standing fifteen minutes. The operation is repeated until powder may be obtained of only half a minute suspension. These powders may now be put in cases where they can be kept clean, and are labelled for use. In this way very fine and excellent working abrasive powders can be made, which I have reason to believe will be especially suited to the work of grinding accurate screws and kindred parts of fine machinery. Professor Rogers succeeds best with some sort of slate. In England, France and Germany many different sorts of slate have been used, especially in the working of surfaces on speculum metal. I think this subject one of much importance and a fruitful

field of research, and I trust that our American mechanics will fully investigate the action of abrasive materials upon metal surfaces, so as to arrive at the very best results. In reference to the question spoken of by my friend Mr. Reese, of the contraction and expansion of metals and other substances, I confess I have had to change materially my ideas of late. With Professor Rogers, I am convinced that the element of *time* plays an important part, and that rapid changes, such as can be readily seen by optical means, are, as a general thing, not changes of the whole mass by distribution of heat or cold throughout the whole substance, but simply changes of surface. Dr. Hastings has devised a beautiful test for optical surfaces by which the almost inconceivably small error of .00000002 of an inch can be detected. This test is called the color test, and is applicable particularly in testing glass surfaces. No human hand can possibly hope ever to work to such accuracy. Let two plates be placed together that we will say are optically flat. We see by angular vision a uniform tint distributed over the surfaces in contact or broad parallel bands. Let the finger or anything slightly different in temperature touch any of the surfaces, and "quick as a flash" the system of bands are disturbed, *i. e.*, in so inconceivably short time that it seems utterly impossible that the disturbance should permeate the mass, but, contracting or expanding the surface, as the case may be, the material is pulled out of shape by this contracting or expanding process. Take the case of a glass speculum, say 2 inches thick. By Foucault's method, errors of .00001 of an inch can be seen. Lay the finger on it in any spot for a moment, and a swelling is instantly seen that affects only the part touched, while the surrounding parts are disturbed very little indeed; gradually the swelling subsides. If, however, the speculum be polished immediately after this disturbance, and is afterwards examined when the speculum comes to its normal condition, a depression will be seen closely correlative to the elevation first formed. Can it then be other than a surface disturbance? These changes call up another question, *i. e.*, the idiosyncrasies of the molecular disturbances in the metal and glass surfaces we have to deal with. These are of such an unstable nature, owing to the difficulty of obtaining *homogeneous* materials, that they will perhaps be the one factor that will forever bar the way to the most perfect results. Thanking you, Mr. President and members of this association, for so kindly listening to my remarks, I will not detain you longer.

THIRTY-NINTH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE DECEMBER 17, 1884, AND LAID BEFORE THE BOARD OF OVERSEERS, JANUARY 14, 1885.

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REPORT.

To the President of the University:

Sir, — The termination of the annual subscription in aid of the Observatory and its partial replacement by the income of a recently subscribed endowment, were mentioned in my last report. The amount of the temporary subscription somewhat exceeded the annual sum of $5,000. Moreover many of the contributors paid their subscriptions for the whole term of five years soon after making them, thus enlarging their donations by a considerable amount of interest since accumulated in the hands of the College Treasurer. The expiration of the subscription, therefore, did not immediately produce its full effect in diminishing the available resources of the Observatory. But the time has now arrived when a considerable restriction of scientific work appears to be inevitable. The recently subscribed endowment, indeed, will permit the continuance of some important researches undertaken during the past five years, which without it must now have been laid aside. Still the decrease during the last ten years in the rate of interest yielded by the funds of the University is so great that the income of the new endowment of $50,000 does not fully replace the diminution in that received from the older endowments. For the ten years ending in 1876 the average rate of interest was 7.21 per cent, while for the last year it has been only 5.17 per cent. The new endowment must therefore have been as much as $70,000 to prevent a reduction in the resources of the Observatory.

The liberal response of the friends of astronomy in this neighborhood to former appeals for assistance prevents a further demand upon their generosity at the present time. The alternative of a reduction in the expenses of the Observatory must therefore be accepted. It has accordingly been found necessary to dispense with the services of five assistants, and to make a corresponding decrease in expenditures of other kinds. The retirement of so many assistants is particularly unfortunate, since the special skill which they had gained in their departments of work by the experience of several years will now be lost to the Observatory. At the same time they will be subjected to the inconvenience of adapting themselves to other pursuits, after having devoted much time and labor to the attainment of their present proficiency in the principal kinds of observation and computation which are required by the researches here pursued.

The place of one assistant has fortunately been supplied without expense to the Observatory by the courtesy of Commodore Walker of the U. S. Navy. He has ordered Ensign E. E. Hayden to duty at this institution, to assist in the work carried on with the Meridian Circle. This arrangement, it is believed, will result in mutual benefit, since an opportunity of acquiring skill in advanced scientific work is thus afforded to an officer of the Navy, while the Observatory enjoys the advantage of the excellent preparation for such work resulting from his previous education and experience.

It is to be hoped that the present restriction of the activity of the Observatory may prove to be only temporary, through the continued addition of unsolicited donations. An important bequest of $5,000 has been received during the past year from the estate of the late Thomas G. Appleton, who has thus given a further evidence of the interest which he showed in the Observatory during his life. The gift of £25 received from Mr. T. W. Backhouse of Sunderland, England, as a contribution to the subscribed endowment, also demands special acknowledgment, since it indicates that the interest taken in the welfare of this Observatory extends beyond its immediate vicinity.

Many valuable additions have been made by astronomers in this country and in Europe to the collection of astronomical photographs undertaken last year. It is hoped that before long a catalogue of the collection may be prepared.

East Equatorial.

Eclipses of Jupiter's Satellites. — Photometric observations of these eclipses have been continued upon the system adopted in 1878. In all, two hundred and eighty-four eclipses have now been observed, forty-seven since the end of October, 1883.

Revision of Zone Observations. — The revision of the observations given in Volume VI. of the Annals of this Observatory, so far as they relate to stars between 50′ and 60′ north of the equator, has been completed during the last year. Some discrepancies in position and magnitude have been found which cannot readily be explained by errors in the observations. The stars thus indicated as interesting objects for further examination will be reobserved.

Standards of Stellar Magnitude. — The charts of twenty-four regions extending 4^m in right ascension and 10′ in declination, and following a series of twenty-four bright stars, as described in the last report, have now been completed. Valuable assistance in revising these charts has been received from Professor E. S. Holden, Director of the Washburn Observatory, Madison, Wisconsin. The telescope of the Washburn Observatory has a slightly larger aperture than that at this Observatory. Similar assistance has been promised by some other observers with still larger telescopes.

Meanwhile the selection of stars for standards of stellar magnitude has been made for many of the regions thus charted, and photometric measurements of their light have been begun. The brighter stars of each region are within the reach of the meridian photometer. To connect these with others of the tenth and eleventh magnitudes the modified wedge photometer is employed in nearly the same manner as in the revision of the zone observations already mentioned. With the addition of a simple shade-glass, stars of the second magnitude may be observed with this wedge, and a further connection is thus made between its work and that of the meridian photometer. The stars in each region which are too faint for observation with the wedge are compared with the bright star, preceding the region under examination, by means of Photometer I, described in Volume XI. of the Annals of the Observatory. Many of the stars observed with the wedge can also be measured with Photometer I, and all the results of the work may thus ultimately be referred to the meridian photometer. Additional photometric methods of measurement are under consideration and it is hoped that the magnitudes of the standard stars which are selected in each region will be determined with a satisfactory precision by these investigations.

Comets. — The observation of comets has been continued by Mr. Wendell with the object of furnishing material for the early computation of their orbits and of providing means for subsequent determinations of greater accuracy. The observations therefore are chiefly confined to the few days following the discovery of a comet and to occasions when it is too faint to be observed with any but powerful telescopes. Comets 1884 I., II., and III. were respectively observed on 6, 6, and 7 nights.

Spectra and Color of Stars. — Two separate series of investigations with regard to stellar spectra have been undertaken. It has been proposed to examine all stars known to have banded spectra with the object of approximately determining the positions of the bands in each upon a uniform system. This would afford means for a more definite and satisfactory classification of these spectra than at present exists. The method of measurement consists in comparing the spectrum with a notched bar beside which it is placed in the field of the telescope. The proper position of the spectrum is secured by a previous reference to an image of the star formed by light allowed to pass beside the prism which forms the spectrum.

For the acquisition of more definite knowledge than at present exists with regard to the color of stars, it has also been proposed to observe all stars to the fourth magnitude inclusive, and north of the thirtieth parallel of south declination, with an instrument designed for the purpose. The spectrum of the star to be observed is properly placed in the field by the same means as in the other instrument just

described. It is then carried by its diurnal motion behind a series of narrow bars placed at right angles to the spectrum, small portions of which are accordingly visible in the narrow spaces between the bars. The successive extinction of these portions of the spectrum is observed in a wedge of tinted glass. In this manner the relative brightness of definite parts of different spectra may be compared.

The diminished resources of the Observatory will probably not permit the completion of these observations upon the original plan. The observations relating to the banded spectra must be somewhat restricted, and those relating to the color of the stars must be confined to stars of the first three magnitudes.

Revision of DM. Magnitudes. — The meridian photometer permits the observation of stars as faint as the magnitude 9.0 and even somewhat fainter. But for the observation of the faintest stars in the Durchmusterung a more powerful instrument is preferable. The modified wedge photometer used with the East Equatorial in the revision of the zone observations already mentioned has accordingly been applied to the observation of the zone between 49° 50′ and 50° 0′ north of the equator. This zone is a part of that which has been observed here since 1870 with the meridian circle, for the determination of the positions of stars. It is also under observation with the meridian photometer, which will accordingly furnish points of reference for the observations made with the wedge photometer applied to the equatorial. Each star belonging to the zone and occurring in the Durchmusterung is twice observed, on different nights, and the interval between its transit over a bar and its disappearance in the wedge is recorded. This work can be carried on by a single observer, although some time is saved by the assistance of a recorder when it can be had. If the results prove satisfactory a more extensive revision of the magnitudes given by the Durchmusterung will be undertaken in the same way.

Meridian Circle.

The observation of fundamental stars has been continued throughout the year, but with less regularity than during former years on account of the interruption caused by the reobservation of scattering zone stars of the Durchmusterung list. During the Autumn Equinox of 1883, 604 observations of fundamental stars were made. The Sun was observed on 34 days, and Polaris on 26 days. During the Spring Equinox of 1884 the Sun was observed on 33 days, Polaris on 30 days, and 261 observations of fundamental stars were obtained. At other times during the year the Sun was observed 46 times, Polaris 59 times, and 301 observations of fundamental stars were made.

At the completion of the reduction of the zone stars, observed be-

tween 1870 and 1879, it was found that about 400 stars required further observation. In the greater number of cases it was found that the wrong star had been observed when two or more were in the same field of the telescope. In other cases it was considered advisable to settle doubtful cases, especially with regard to the minute of declination, by a reobservation. This revision of the zone observations was begun October 9, 1883, and was completed August 9, 1884. 1463 observations were made of zone stars, and 671 corresponding fundamental stars were observed in addition to the observations enumerated above. It will be seen from this summary that the total number of observations made with the meridian circle is 3528, and does not materially vary from the number in former years.

The investigation of the errors of the east circle has occupied a large portion of the time since July of the present year. Eighteen determinations of the stars of the 30° divisions of the circle have been made with very satisfactory results. Two independent determinations of the 15° divisions have also been made. It is proposed to continue the investigation of the main subdivisions of the circle at regular intervals throughout the coming year. This work will for the most part occupy the entire time of Professor Rogers for several months, and will take the place of the ordinary observations. During the year considerable progress has been made in printing Volume XV. The catalogue of all the primary and secondary stars observed between 1870 and 1879 for the epoch 1875.0, together with the mean values of the coördinates of each star for each year of observation, is printed, and will shortly be issued in advance of the publication of the entire volume. The reductions of the zone stars observed during the past year are about one half completed.

Meridian Photometer.

During the past year 141 series of observations have been taken with the meridian photometer by Mr. Wendell and myself. The total number of separate settings is about 27,500, an increase of more than a third over the previous year. The accordance of the results also shows a decided improvement in the accuracy of the measures. The average deviation of the separate measures of the one hundred circumpolar stars to which all the others are referred, amounted to about .18 of a magnitude with the first meridian photometer. The new instrument gives .15, .12 and .11 for the same quantity in the three years in which it has been employed. Since the value of the observations is proportional to the squares of these quantities, it appears that one of our present observations is nearly equal to three with the smaller instrument employed in the observations which have been published in Volume XIV.

The principal investigation now carried on with the meridian photometer consists in the measurement of stars the magnitude of which has been estimated at each of two observations during the recent revision of the Durchmusterung. The number of such stars is about eight thousand. Besides the two estimates just mentioned, two others were made in the original formation of the Durchmusterung, and additional estimates for many of the stars also occur in other catalogues. The results obtained by the meridian photometer will therefore have a wide application in the determination of scales of magnitude. In connection with the photometric observations, estimates of magnitude are made by the observers.

Another class of stars observed with the meridian photometer comprises a part of those already measured with the smaller instrument of the same kind employed here during the years 1879 to 1883. The stars selected for remeasurement are those for which the magnitude resulting from any single observation differed more than 0.5 from the mean of the other results. All stars belonging to Flamsteed's British Catalogue, and not already measured with the first meridian photometer, are likewise observed with the new intrument. These observations will serve to complete the reduction of the comparisons made by Sir William Herschel. Other observations made with the meridian photometer relate to asteroids, to known and suspected variables, to stars with peculiar spectra, and to objects having any special characters which may make it a matter of interest that their light should be measured.

Miscellaneous.

Almucantar. — It was mentioned in the last report that a small instrument bearing this name and devised by Mr. Chandler had been employed by him with very satisfactory results in the observation of transits over given parallels of altitude. He accordingly decided upon the construction of an instrument of the same kind and sufficiently large for the execution of researches similar in importance and delicacy to those usually undertaken with the best meridian instruments. The telescope of this large almucantar is 4 inches in aperture and 43.8 inches in focal length. The frame to which it is attached floats in a shallow trough of mercury, as in the previous instrument, and the entire apparatus revolves upon a vertical axis. Various difficulties which occurred in its construction were removed by the ingenuity of the inventor, and it was mounted during April, 1884. After about a month of experimental observations it was applied to the determination of the corrections required by the right ascensions of certain fundamental stars near the pole, and the declinations of others near the equator. It is hoped that the results of this investigation will estab-

lish the value of the new principles of construction and of observation employed in the undertaking. Up to November 1, about 700 observations of 190 stars had been made. Many additional observations have been made for other purposes; in particular, the determinations of clock error required by the time service of the Observatory have been frequently made by the almucantar.

The results thus far secured seem to justify the expectations entertained of the capacity of the almucantar to determine with exactness the relative positions of stars widely separated from each other. It will be useful, accordingly, not only in observations of time and latitude but also in various higher and more delicate problems of practical astronomy, for the solution of which it furnishes a new method.

The probable error of a single observation of zenith distance is in general $\pm 0.''37$, and of a single observation of right ascension $\pm 0.^s025$. Certain minor improvements in the apparatus, now under consideration, are expected to afford a still further increase in accuracy.

The latitude of the dome of the Observatory given by three nights' work with the new almucantar is $42° 22' 47.''58 \pm 0.''09$, which closely agrees with the value $42° 22' 47.''63$ found last year by the small almucantar.

The character of the various results given above encourages the hope that the use of the almucantar may become a permanent part of the work done at this Observatory. Should the results prove no more accordant than those of other instruments, the fact that they are obtained by an entirely independent method would free them from many of the errors which are commonly repeated in meridian observations.

Variable Stars. — The observation of telescopic variables was continued by Mr. Chandler upon the same system as before until May, since which time he has been chiefly occupied with the almucantar. He obtained 1240 observations of the light of variable stars between November 1 and June 30. He also made 952 observations of the color of 108 variable stars. The color was generally estimated upon an arbitrary scale, but many determinations were also made by the estimation of magnitudes with and without the use of a blue shade glass. The results of this work seem to establish the law that the redness of variable stars increases with the length of their periods.

Three observers of variable stars have acted in coöperation with this Observatory during the past year: Mr. H. M. Parkhurst, of New York, Rev. J. Hagen, S. J., of Prairie du Chien, Wisconsin, and Mr. J. H. Eadie, of Bayonne, N. J. Mr. Parkhurst, in addition to estimates of relative brightness, has made many photometric observations with apparatus devised by himself. He has recently undertaken the observation of the comparison stars employed for several variables by previous observers. Father Hagen, assisted by Messrs. Zwack

and Zaiser, besides observing many known variables, has examined a number of bright stars suspected of variability. Mr. Eadie is acting in special coöperation with Mr. Parkhurst, in such a manner as to obtain the best results without needless repetition of work. The labors of all these gentlemen have secured a large amount of valuable knowledge.

Under the title "Recent Observations of Variable Stars" (Proc. Amer. Acad. XIX. 296) information was brought together for the guidance of observers in that branch of astronomy with regard to the selection of objects for observation. The pamphlet contains a list of variable stars, with indications of the work recently done in observing them, so that the relative importance of further observations of any object in the list may be apparent. The publication will be continued annually if it appears to meet a want among astronomers, and if they comply with the request expressed in the pamphlet to forward to the Observatory such notices of their observations of variable stars as may serve to render the list more complete.

Time Signals. — The new standard time referred to in my last report was introduced in November, 1883. At Greenwich midnight on the seventeenth of that month the signals according to the meridian of the Boston State House were discontinued; at the next Greenwich noon signals over our lines were sent conforming to the minute and second of Greenwich mean time; and at mean noon on the seventy-fifth meridian on Sunday the eighteenth, the Boston time-ball was dropped according to Eastern time, five hours slow of Greenwich mean time.

The office of the U. S. Signal Service having been moved on October 1, 1884, from the Equitable building to the Boston Post-Office and Sub-Treasury building, the time-ball has been dropped since that date by an officer detailed by the Police Commissioners. A new ball, however, is to be erected on the Post Office building to be dropped by the Signal Office. The public has been much indebted to the management of the Equitable Life Assurance Company for accommodating the ball upon the roof of their building and for furnishing and maintaining the apparatus used there.

Several companies having recently begun operations for distributing time by electric clocks and by striking the hour and minute to the listener at a telephone, our time has been furnished them during their experimental steps either gratuitously or for a small compensation. The Observatory however has not been responsible for ascertaining that the companies have furnished their patrons with time conforming to the standard. Negotiations are pending for a more intimate connection of one or more of these enterprises with us. The Western Union Telegraph Co. have received our signals and have been per-

mitted to transmit them automatically to others. The following list comprises those who receive our time for regulating their time-pieces and similar purposes, but who are not authorized to directly transmit or automatically repeat the signals to others: American Watch Co., C. W. Beals, Bigelow, Kennard & Co., Wm. Bond & Son, Boston Electric Time Co., C. A. W. Crosby, Equitable Life Assurance Co., E. Howard & Co., Kattelle Bros., J. V. Kettell & Co., Geo. H. Morrill & Co., Jas. Munroe & Son, N. E. Telegraph and Telephone Co., G. H. Richards, Jr., R. I. Electric Co., Shreve, Crump & Low, Alvah Skinner & Son, Ira P. Stere & Co., U. S. Signal Service Office. The large subscribers, on whom depends the existence of the time service as now conducted, are the city of Boston and all the steam railroads having stations in Boston, except the Boston & Lowell R. R. and the Revere Beach R. R.

Telegraphic Announcements. — This department of the work of the Observatory is conducted by Mr. Ritchie upon the enlarged system mentioned in the last report. During the year announcements have been sent to thirty institutions or individuals in this country. The total number of messages is 246, of which 107 related to asteroids, 119 to comets, and 20 to the elements of cometary orbits. The number of asteroids discovered was nine, but with regard to one of these discoveries no telegraphic information was received from Europe. The number of comets discovered was four. All the notices were distributed by means of the Associated Press and the local newspapers, as well as by telegraph. The "Science Observer" also issued thirteen special circulars relating to the various announcements.

PUBLICATIONS.

The chief publication of the Observatory during the past year has been Volume XIV., Part I. of its Annals. This gives the principal results of the work done with the first meridian photometer. It includes a catalogue of 4,260 stars with their magnitudes as determined by the photometer and according to the estimates or measures of seventeen other authorities. As uniformity in reference to catalogues is always desirable, it has been suggested in the volume that this catalogue should be known as the "Harvard Photometry" and designated by the abbreviation H. P. Since it contains the approximate places for 1880 of all stars visible to the unaided eye in this latitude, as well as their magnitudes, its principal columns have been separately reprinted and offered for sale at the cost of publication, under the title of Harvard Photometry. The stereotype plates of this portion of the catalogue will be preserved, so that additional copies may be printed at any time when they may be needed.

The second part of Volume XIV. is now in course of publication. It contains the discussion of many special topics connected with the photometric observations recorded in the first part of the volume, and with the corresponding estimates or measures of other observers since the time of Ptolemy.

The publications named below have appeared since the similar list in the last report was drawn up, either as official communications from the Observatory or as papers prepared by its officers individually: —

Thirty-eighth Annual Report of the Astronomical Observatory of Harvard College.

On the Possible Connection of the Comet Pons-Brooks with a Meteor-stream. By S. C. Chandler, Jr. Astronomische Nachrichten, cvii. 275.

Observations and Ephemeris of U Ophiuchi. By S. C. Chandler, Jr. Id. cviii. 55.

Observations of Meteors, 1883, December 6. By O. C. Wendell. Id. cviii. 433.

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EDWARD C. PICKERING, *Director*.

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From Volume VII. Transactions.
[*Read at the Boston Meeting of the Amer. Soc. of Mechanical Engineers.*]

CXCIII.

THE MICROSCOPE IN THE WORKSHOP.

BY WM. A. ROGERS, CAMBRIDGE, MASS.

It is a sound principle of mechanical construction that a secondary tool should never be used when the work can be as well done by a primary tool. If the capacity and the efficiency of the primary tool can be increased so as to meet every requirement, it is better to dispense with the secondary tool altogether.

In the ordinary operations of the workshop, the lathe and the planer are the primary tools, while the calliper, with the graduated scale, is the secondary tool. Let us take the most simple case. It is required to turn down a piece of metal to a given diameter. In order to make the assumed case as simple as possible, we will assume the required diameter to be an even inch. The calliper is set for this unit of length, either from a graduated scale or, more accurately, from an end-measure inch with parallel faces. The setting in the latter case is done by the sense of feeling. We thus introduce an additional element of complexity, since sight is at once the primary sense and the ultimate test of a given limit of extension upon which the workman must rely. When the market is supplied with graduated scales from which any required length may be taken by the sense of feeling, it will be in order to defend the practice of relying upon this sense as a final test in measurements of extension. As a differential test, it is both useful and accurate. As an absolute test it had better be abandoned. It is a make-shift at best.

Assuming that the calliper has been set to an exact inch, the workman turns the piece of metal to the required size by a series of approximations, with the ever-present risk of going beyond the required limit. During the final part of the operation he stops the lathe to test the diameter with his calliper. He then takes another chip, stops, tries, starts, stops, tries until the subtle and ever-varying sense of feeling satisfies him that he has obtained the correct diameter. But after all, the uncertainty in the setting of

the calliper remains, and this uncertainty is generally greater than that which would be found to exist in the comparative trials of the diameter. If now we increase the required unit, and especially if fractional increments are added, the problem of transferring a required length from a scale to a calliper becomes a most serious one.

Every machinist must admit that there would be a great gain both in time and in accuracy if he could be sure of knowing the exact amount of work done at any instant, if he could see and measure the varying diameter of his cylinder, and at the same time control the amount of work to be done by the manipulation of his lathe, stopping at the exact instant when the required diameter has been obtained—if he could turn two shoulders upon a cylindrical shaft to any required length in one operation, stopping the last chip at the instant the correct length has been obtained—if he could turn a shaft to a required taper, and be sure that the correct angle of inclination has been maintained during every part of the operation—if he could—but I forbear further enumeration, lest the enthusiasm of inexperience may lead you to overlook the gravity of the demand for a radical change in our present methods not only of obtaining, but of applying measurements of length; a demand made in the interest of *accuracy*, *uniformity* and *economy*.

It is quite worth our while, therefore, to discuss the question whether the microscope considered as an attachment to the lathe and to the planer will not enable us to dispense with that secondary tool, the calliper, in a majority of cases, and at the same time increase the precision and the economy of mechanical construction.

The microscope has been generally accounted a delicate instrument, especially adapted to the minute study of delicate organisms and to the measurement of minute dimensions. By common consent it has been relegated to the laboratory of the investigator and has been considered quite unsuited to the every-day operations of a machine shop. One reason for this view formerly had great force. Until the invention of the opaque illuminator, by the late Robert B. Tolles—a single prism inserted between the two front lenses of an objective—the illumination of objects in the field of the microscope was for the most part obtained by transmitted light, thus requiring a transparent substance. A previous invention by Professor Hamilton L. Smith, of Geneva, N. Y., and since patented under a slightly different form by Beck, of London, gives

equally good results, but the care and the time required in adjustment and the difficulty in manipulation would prevent its use in the workshop. With Tolles' illuminator, however, it is easy to obtain at once a perfect illumination of a metal surface under almost any given conditions. It is only required that one face of the prism shall be presented toward the source of light, whether it be an artificial flame or the open sky.

It has been assumed also that a machine to which a microscope is attached must be most firmly mounted upon solid piers insulated from the building and in a room in which a steady temperature can be maintained. This is by no means necessary in ordinary workshop practice. The difficulty with regard to solidity of foundation can be practically overcome by adding mass to the machine, and the question of temperature will be taken care of by having separate standards of length of the different metals in ordinary use.

Only one other objection remains to be overcome. It is the common impression that the delicate adjustments of the microscope which are continually demanded—especially the adjustment for focus—can only be made by the most delicate and sensitive means. No impression could be more erroneous. Give me a small lead hammer and I will set the stop of my comparator to a given line in half of the time and with greater precision than it can be set by means of a screw movement. Give me a vertical movement by means of an excentric disc and a long lever-arm, and I will bring the surface of a plate weighing 100 pounds into the focus of the objective quite as quickly and quite as accurately as a similar adjustment could be made in the hands of a professional microscopist.

Having thus cleared away some of the objections which would be very properly made *a priori* against the proposal to use the microscope as an essential part of the lathe and of the planer, it will now be in order to point out some of the ways in which it can be most effectively used in the interest of accuracy and of economy.

I shall, in this paper, limit the illustrations of the applications of the microscope, to four operations in lathe work, viz.:

(*a*) Turning shoulders upon a shaft to a required length.

(*b*) Turning a face plate to a required diameter.

(*c*) Turning a shaft to a required diameter.

(*d*) Turning a shaft to a required taper.

In Fig. 67, f^1 represents a microscope which is attached to the

carriage of the lathe. The microscope f, also attached to the carriage, is adjusted over a scale g, which rests upon a plate h. This plate is attached to the lathe bed. It has a fulcrum at h^1, and an adjustable movement in elevation by means of a screw h^2. Two or more flexure screws secure a parallelism of the upper surface of the plate at every point with the horizontal plane described by the carriage.

The shoulder next to the head stock is supposed to be already turned. The micrometer line of microscope f^1 is then set upon the limiting edge and the zero of the graduations upon the bar g is brought into coincidence with the micrometer line of microscope f.

It is obvious that if the relative positions of the two microscopes remain unchanged, the distance measured on the scale by the

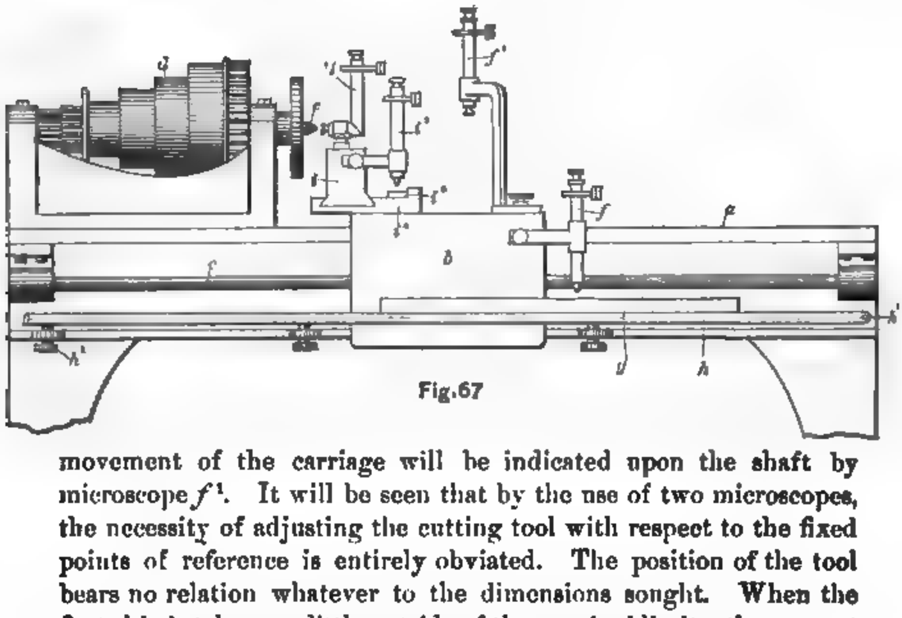

Fig. 67

movement of the carriage will be indicated upon the shaft by microscope f^1. It will be seen that by the use of two microscopes, the necessity of adjusting the cutting tool with respect to the fixed points of reference is entirely obviated. The position of the tool bears no relation whatever to the dimensions sought. When the first chip is taken—a little outside of the required limit—the amount by which the carriage must be moved will be indicated by the micrometer line of microscope f^1. It is to be noted, however, that the tool will do its work for one-quarter of a revolution before the amount of work done is indicated by the microscope, but the proper allowance can be quickly made by means of a graduated scale in the eye-piece of the micrometer of microscope f^1.

In order to turn a face plate to a required diameter, adjust vertically the micrometer line of microscope i over the point of the face

plate which is stationary during its revolution. Adjust microscope i^2 upon the zero of the transverse scale. The required diameter will then be indicated by the movement of i^2 over the scale and the indicated limit of the circumference through microscope i.

In order to turn a shaft to a given diameter, it is necessary to set the micrometer line of microscope i^2 in the line between the centers of the lathe. Since it is not possible to do this directly, we introduce an auxiliary measuring gauge k k^1, Fig. 68, which will also be found to be of great service in testing the various adjustments of the lathe. k is a cylindrical shaft, ground to a true cylinder, *e. g.*, in a Brown & Sharpe grinding machine, while supported at its centers. k^1 is a ring which slides freely upon the shaft and is capable of being firmly secured to k by projecting flanges (not

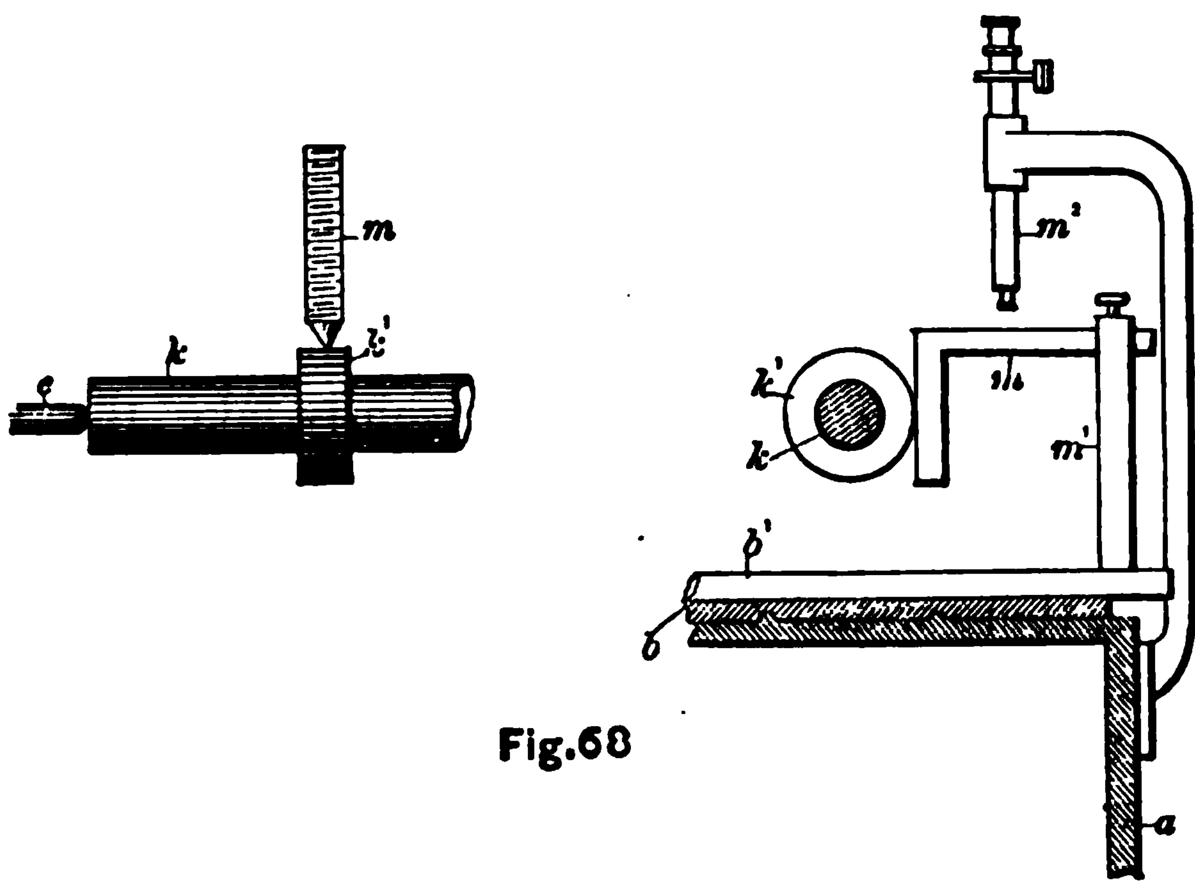

Fig.68

shown in the figures). This ring is also ground to a true cylinder and has a known diameter, *e. g.*, of 3 inches.

When this shaft is placed between the centers of a lathe and k^1 is set near the tail-stock, the projecting arm of a sliding arm m, Fig. 68, is brought into contact with the outside surface of k^1, and the micrometer line of microscope $m^2 = i^2$ is set upon the zero of the scale upon the upper surface of the arm m. The shaft is now removed and the carriage is moved inwards through the space of 3 inches, as indicated by the scale. The micrometer line of the microscope will now be coincident with the line between the centers of the lathe, if the proper adjustment of the tail-stock has been made.

It is not probable that this adjustment would need to be tested very often, if the microscope is firmly attached to the carriage.

If the point of the cutting tool is brought into adjustment under

the micrometer line of the microscope, the required diameter can be read off directly from the scale. Since, however, the wear of the cutting tool would probably be appreciable, this direct method of measurement would not probably be as satisfactory as the more indirect method of employing the additional microscope f^1 in connection with an auxiliary calliper scale.

For any diameter up to about one inch with a 1-inch objective, we may proceed as follows. When the contact of the arm with the surface of the ring k^1 is made, set the micrometer line of microscope f^1 tangent to the ring on the same side. Then, for any radius of the shaft to be turned, less than the working distance of the microscope, we shall have, after an inward movement of 3 inches, the micrometers of both microscopes coincident with the line between the centers of the lathe—one set upon the scale and the other over the shaft. The required diameter will then be obtained when the micrometer line of the microscope i^2 reaches the required point on the scale and the micrometer line of f^1 is tangent to the circumference of the shaft.

With a slight vertical movement at right angles to the plane of the ways, microscope i might advantageously take the place of microscope f^1. It would be necessary to raise the microscope in passing the center of the lathe, but since it would fall back upon the same seat, there would be little danger of disturbing the relative positions of the two microscopes by this movement.

In order to set the tail-stock for turning any required taper, set the ring at the end of the shaft adjacent to the head-stock and set the microscopes as described above. First set the ring at the point where the largest diameter is required, and then adjust the tail-stock in the usual way, so that the reading on the transverse scale shall indicate the lesser diameter.

It is obvious that two microscopes attached to the tool carriage of a planer, in connection with longitudinal and transverse scales, may be made to serve the same purpose as in the lathe. The microscope may be made especially useful in leveling up the bed of a planer. Place beside the lathe a shallow dish of mercury extending its entire length with means of adjustment similar to plate h, Fig. 67. Attach a microscope first to one end and then to the other end of the planer, and make the adjustment for level such that the surface of the mercury is sharply in focus under the microscope in the two positions. The bed-plate can then be blocked up at the intermediate points until every point is in focus.

With regard to the expense of fitting up a lathe with the microscopic attachments indicated above, it may be estimated at about $125, exclusive of the cost of the graduated scales. Only two objectives would be needed, at a cost of $20 each.

If it is urged that this direct process of applying dimensions in mechanical construction is not practical, or not well adapted to ordinary machine-shop practice, if it is insisted that the calliper *must* be used, I shall still maintain that a reform is needed in the method of setting the calliper for a required measure of length, and that there is a better way than that ordinarily followed. It is simply impossible to set a calliper with any degree of precision from a

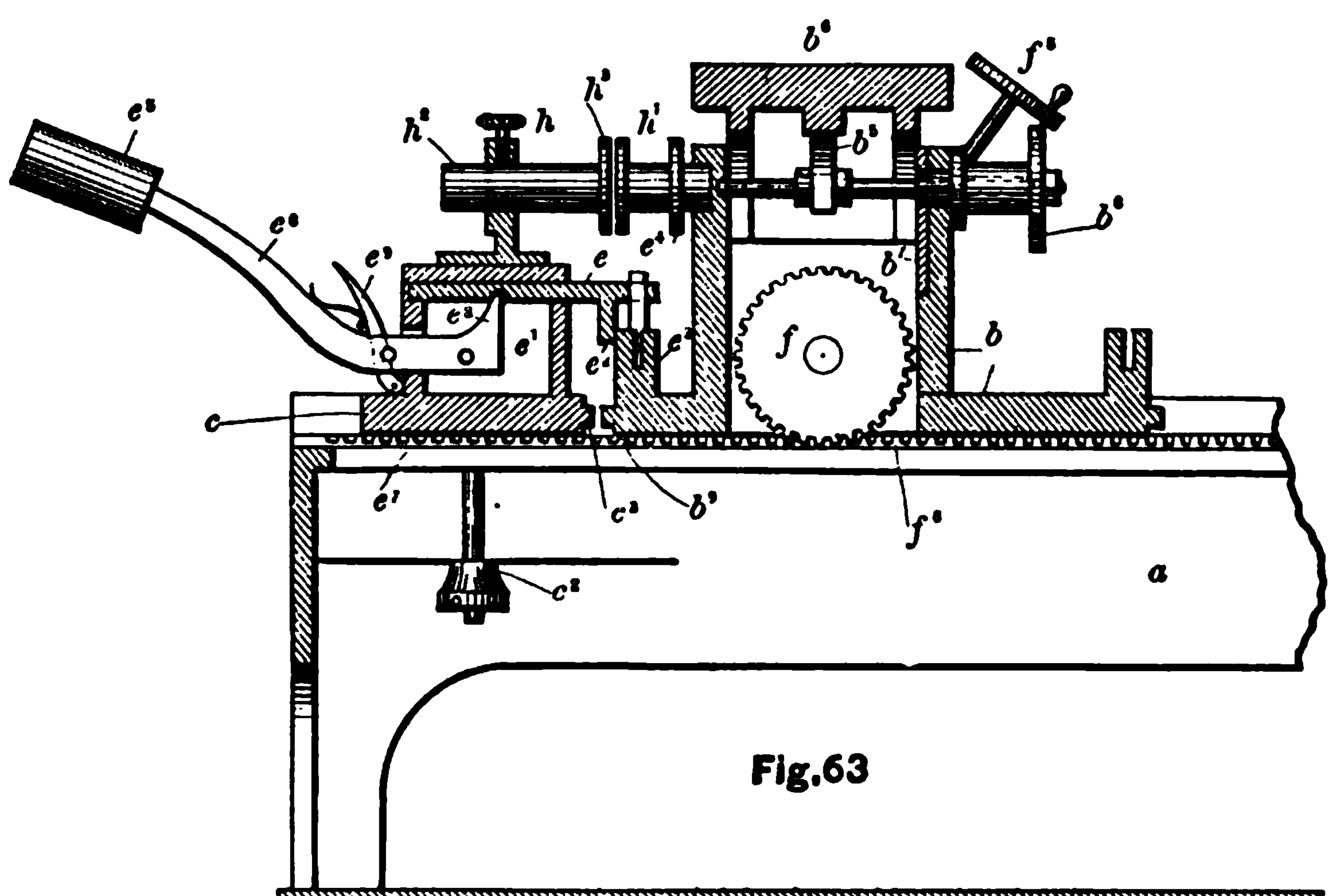

Fig.63

line scale. End-measure gauges are expensive and they require the most careful manipulation. Moreover, as ordinarily made, they can only be used for aliquot parts of a given unit. They are useless except for a few special lengths, and the extreme length seldom exceeds 6 inches.

There are four requirements which ought to be met absolutely in any proposed system of obtaining and transferring measures of length.

First.—All measures of length must be referred to one line-measure standard. This standard should be at least one yard in length, and the subdivisions should be such that any required length can be taken from it directly to $\frac{1}{1000}$ inch. Subdivisions less than

this limit can be better obtained with the aid of an eye-piece micrometer in the microscope. The yard should be standard at 62.0° Fahr., and the subdivisions should be so exact that there would be no necessity of applying corrections.

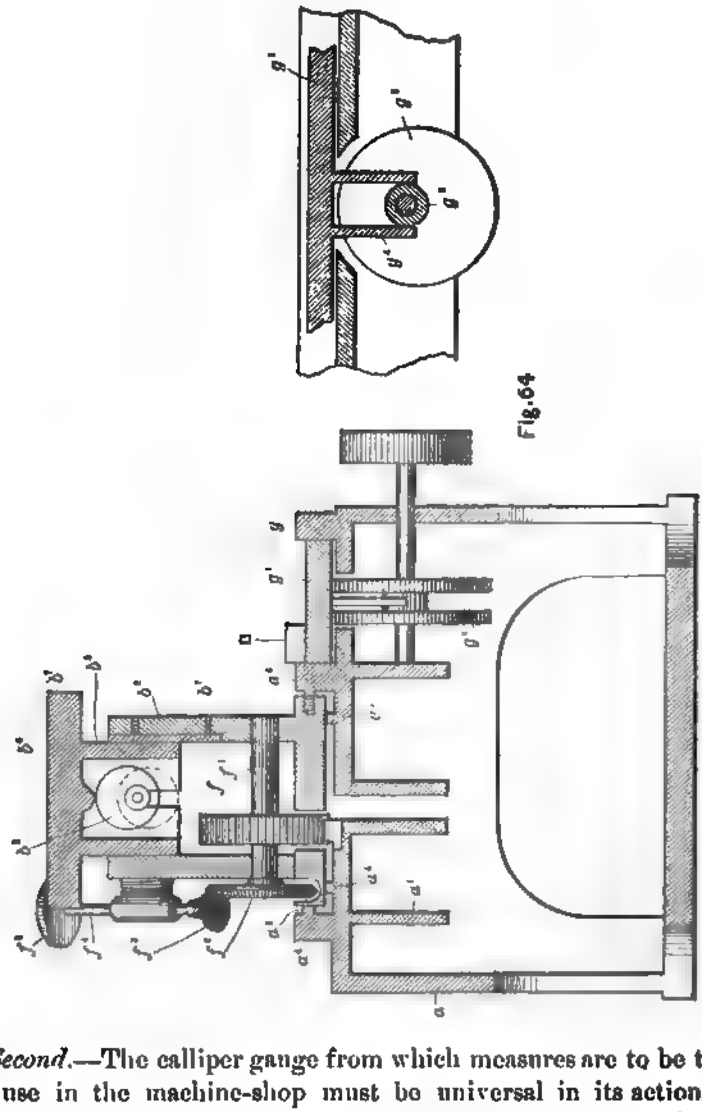

Fig. 64

Second.—The calliper gauge from which measures are to be taken for use in the machine-shop must be universal in its action. It must be capable of being set to correspond to any required length, aliquot or fractional, as indicated upon the line standard.

Third.—It must be so simple in form, so direct and so sure in

its action, that the amount of time required in its manipulation shall be less than that required in the present practice of obtaining measures of length.

Fourth.—The cost of construction must be such that any shop of moderate capacity can afford the outlay.

It is the experience of the writer that these conditions are fulfilled in the universal microscopic calliper shown in the Figures 62, 63 and 64. This machine has been in constant use in the comparing room at Harvard College Observatory for nearly a year, and while it has less conveniences than the larger universal comparator, it has been found to be capable of doing quite as accurate work. Its first cost, not including the calliper attachment, but including cost of patterns, was $320.

The main features of this apparatus and the method of operation will be seen from the following outline references. In Fig. 62, the microscope slide b^2, which is closely fitted to the projecting side bearings a^4 and a^5 and to similar elevated bearings beneath, is carried the entire length of the bed by the rack f^4 and the bevel gear pinions $f^2 f^3$ and the pinion f, Fig. 64. The microscope plate b^4 has a slow motion adjustment in elevation by means of an excentric b^5, Fig. 63.

The stops c c^1 can be set at any desired position upon the bed. They can be firmly secured without the slightest disturbance of the stops by means of large circular clamps beneath the bed plate at e^2, Fig. 63.

The plate r extends the entire length of the bed and is closely fitted between the walls g and g^1, Fig. 64. It rests upon two excentrics opposite g^5 and g^6, Fig. 62, and shown in the end view at g^1 g^2, Fig. 64. The adjustment in elevation is made by means of levers inserted in the wheels g^5 g^6.

The gravity lock of the microscope plate against the stops, is shown in Fig. 63. The weighted lever e^6 can be thrown out of connection by means of the spring catch e^9, when it is desired to make the contact with the stops by means of the rack and pinion movement.

The graduated bar B rests upon the bed of the machine and against a vertical ledge which extends the entire length.

The universal calliper s s^1 rests upon the plate r, and can be placed in any desired position. The two parts s s^1 move independently; s^1 being carried by two arms attached to the microscope slide b^2.

The operation of setting the calliper for any required length will be as follows:

(*a*) Clamp section *s* in any convenient position upon the plate *r*.

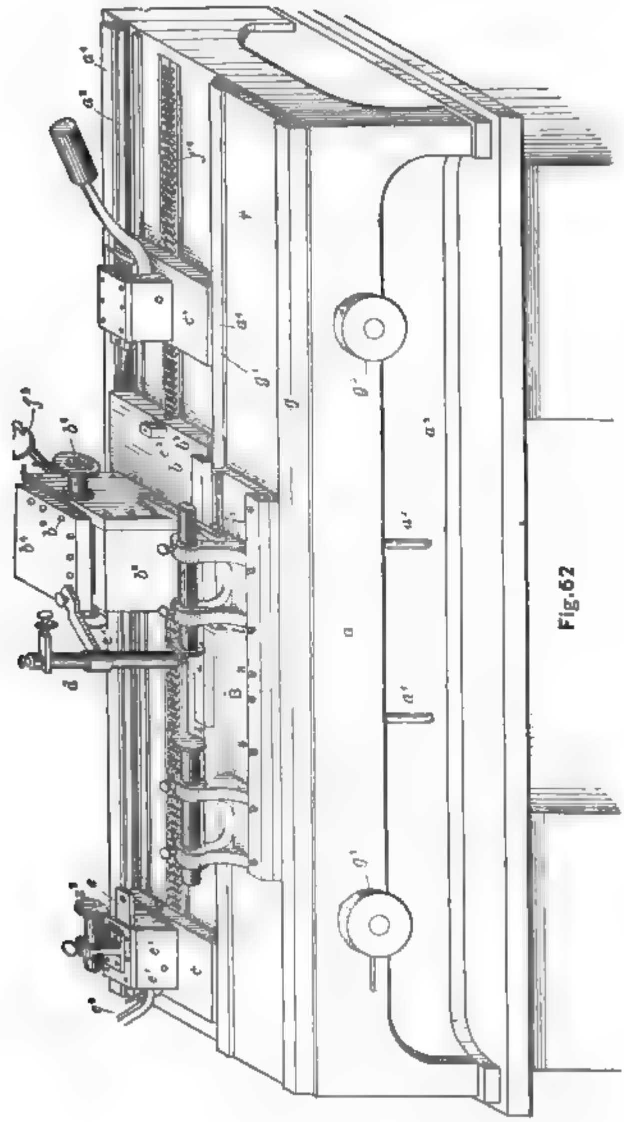

Fig. 62

(*b*) Bring section s^1 into a position such that the stops *s* and s^1 will be in contact with a clear space between the bases *s* s^1.

(*c*) Make the connection between b^3 and s^1 by means of the screws at *l* and l^1.

(*d*) Set the micrometer line of the microscope in coincidence with the zero line of the graduated bar *B*.

(*e*) Then, when the microscope slide *b* moves over any required distance as indicated by the graduated scale, the stop z^1 will move the same distance away from the stop *z*. The cylinders which form the stops are hollow, and a rod (not shown in the figure) passes through both, which serves as a support for the transferring calliper for inside measures. For outside measures, allowance is made for the thickness of the two face plates in setting the microscope slide. For support, the transferring calliper rests upon the two cylinders.

In some kinds of work it will be found quite as convenient to attach one stop z^1, to one stop of the comparator and the other to the vertical face of the microscope slide b^2.

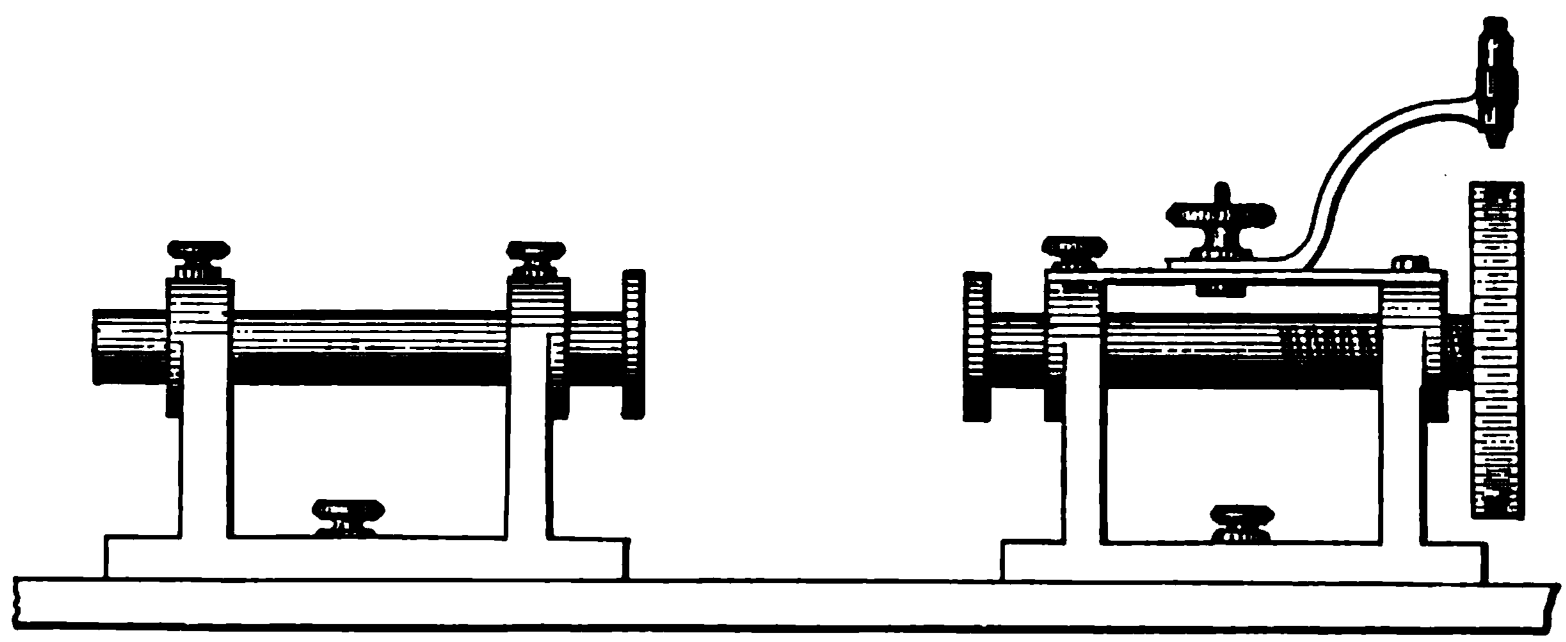

Fig.66

Another convenient form of the adjustable stops is shown in Fig. 66. They can be substituted for stops c c^1 with advantage when minute *differences* in length are to be measured.

This machine can be used to the best advantage by adopting the system of delivering measures of length from a "standards room," just as tools are now delivered from a tool room. This can be most economically done by the aid of the small boy. If a workman wishes his calliper set for any distance, he calls "boy." The messenger boy receives the calliper and a card on which is written the dimension required. He takes it to the "standards room" and after a brief delay he receives back the calliper properly set, together with the card on which are written the figures which have actually been used, and delivers both to the workman in half the time it would take him to set it from a scale with anything like equal precision.

There can be no more convincing way of demonstrating the feasibility of this method of obtaining and transferring measures of length than by performing some of the operations indicated. I enumerate below a few of them, with an estimate of the time which each operation requires.

It is required:

(*a*) To bring the surface of the graduated scale into a plane parallel with the plane described by the microscope carriage. Time, 30s.

(*b*) To set the longitudinal line of a graduated scale parallel with the ways of the comparator, 45s.

(*c*) To "set for focus" upon a given line. By lever movement, 6s. With lead hammer, 4s.

(*d*) To set for exact coincidence with a line of the scale by tapping the bar with a lead hammer, 6s.

(*e*) To set the stops c c^1 to correspond with any given distance. For short distances, 50s. For long distances, 1m. 10s.

(*f*) To compare a Whitworth end-measure yard with a line standard and with a limit of error not exceeding one ten-thousandth of an inch, 2m. 30s.

(*g*) To compare Whitworth 12 inch, 6 inch, and 1 inch end-measures with corresponding line measures, 1m. 45s. each.

(*h*) To compare a standard end-measure inch with corresponding line measures, 1m. 30s.

(*i*) To illustrate the limit of accuracy in measurements by the sense of feeling by adding to or subtracting from an end-measure inch one twenty-five-thousandth of an inch, 2m.

(*j*) To set a calliper for the distance one inch, upon the machine and compare it with a Pratt & Whitney inch, 2m.

(*k*) To illustrate the method of delivering calliper measures for use in the machine shop.

As the result of experiment, it has been found that the actual time required to perform the operations indicated is less than one-half of the limit given above. For the experiment under division (*h*), the writer is under obligations to the Brown & Sharpe Mfg. Co. for the loan of the five end-measure gauges of their manufacture.

From a comparison of the different end-measure inch gauges, the following results were obtained:

A plus sign indicates that the corresponding space is *too short;* a minus sign, that it is *too long.*

Space.	Correction.	Space.	Correction.	Space.	Correction.
1	−.000006 inch.	7	+.000005 inch.	13	+.000000 inch.
2	+.000024	8	−.000004	14	+.000020
3	+.000000	9	−.000013	15	−.000016
4	+.000010	10	−.000042	16	+.000014
5	−.000024	11	−.000026	17	+.000016
6	+.000005	12	+.000036	18	+.000010

It must be understood that any apparent degree of accuracy expressed by a figure in the sixth decimal place in which the unit is one inch, is probably fictitious.

(ADDED SINCE THE MEETING.)

Since this paper was put in type, the writer has received from the Pratt & Whitney Co. an end-measure inch, kindly loaned for this occasion. This standard was constructed from a four-inch line standard, graduated by the writer in 1881, and which has served as the basis of all the gauges made by this company. It has no correction at 62.0° Fahr. The Betts Mfg. Co. also kindly loaned a standard inch, made especially for the present purpose. They also sent the Whitworth inch, which has served as the basis of their gauges of this dimension. The same company some time ago sent two additional standards for comparison to with my line standard. Thus, with my own Whitworth standard, bought in London in 1880, we have five independent standard inches for comparison under division (*h*). With the aid of a recorder, it has been found quite easy to make all of these comparisons without haste, inside of the limit given for a single comparison as given under (h).

It will perhaps be worth while to give the results obtained. The particular inch with which the comparison was made, is the first inch of a standard one-half yard, subdivided to tenths of inches. In order to obtain a standard for which no sensible corrections of any kind would be required, 96 separate trials and corresponding investigations were made.

The relative errors of the separate inches are given below. The half yard has no correction for errors of total length at 62.0° Fahr.

The Pratt & Whitney Co. inch is 1 millionth of an inch *too short.*

The new Betts inch (probably a copy of the P. & W. inch) is 21 millionths of an inch *too long.*

The Betts-Whitworth is 202 millionths of an inch *too short.*

The Rogers-Whitworth is 236 millionths of an inch *too short.*

The Betts inch, No. 1, made in 1883, is 202 millionths of an inch *too short.*

The Betts inch, No. 2, made in 1883, is 196 millionths of an inch *too short.*

It will be seen that these comparisons bear out the claim of the Betts Mfg. Co. that their gauges are practically exact copies of the Whitworth standard. *All* of the Whitworth gauges which have been examined by the writer have been found *too short.*

Referring to division (*i*), it may be said that the difference in the length between the first two gauges in the list can be detected by the sense of feeling with considerable certainty.

(*The calliper, with its microscopes, although set up to be exhibited when the paper was read, was not described before the meeting, the author being prevented from attendance. For the same reason, the paper received no discussion.*)

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XV.

AN EXAMINATION OF THE STANDARDS OF LENGTH CONSTRUCTED BY THE SOCIÉTÉ GÉNEVOISE.

BY PROFESSOR W. A. ROGERS.

Communicated December 10th, 1884.

SEVERAL physical laboratories in this country have recently received from the Société Génevoise instruments of precision of various kinds, which appear to have decided merit, both in regard to design and workmanship. The Society has among other things undertaken the construction of standards of length, and of a cathetometer, which is designed to take a high rank as an instrument of precision. Through the kindness of Professor Wright of Yale College, the writer has been permitted the opportunity of a somewhat extended study of one of the standard meters of the Society. Through the courtesy of J. W. Queen & Co. of Philadelphia, the opportunity was at the same time offered of an examination of three other standard meters, and of the meter graduated upon the upright bar of a cathetometer.

On account of the somewhat extended introduction of these standards, it has seemed to the writer worth while to place upon record the results of this examination.

EXAMINATION OF METERS.

The meter belonging to Professor Wright is designated W. The meter of similar form and dimensions received from J. W. Queen & Co. is designated Q_1. A second meter, in which the graduations are nearly along the centre of gravity of a cross section of the bar, is designated Q_2. The graduations upon W, Q_1, and Q_2 are upon silver inlaid in the brass, which is the material of the bars. A third meter, designated Q_3, has the graduations upon the brass. The meter of the cathetometer is designated Q_4.

COMPARISON OF METER W WITH BRONZE STANDARD METER $R_2^{a^2}$ AND STEEL STANDARD METER R_3,

in which, designating the Mètre des Archives by A_0,

$$R_2^{a^2} - 1.6\,\mu = A_0$$
$$R_3 \quad - 1.2\,\mu = A_0$$

The coefficient of expansion of R_2 for 1° C. in 1 metre is assumed to be 17.17 μ.* The coefficient of R_3 for 1° C. is assumed to be 10.28 μ. At 0° C. these relations become, with these coefficients,

$$R_2^{a^2} + 284.6\,\mu = A_0$$
$$R_3 \quad + 170.2\,\mu = A_0$$

The values $R_2 - W$ and $R_3 - W$ are given in divisions of the micrometer of the microscope employed, in which

$$1 \text{ div.} = 0.503\,\mu.$$

The thermometer employed is No. 8612 Baudin, and the readings τ have been reduced to the Yale standard.

The observations extend from November 15 to December 3, 1884.

EQUATIONS OF CONDITION BETWEEN R_2 AND W.

No. Obs.	$R_2 - W$		$(16°.67 - \tau)$	$R_2 - W$ at 16°.67	Residuals.	
3	—490.6 div.	$= a$	$+19.41\,b$	—526.1 div.	—3.2 div.	= —1.6 μ
3	—496.9	$= a$	$+14.79\,b$	—524.0	—1.1	= —0.6
3	—499.2	$= a$	$+13.44\,b$	—523.8	—0.9	= —0.5
6	—501.0	$= a$	$+10.94\,b$	—521.0	+1.9	= +1.0
2	—508.4	$= a$	$+\ 8.07\,b$	—523.2	—0.3	= —0.2
4	—512.2	$= a$	$+\ 5.31\,b$	—521.7	+1.2	= +0.6
5	—513.3	$= a$	$+\ 1.08\,b$	—516.4	+6.5	= +3.3
4	—552.5	$= a$	$-14.06\,b$	—526.8	—3.9	= —2.0
			Mean	—522.9		

Normal Equations.

$$-\ 4073.9 = 8\,a + 59.58\,b$$
$$-28077.3 = 59.58\,a + 1189.62\,b$$

Whence

$$a = -522.9 \text{ div.} = -263.0\ \mu$$
$$b = +\ 1.83 \text{ div.} = +\ 0.92\ \mu$$

* Proceedings of the American Academy, Vol. XVIII. p. 341.

EQUATIONS BETWEEN R_3 AND W.

No. Obs.	$R_3 - W$		$R_3 - W$ at 16°.67	Residuals.	
3	—220.4 div.	$= a + 19.41\,b$	—522.0 div.	+2.1 div.	$= +1.1\,\mu$
4	—301.4	$= a + 14.79\,b$	—531.2	—7.1	$= -3.6$
3	—313.7	$= a + 13.44\,b$	—522.6	+1.5	$= +0.8$
6	—351.2	$= a + 10.94\,b$	—521.2	+2.9	$= +1.5$
2	—397.1	$= a + 8.07\,b$	—522.5	+1.6	$= +0.4$
4	—430.1	$= a + 5.31\,b$	—521.6	+2.5	$= +1.3$
5	—501.5	$= a + 1.68\,b$	—527.6	—3.5	$= -1.8$
4	—742.3	$= a - 14.06\,b$	—523.8	+0.3	$= +0.2$
		Mean	—524.1		

Normal Equations.

$$-3266.7 = 8\,a + 59.58\,b$$
$$-12736.4 = 59.58\,a + 1189.62\,b$$

Whence

$$a = -524.1 \text{ div.} = 263.6\ \mu$$
$$b = +15.54 \text{ div.} = 7.82\,\mu$$

It appears from these observations, that the coefficient of W for 1° C. from comparison with R_2 is

$$17.17\,\mu + 0.92\,\mu = 18.09\,\mu;$$

and that the coefficient of W for 1° C. from comparison with R_3 is

$$10.28\,\mu + 7.82\,\mu = 18.10\,\mu.$$

On account of the large deviation from the value communicated by the Society, viz. 19.155 μ, a series of comparisons was instituted beween W and a steel end-meter S immersed in melting ice, according to the method described in the Proceedings of the American Academy, Vol. XVIII. p. 341.

EQUATIONS OF CONDITION BETWEEN W AND S IN MELTING ICE.

No. Obs.	$S - W$	$(16°.67 - \tau)$	$S - W$ at 16°.67	Residuals.	
5	+ 30.0 div.	$= a + 17.75\,b$	—619.1 div.	—0.8 div.	$= -0.4\,\mu$
5	— 7.5	$= a + 16.66\,b$	—616.8	+1.5	$= +0.8$
10	— 177.0	$= a + 12.24\,b$	—624.6	—6.3	$= -3.2$
9	— 198.3	$= a + 11.58\,b$	—621.8	—3.5	$= -1.8$
5	— 260.9	$= a + 9.58\,b$	—611.2	+7.1	$= +3.6$
6	— 327.4	$= a + 7.94\,b$	—617.8	+0.5	$= +0.3$
4	— 369.8	$= a + 6.73\,b$	—615.9	+2.4	$= +1.2$
10	— 571.5	$= a + 1.28\,b$	—618.3	+0.0	$= +0.0$
18	—1124.7	$= a - 13.83\,b$	—618.9	—0.6	$= -0.3$
		Mean	—618.3		

Normal Equations.

$$-3007.1 = 9a + 69.93b$$
$$+8180.1 = 69.93a + 1269.56b$$

Whence

$$b = 36.57 = 18.39\,\mu$$

Combining this value with the values 18.09 μ derived from R_2, and 18.10 μ derived from R_3, we have:

Coefficient of expansion of bar W in one meter $= 18.19\,\mu$ for each degree Centigrade.

For the relation between W and A_0 at 16°.67 C. = 62°.0 Fahr., we have:

$$W - 263.0\,\mu = R_2 = A_0 + 1.6\,\mu \qquad W - 263.6\,\mu = R_3 = A_0 + 1.2\,\mu$$
$$W - 264.6\,\mu = A_0 \qquad W - 264.8\,\mu = A_0$$

And finally, $$W - 264.7\,\mu = A_0$$

Reducing to 0° C., with the coefficient 18.19 μ, we have at 0°

$$W + 38.5\,\mu = A_0$$

A slightly different but probably more accurate value of this relation will be obtained by selecting only those comparisons which were made near 0°. From these data we have:

No. Obs.	8612 °	$R_2 - W$	$R_2 - W$ at 0° C.	No. Obs.	8612 °	$R_3 - W$	$R_3 - W$ at 0° C.
3	−2.74	−490.6 div.	−249.6 μ	3	−2.74	−220.4 div.	−132.6 μ
3	+1.88	−496.9	−247.9	4	+1.88	−301.4	−136.9
3	+3.43	−499.2	−247.6	3	+3.00	−313.7	−132.4
6	+5.73	−501.0	−246.2	6	+5.73	−351.2	−132.2
		Mean	−247.8				−133.5

Whence $W + 36.1\,\mu = A_0$ $\qquad$ $W + 36.0\,\mu = A_0$

COMPARISON OF STANDARD Q_1 WITH STANDARDS R_2 AND R_3.

EQUATIONS OF CONDITION BETWEEN R_2 AND Q_1.

No. Obs.	$R_2 - Q_1$		$R_2 - Q_1$ at 16°.67	Residuals.	
6	−580.2 div.	$= a + 16.02b$	−604.9 div.	+1.0 div.	$= +0.5\,\mu$
4	−586.0	$= a + 12.54b$	−606.2	−0.3	$= -0.2$
2	−597.1	$= a + 6.59b$	−607.2	−1.3	$= -0.6$
4	−627.1	$= a - 14.06b$	−605.4	+0.5	$= +0.3$
		Mean	−605.9		

Normal Equations.

$$-2391.30 = 4a + 21.09b$$
$$-11772.39 = 21.09a + 655.00b$$

Hence

$$a = -605.9 = 304.8\ \mu$$
$$b = +\ 1.54 = 0.78\,\mu$$

$$Q_1 - 304.8\,\mu = R_2 = A_0 + 1.6\,\mu$$
$$Q_1 - 306.4\,\mu = A_0$$

$$\text{Coefficient} = 17.17\,\mu + 0.78\,\mu = 17.95\,\mu$$

EQUATIONS OF CONDITION BETWEEN Q_1 AND R_3.

No. Obs.	$R_3 - Q_1$		$R_3 - Q_1$ at 16°.67	Residuals.		
6	—364.2 div.	$= a + 16.02b$	—608.4 div.	+3.8 div.	=	+1.7 μ
4	—424.2	$= a + 12.54b$	—611.4	—4.7	=	—2.4
2	—506.0	$= a + 6.59b$	—605.4	+1.3	=	+0.7
4	—816.5	$= a - 14.06b$	—606.6	+0.1	=	+0.1

Normal Equations.

$$-2111.8 = 4a + 21.09b$$
$$-3014.4 = 21.09a + 655.00b$$

Hence

$$a = -606.7 = -305.2\ \mu$$
$$b = +\ 14.93 = +\ 7.51\,\mu$$

$$Q_1 - 305.2\,\mu = R_3 = A_0 + 1.2\,\mu$$
$$Q_1 - 306.4\,\mu = A_0$$

$$\text{Coefficient} = 10.28\,\mu + 7.51\,\mu = 17.79\,\mu$$

COMPARISON BETWEEN STANDARDS Q_2, R_2, AND R_3.

EQUATIONS OF CONDITION BETWEEN R_2 AND Q_2.

No. Obs.	$R_2 - Q_2$		$R_2 - Q_2$ at 16°.67	Residuals		
6	—530.8 div.	$= a + 16.03b$	—574.5 div.	—3.8 div.	=	—1.9 μ
4	—533.9	$= a + 12.96b$	—569.2	+1.5	=	+0.8
3	—539.1	$= a + 10.40b$	—567.5	+3.2	=	+1.6
4	—609.9	$= a - 14.06b$	—571.5	—0.8	=	—0.4

Normal Equations.

$$-2213.7 = 4a + 25.33b$$
$$-12459.4 = 25.33a + 730.76b$$

Hence

$$a = -570.7 = -287.1\ \mu$$
$$b = +\ 2.73 = +\ 1.45\,\mu$$

$$Q_2 - 287.1\,\mu = R_2 = A_0 + 1.6\,\mu$$
$$Q_2 - 288.7\,\mu = A_0$$

$$\text{Coefficient} = 17.17\,\mu + 1.45\,\mu = 18.62\,\mu$$

Equations of Condition between Q_2 and R_3.

No. Obs.	$R_3 - Q_2$		$R_3 - Q_2$ at 16°.67	Residuals.		
6	—324.4 div.	$= a + 16.03\,b$	—579.4 div.	—5.5 div.	=	$-2.8\,\mu$
4	—361.6	$= a + 12.96\,b$	—567.7	+6.2	=	+3.1
3	—408.3	$= a + 10.40\,b$	—573.8	+0.1	=	+0.1
4	-798.3	$= a - 14.06\,b$	—574.6	—0.7	=	—0.4

Normal Equations.

$$-1892.6 = 4a + 25.33\,b$$
$$-2908.6 = 25.33\,a + 730.76\,b$$

Hence

$$a = -573.8 \text{ div.} = 288.6\ \mu$$
$$b = +15.91 \text{ div.} = 8.00\,\mu$$

$$Q_2 - 288.6\,\mu = R_3 = A_0 + 1.9\,\mu$$
$$Q_2 - 289.8\,\mu = A_0$$

$$\text{Coefficient} = 10.28\,\mu + 8.00\,\mu = 18.28\,\mu$$

Comparison of Q_3 with R_2.

	8612 °	$R_2 - Q_3$
	+2.12	—600.7 div.
	+2.28	—580.2
	+3.22	—586.9
	+4.70	—579.6
	+3.60	—576.2
Means	+3.17	—584.7

Reduced to 0° with coeff. 1.83 div., we have

$$R_2 - Q_3 \text{ at } 0^\circ = -579.2 = -301.3\,\mu$$

Hence

$$Q_3 - 301.3\,\mu = R_2 = A_0 - 284.6\,\mu$$
$$Q_3 - 16.7\,\mu = A_0$$

COMPARISON OF CATHETOMETER METER Q_4 WITH R_2 AND R_3.

Equations of Condition between Q_4 and R_2.

No. Obs.	$R_2 - Q_4$		$R_2 - Q_4$ at 16°.67	Residuals.		
7	—644.1 div.	$= a + 16.19\,b$	—631.0 div.	—0.6 div.	=	$-0.3\,\mu$
7	—637.5	$= a + 12.69\,b$	—627.2	+3.2	=	+1.6
7	—638.4	$= a + 10.68\,b$	—629.7	+0.7	=	+0.4
3	—639.4	$= a + 6.24\,b$	—634.3	—3.9	=	—2.0
8	—618.9	$= a - 13.24\,b$	—629.6	+0.8	=	+0.4

Normal Equations.

$$-3178.3 \text{ div.} = 5a + 32.56\,b$$
$$-21131.7 \text{ div.} = 32.56\,a + 751.46\,b$$

Hence

$$a = -630.4 = 317.1\ \mu$$
$$b = -\ 0.81 = \ 0.41\ \mu$$

$$Q_4 - 317.1\,\mu = R_2 = A_0 + 1.6\,\mu$$
$$Q_4 - 318.7\,\mu = A_0$$

$$\text{Coefficient} = +17.17\,\mu - 0.41\,\mu = 16.76\,\mu$$

Equations of Condition between Q_4 and R_3.

No. Obs.	$R_3 - Q_4$		$R_3 - Q_4$ at 16°.67	Residuals.	
7	—423.2 div.	$= a + 16.19\,b$	—625.6 div.	+2.8 div.	= +1.4 μ
4	—472.2	$= a + 12.63\,b$	—630.1	—1.7	= —0.9
3	—552.1	$= a + \ 6.24\,b$	—630.1	—1.7	= —0.9
4	—800.2	$= a - 13.81\,b$	—627.6	+0.8	= +0.4

Normal Equations.

$$-2247.7 = \quad 4\,a + \ 21.25\,b$$
$$-5209.8 = 21.25\,a + 651.30\,b$$

Hence

$$a = -628.4 = -316.1\ \mu$$
$$b = +\ 12.50 = +\ \ 6.29\,\mu$$

$$Q_4 - 316.1\,\mu = R_3 = A_0 + 1.2\,\mu$$
$$Q_4 - 317.3\,\mu = A_0$$

$$\text{Coefficient} = +10.28\,\mu + 6.29\,\mu = +16.57\,\mu$$

For the mean value we have:

$$Q_4 - 318.0\,\mu = A_0$$
$$\text{Coefficient} = 16.66\,\mu$$

For the relations between Q_1, Q_2, Q_4, and A_0, at 0° C., we have, from the comparisons made near 0°, as follows:

8612 Reading at Observation.	$R_2 - Q_1$ at 0°	$R_3 - Q_1$ at 0°	8612 Reading at Observation.	$R_2 - Q_2$ at 0°	$R_3 - Q_2$ at 0°
° + 0.65	—291.3 μ	—178.3 μ	° +0.64	—266.2 μ	—158.0 μ
+ 4.13	—292.3	—182.2	+3.71	—263.7	—151.7
+10.08	—293.1	—178.9	+6.27	—283.0	—154.6
Means	—292.2	—179.8		—264.3	—154.8

$$Q_1 - 292.2\,\mu = R_2 \qquad\qquad Q_1 - 179.8\,\mu = R_3$$
$$= A_0 - 284.6\,\mu \qquad\qquad = A_0 - 170.2\,\mu$$
$$Q_1 - \ 7.6\,\mu = A_0 \qquad\qquad Q_1 - \ 9.6\,\mu = A_0$$

For the mean $\qquad Q_1 - 8.6\,\mu = A_0$

$$Q_2 - 264.3\,\mu = R_2 \qquad Q_2 - 154.8\,\mu = R_3$$
$$= A_0 - 284.6\,\mu \qquad = A_0 - 170.2\,\mu$$
$$Q_2 + 20.3\,\mu = A_0 \qquad Q_2 + 15.4\,\mu = A_0$$

For the mean $Q_2 + 17.8\,\mu = A_0$

8612 Reading at Observation.	$R_2 - Q_4$ at 0°	8612 Reading at Observation.	$R_3 - Q_4$ at 0°
°		°	
+ 0.48	−324.2 μ	+ 0.48	−209.8 μ
+ 3.98	−322.6	+ 3.98	−212.2
+ 5.94	−324.0	+10.48	−211.4
+10.63	−326.7		
Means	−324.4		−211.1

$$Q_4 - 324.4\,\mu = R_2 \qquad Q_4 - 211.1\,\mu = R_3$$
$$= A_0 - 284.6\,\mu \qquad = A_0 - 170.2\,\mu$$
$$Q_4 - 39.8\,\mu = A_0 \qquad Q_4 - 40.0\,\mu = A_0$$

For the mean $Q_4 - 40.3\,\mu = A_0$

Collecting results, we have:

At 0°	At 16°.67	Coefficient.
$W + 36.1\,\mu = A_0$	$W - 264.7\,\mu = A_0$	18.19 μ
$Q_1 - 8.6 = A_0$	$Q_1 - 306.4 = A_0$	17.87
$Q_2 + 17.8 = A_0$	$Q_2 - 289.2 = A_0$	18.45
$Q_3 - 16.7 = A_0$	. . . . . .	. . .
$Q_4 - 40.3 = A_0$	$Q_4 - 318.0 = A_0$	16.71

It will be seen that every value of the coefficient is less than the value communicated by the Society, viz. 19.155 μ. It seems impossible to resist the evidence given by this determination, that this value is certainly too large. It is possible that the solution of the discrepancy may be found in the fact shown by these observations, that when the graduations are upon silver inlaid in brass, the coefficient of the brass controls that of the silver somewhat in proportion to the relative masses of the two metals. In every one of the five cathetometers standardized by the writer, the brass has carried the silver with it, that is, the coefficient has been found to be that of brass, and not that of silver.

It will be seen that the cathetometer has a less coefficient than the bronze bar R_2, which is identical in composition with the Imperial Yard.

It is probable, also, that the resultant coefficient of two metals which have a mechanical junction depends somewhat upon the perfection with which the junction is made. The writer has not found so large differences as here given in the values of the coefficient for different specimens of the same metal, but in this case the differences may be

due to accidental errors of observation, on account of the limited number of comparisons made.

It should be stated also, that, judging by the color of the metal, the composition of W and Q_1 differs from that of Q_3 and Q_4.

It now only remains to describe some observations which were made to determine the extent to which the variations in the length of the standards under variations of temperature correspond with the computed values from the indicated readings of the thermometer when placed upon the upper surface of the bars.

First, a series of comparisons were made with a rising temperature. When reduced to 16°.67, the value of $R_3 - W$ should be 522.9 div. The following are the observed values.

Time. h m.	8612 °	$R_3 - W$ at 16°.67	Deviation from 522.9 div.		
A.M. 7 10	6.20	—516 5 div.	— 6.4 div.	=	—3.2 μ
" 8 20	7.30	—517.7	— 5.2	=	—2.6
" 9 30	7.92	—522.1	— 0.8	=	—0.4
" 11 20	12.07	—517.1	— 5.8	=	—2.9
" 12 50	13.67	—511.3	—11.6	=	—5.8

It therefore appears that, for a rising temperature of about 1° for each hour of time, the length derived from the reading of the thermometer will be from 2 μ to 3 μ too short.

The length of W was then compared with an end-meter in melting ice with a falling temperature. The temperature of the comparing-room had for several hours remained at 15°.4 C. At 8h. 27m. A. M. both windows were opened. The relation $S - W$ had been previously found to be —618.3 div. The comparisons were now continued every three minutes with the following results.

Time. A. M. h. m.	8612 °	$S - W$ at 16°.67 div.	Deviation from —618.3 div. div.	μ	Time. A. M. h. m.	8612 °	$S - W$ at 16°.67 div.	Deviation from —618.3 div. div.	μ
8 30	14.55	—634.2	+15.9	=+ 8.0	8 54	11.97	—631.0	+12.7	=+ 6 4
8 33	14.25	—633.0	+14.7	=+ 7.4	8 57	11.39	—644.7	+26.4	=+13.3
8 36	13.67	—645.6	+27.3	=+13.7	9 0	11.10	—640.2	+21.9	=+11.0
8 39	13.41	—639.8	+21.5	=+10.8	9 3	10.94	—636.0	+17.7	=+ 8.9
8 42	13.07	—639.3	+21.0	=+10.6	9 6	10.95	—623.3	+ 5.0	=+ 2.5
8 45	12.76	—646.1	+27.8	=+14.1	9 9	10.90	—623.5	+ 5.2	=+ 2.6
8 48	12.57	—639.9	+21.6	=+10.9	9 12	11.00	—615.9	— 2.4	=— 1.2
8 51	12.19	—643.1	+24.8	=+12.5					

It appears, therefore, that the measured length will be found to be too great under a rapidly falling temperature by about 10 μ or 12 μ for at least half an hour after the change of temperature has occurred.

Under circumstances similar to those described above, it will not be safe to make the comparisons under about one hour after the change of temperature. For a bronze bar having a cross section of one inch, the writer has found that this limit of time is about five hours. This form of the metres of the Society, therefore, seems to be well adapted for ordinary use. But it should be stated that the small depth of the bars W and Q_1 requires that they shall rest upon a flat surface during comparison, — something not easy to obtain.

In the third test, the temperature of bar W was raised to about 28° C. by heating over a register. At 3h. 19m. P. M. it was removed to the comparing room within which the temperature remained very constant at about 19°.0 C. Comparisons were then made every three or four minutes with the end-meter in melting ice, as follows. The comparisons were begun when the thermometer placed upon the bar ceased to rise.

Time. P. M. h. m.	8612 °	$S-W$ at 16°.67	Deviation from −618.8 div.		Time. P. M. h. m.	8612 °	$S-W$ at 16°.67	Deviation from −618.8 div.	
3 21	12.45	−620 div.	+17 div.	+ 1 μ	3 42	6.40	−647 div.	+29 div.	+15 μ
3 25	8.31	−676	+58	+29	3 44	6.21	−649	+31	+16
3 29	7.32	−676	+58	+29	3 46	6.09	−636	+18	+ 9
3 33	6.85	−667	+49	+25	3 49	6.00	−638	+20	+10
3 36	6.55	−660	+42	+21	3 51	5.77	−638	+20	+10
3 39	6.46	−658	+40	+20	3 54	5.49	−642	+24	+12

It will be noticed from these results, that the bar remained nearly two minutes in the comparing-room before the change of length was decidedly apparent. After that, the computed length was too great by a maximum amount of 29 μ, and notwithstanding the fact that after about fifteen minutes the thermometer readings decreased very slowly and with considerable regularity, the indicated length of the meter was at the expiration of 33 minutes still 12 μ too great. It is apparent, therefore, that, under circumstances similar to those described above, it will not be safe to make comparisons until the bar shall have remained at a nearly constant temperature for at least one hour.

The lines traced upon the silver surface are, in all these standards, of the best quality. The edges are not rounded, and it is possible to focus upon the lines with great sharpness. The writer has never before seen heavy lines of as good quality as these. The width of the lines in W, Q_1, and Q_2 is about 20 μ, and in Q_3 about 50 μ. In Q_4 the width is slightly different at the two ends, the mean value being about 25 μ.

The only serious criticism to be made upon these standards is, that

the surfaces of W and Q_1 are slightly convex between centimeters 15 and 45, while Q_3 is convex to a less degree between centimeters 70 and 85. The surface of Q_4 is nearly plane when the cathetometer is supported at its neutral points.

RELATIVE ERRORS OF THE SUBDIVISIONS.

A plus sign indicates that the measured space is too short.

Decimeters.

Spaces.	W	Σ	Q_1	Σ	Q_3	Σ	Q_4	Σ
1	+ 0.7 μ	+ 0.7 μ	− 5.8 μ	− 5.8 μ	+ 4.7 μ	+ 4.7 μ	+ 0.0 μ	+ 0.0 μ
2	+11.3	+12.0	+ 1.9	− 3.9	+ 0.2	+ 4.9	− 2.5	− 2.5
3	+ 6.0	+18.0	−23.0	−26.9	−18.4	−13.5	− 1.1	− 3.6
4	−11.0	+ 7.0	+ 4.0	−22.9	− 0.1	−14.6	− 1.7	− 5.3
5	− 1.6	+ 5.4	+17.7	− 5.2	+ 0.9	−13.7	+ 9.1	+ 3.8
6	+ 3.0	+ 8.4	+ 5.3	+ 0.1	+ 0.2	−13.5	+ 1.4	+ 5.2
7	− 8.7	− 0.3	−11.1	−11.0	− 0.9	−14.4	− 9.3	− 4.1
8	+ 4.6	+ 4.3	−17.7	−28.7	+ 8.5	− 5.9	+ 8.6	+ 4.5
9	+ 1.3	+ 5.6	+11.4	−17.3	+ 4.3	− 1.6	+ 6.3	+10.8
10	− 5.6	+ 0.0	+17.3	+ 0.0	+ 1.6	+ 0.0	−10.8	+ 0.0

The relative errors of the 10 centimeters of the first decimeters of Q_3 and Q_4 were found to be as follows. The centimeter subdivisions of W and Q_1 were not investigated.

Spaces.	Q_3	Σ	Q_4	Σ
1	+0.5 μ	+0.5 μ	+1.4 μ	+1.4 μ
2	−5.0	−4.5	−1.1	−0.8
3	−2.8	−7.3	−4.7	−5.0
4	−0.8	−8.1	−4.0	−9.0
5	+1.9	−6.2	+4.1	−4.9
6	+0.3	−5.9	+5.1	+0.2
7	+1.1	−4.8	−3.1	−2.9
8	+3.1	−1.7	+0.4	−2.5
9	+1.3	−0.4	+4.9	+2.4
10	+0.4	+0.0	−2.4	+0.0

The errors of the one-tenth millimeter subdivisions seem to be inappreciable, indicating that the screw employed has no sensible periodic error which is a function of a single revolution.

With regard to the decimeter subdivisions, it should be said that the graduation was done in three operations, the second zero being at about the thirty-third centimeter, and the third at about the sixty-sixth centimeter.

XVII.

OBSERVATIONS OF VARIABLE STARS IN 1884.

BY EDWARD C. PICKERING.

Communicated March 11, 1885.

IN the communication entitled "Recent Observations of Variable Stars,"[1] it was stated that a similar circular would be published early in 1885. The friendly co-operation of several astronomers interested in the subject makes it practicable to present on this occasion a much fuller view of the progress of observation, in Europe as well as in America, than could be given last year. The various observers are named below in alphabetical order, with the abbreviations employed to designate them in the subsequent tabular statements.

B. These observations were made by Mr. T. W. Backhouse, at Sunderland, England. The instruments employed were a refracting telescope by Cooke, aperture 4½ inches, with magnifying powers 38 and 75; the finder of this telescope, power 9; a field-glass and an opera-glass, with powers 4 and 2.2 respectively; other observations were made with the naked eye. The comparisons were made either in grades, in fractions of the interval between two comparison stars, or by approximate differences. A copy of the observations for 1884 has been received at the Harvard College Observatory.

C. These observations were made by Mr. S. C. Chandler, Jr., at the Harvard College Observatory. The telescope is by Clacey; aperture 6¼ inches, magnifying power generally 45, sometimes 125 or 200. The observations were made by Argelander's method. Most of them were made before April 28, and they were discontinued after June 30, owing to the requirements of other researches. They are not likely to be resumed at present.

D. These observations were made by Dr. N. C. Dunér, at the Observatory of Lund, Sweden, according to the method of Argelander.

E. These observations were made by Mr. John H. Eadie, at Bayonne, New Jersey. The telescope employed was made by John Byrne; its aperture is 3¼ inches, and the lowest magnifying power

[1] Proc. Amer. Acad. of Arts and Sciences, XIX. 296.

about 50. Argelander's method of comparison is used. A copy of the observations has been furnished to the Harvard College Observatory.

Hg. These observations were made by Dr. E. Hartwig, formerly of Strassburg, Germany, at present of Dorpat, Russia. Since his removal to Dorpat, circumstances have prevented Dr. Hartwig from making frequent observations of variable stars.

Hn. These observations were made by the Rev. J. Hagen, S. J., at the College of the Sacred Heart, Prairie du Chien, Wisconsin. The instrument is a telescope by Merz; its aperture is 3 inches. The observations were made by the division into tenths of the interval between two comparison stars. A copy of the observations has been furnished to the Harvard College Observatory. Messrs. Zwack and Zaiser have taken part in the work as assistants.

K. These observations were made by Mr. George Knott, at Knowles Lodge, Cuckfield, Hayward's Heath, England. The telescope employed was made by Alvan Clark and Sons; its aperture is 7½ inches, and that of the finder 2 inches. The variable is compared with stars differing little from it in brightness; the magnitudes of the comparison stars, and sometimes the magnitude of the variable, are determined by the method of limiting apertures.

P. These observations were made by Mr. H. M. Parkhurst, at Brooklyn, N. Y. The instrument is a telescope made by Fitz; its aperture is 9 inches, and the magnifying powers employed are 56 and 150. Many of the observations were made by Argelander's method, and the remainder with photometric apparatus devised by Mr. Parkhurst, and partially described in the previous circular. A copy of the observations has been furnished to the Harvard College Observatory.

Sk. Professor Safarik of Prague, Austria, has published a notice of his observations of variable stars in the Vierteljahrsschrift der Astronomischen Gesellschaft, XIX. 144, from which the memoranda given in this circular are derived.

Sr. These observations were made according to Argelander's method by Mr. E. F. Sawyer, at Cambridgeport, Massachusetts, by means of an opera-glass for the brighter stars and of a field-glass for the others. The same plan of observation will be followed during 1885.

W. These observations were made by Dr. F. Wilsing, at the Astrophysikalisches Observatorium, Potsdam, Germany. The wedge photometer was employed in part of the comparisons, but in such cases estimates in grades of the difference in brightness between the stars compared were almost always added. These estimates appear to be somewhat more accurate than the photometric observations, which will

therefore be employed in future chiefly in determining the brightness of the comparison stars.

Zk. The observations of Assistant G. Zwack have already been mentioned under the heading Hn.

Zr. The observations of Assistant Zaiser have already been mentioned under the heading Hn.

The summary of the progress of observation during 1884 is contained in the last column of Table I. The preceding columns are repeated, after correcting some numerical errors, from the corresponding table in the statement published last year. The first column of the left-hand page gives a provisional number for designating the star. This number is taken from Schönfeld's Catalogue when the star occurs there; in other cases, a letter is added to the number. Other letters may be employed in effecting additional interpolation. The second column contains numbers from the Photometric Catalogue called Harvard Photometry, and published in Volume XIV. of the Annals of the Harvard College Observatory. The following columns contain the usual designation of the star, its right ascension and declination for 1875, magnitude at maximum and minimum, and period in days.

The first column of the right-hand page repeats the number to be used for the provisional designation of the star. The second gives the class to which the star belongs, upon the system of classification employed in the Proceedings of the American Academy of Arts and Sciences, XVI. 257. Upon this system, Class I. includes temporary stars; Class II., stars undergoing large variations in periods of several months; Class III., irregularly variable stars undergoing but slight changes in brightness; Class IV., variable stars of short period, like β *Lyræ* or δ *Cephei;* Class V., Algol stars, or those which at regular intervals undergo sudden diminutions of light, lasting for but a few hours. The third column gives the name of the discoverer, and the fourth column the date.

The last column, as above stated, contains the number of nights on which each star was observed by the astronomer whose designation is attached to the number. A dash preceding a designation shows that the star has been observed, but that the number of nights has not been furnished. The abbreviations employed have been explained above. The letter K. is preceded by two numbers, the first of which relates to observations made in 1883.

Table I. is followed by a series of remarks containing observed dates of maximum and minimum, and other information received from the observers with regard to particular stars.

TABLE I. — VARIABLE STARS.

No.	H.P.	Name.	R. A. 1875.			Dec. 1875.		Max.	Min.	Per.
			h.	m.	s.	°	′	m.	m.	d.
0a	—	Ceti	0	15	26	—20	45.1	5.2	7.0	—
1	51	T Cassiopeiæ		16	29	+55	5.9	6.5 — 7.0	11 — 11.2	436
2	54	R Andromedæ		17	28	+37	58.0	5.6 — 8.6	<12.8	404.7
3	—	S Ceti		17	42	—10	1.3	7.0 — 8.0	<10.7	323.6
4	—	B Cassiopeiæ		17	52	+63	27.2	>1	?	—
5	—	T Piscium		25	31	+13	54.6	9.5 — 10.2	10.5 — 11.0	Irr.
6	94	α Cassiopeiæ		33	25	+55	51.1	2.2	2.8	Irr.
6a	—	U Cephei		51	18	+81	12.1	7.0	9.5	2.5
7	—	S Cassiopeiæ	1	10	30	+71	57.2	6.7 — 8.5	<13	615
8	—	S Piscium		11	2	+ 8	16.3	8.8 — 9.3	<13	406.6
8a	—	Piscium		16	22	+12	12.7	10	14	—
8b	—	Ceti		19	31	— 4	30.6	6.5	7.8	—
8c	—	R Sculptoris		21	13	—33	11.5	5½	7½	207
9	—	R Piscium		24	12	+ 2	14.1	7.4 — 8.3	<12.5	345
10	—	S Arietis		57	55	+11	55.5	9.1 — 9.8	<13	288.8
11	—	R Arietis	2	9	1	+24	28.4	7.6 — 8.5	11.9 — 12.7	186.2
12	370	ο Ceti		13	1	— 3	32.7	1.7 — 5.0	8 — 9	331.3
13	—	S Persei		13	54	+58	0.8	8.5 ?	<9.7	—
14	—	R Ceti		19	39	— 0	44.6	7.9 — 8.7	<12.8	167.1
15	—	T Arietis		41	22	+16	59.8	7.9 — 8.2	9.4 — 9.7	324
16	489	ρ Persei		57	10	+38	21.3	3.4	4.2	Irr.
17	496	β Persei	3	0	2	+40	28.4	2.2	3.7	2.9
18	—	R Persei		22	6	+35	14.3	8.1 — 9.2	12.5	208.8
19	657	λ Tauri		53	45	+12	8.2	3.4	4.2	4.0
20	—	T Tauri	4	14	43	+19	14.3	9.2 — 11.5	12.8 — <	Irr
21	—	R Tauri		21	27	+ 9	52.9	7.4 — 9.0	<13	325.6
22	—	S Tauri		22	22	+ 9	40.1	9.9	<13	378
22a	—	Doradus		35	19	—62	19.4	5½	6½	—
23	—	V Tauri		44	48	+17	19.6	8.3 — 9.0	<12.8	168.6
24	—	R Orionis		52	13	+ 7	56.3	8.7 — 8.9	<13	378.8
25	877	ε Aurigæ		53	0	+43	38.2	3.0	4.5	Irr
26	880	R Leporis		53	55	—14	59.7	6 — 7	8.5 ?	437.8
27	—	R Aurigæ	5	7	12	+53	20.6	6.5 — 7.4	12.5 — 12.7	465
27a	—	S Aurigæ		18	52	+34	2.3	9.4	<13	—
28	—	S Orionis		22	50	— 4	47.5	8.3 ?	<12.3	—
29	1005	δ Orionis		25	37	— 0	23.6	2.2 ?	2.7	Irr.
29a	—	Orionis		29	42	— 5	33.5	10	13	—
30	1091	α Orionis		48	24	+ 7	23.3	1	1.4	Irr.
31	1160	η Geminorum	6	7	20	+22	22.4	3.2	3.7 — 4.2	229.1
31a	—	Monocerotis		16	25	— 2	8.1	7	<10	—
32	1205	T Monocerotis		18	29	+ 7	9.1	6.2	7.6	26.8
33	—	R Monocerotis		32	21	+ 8	50.7	9.5	11.5	Irr.
34	1256	S Monocerotis		34	6	+10	0.5	4.9	5.4	3.4
35	—	R Lyncis		50	50	+55	30.2	9 ?	<12.3	—
36	1334	ζ Geminorum		56	41	+20	45.1	3.7	4.5	10.2
37	—	R Geminorum		59	40	+22	58.8	6.6 — 7.3	<12.3	371.0
38	—	R Canis min.	7	1	50	+10	13.1	7.2 — 7.9	9.5 — 10.0	335.0
38a	—	Puppis		9	48	—44	26.2	3½	<6	135
38b	—	V Geminorum		16	10	+13	21.8	8.5	12 — 13½	276
38c	1417	U Monocerotis		24	50	9	31.0	6.0	7.2	46.0
39	—	S Canis min.		25	50	+ 8	35.0	7.2 — 8.0	<11	332.2
40	—	T Canis min.		27	3	+12	0.6	9.1 — 9.7	<13	336.2
40a	—	Canis min.		34	34	+ 8	40.2	8½	13.5	405

TABLE I.—VARIABLE STARS.

No.	Class.	Discoverer.	Date.	Observations, 1884.
0*a*	—	Chandler	1881	35 Sr.
1	II.	Krüger	1870	4 C. 9 E. - Sk. 4 W.
2	II.	Argelander	1858	7 C. 3 D. 7 E. - Sk. 44 Sr. 8 W.
3	II.	Borelly	1872	1 C. 13 E. 2 P. - Sk.
4	I.	Tycho Brahe	1572	—
5	II.	Luther	1855	3 C. 7 E. 2 P.
6	III.	Birt	1831	1 B. 18 E. 27 Sr.
6*a*	V.	Ceraski	1880	4 Hg. 10 Hn. 7 W. 3 Zk.
7	II.	Argelander	1861	4 C. 26 P. - Sk.
8	II.	Hind	1851	2 C. 6 E.
8*a*	—	Peters	1880	10 P.
8*b*	—	Gould	1874?	4 Hn.
8*c*	II.	Gould	1872?	—
9	II.	Hind	1850	2 C. 1 E. - Sk.
10	II.	Peters	1865	4 C. 5 P.
11	II.	Argelander	1857	6 C. 18 Hn. 8 P. - Sk. 12 Zk.
12	II.	Fabricius	1596	17 B. 9 C. - Hg. - Sk. 24 Sr.
13	II.	Krüger	1873	5 C. 9 Hn. - Sk. 2 Zk.
14	II.	Argelander	1866	9 C. 5 P. - Sk.
15	II.	Auwers	1870	3 C. 22 Hn. - Sk. 12 Zk.
16	II.?	Schmidt	1854	47 Sr.
17	V.	Montanari	1669	6 B. 3 Hg. 1 Sr.
18	II.	Schönfeld	1861	10 C. 9 E. 19 Hn. - Sk. 4 W. 12 Zk.
19	V.	Baxendell	1848	2 B. 7 Zr.
20	—	Hind	1861	1 C. 4,1 K. 3 P. - Sk.
21	II.	Hind	1849	5 C. 10 E. 3 P. - Sk.
22	II.	Oudemans	1855	5 C. 8 E. 2 P. - Sk.
22*a*	—	Gould	1874?	—
23	II.	Auwers	1871	6 C. 9 P.
24	II.	Hind	1848	7 C. 1 E. 1,0 K.
25	III.	Fritsch	1821	6 P. 48 Sr.
26	II.	Schmidt	1855	5 C. 2,0 K. - Sk. 11 Sr.
27	II.	At Bonn	1862	7 C. 21 Hn. 2,0 K. - Sk. 14 Zk.
27*a*	II.	Dunér	1881	7 C. 5 D. 9 P.
28	II.	Webb	1870	6 C. 8 E. 7,6 K. 1 P. - Sk.
29	III.	J. Herschel	1834	28 Sr.
29*a*	—	Bond	1868	1 E. 1 P.
30	III.	J. Herschel	1836	3 Zr.
31	II.?	Schmidt	1866	6 B. 12 Sr.
31*a*	—	Schönfeld	1883	12 Sr.
32	IV.	Gould	1871	83 Sr.
33	II.	Schmidt	1861	3 C.
34	IV.	Winnecke	1867	29 Sr.
35	II.	Krüger	1874	11 C. 11 P.
36	IV.	Schmidt	1844	1 B. 24 Sr. 1 Zr.
37	II.	Hind	1848	6 C. 1,2 K. 25 P. 3 Sr.
38	II.	At Bonn	1854	9 C. 19 Hn. 2,0 K. 7 W. 17 Zk.
38*a*	II.	Gould	1872	7 C.
38*b*	II.	Baxendell	1880	23,9 K.
38*c*	II.?	Gould	1873	49 Sr.
39	II.	Hind	1856	9 C. 6,4 K. 6 W.
40	II.	Schönfeld	1865	4 C. 1,0 K.
40*a*	II.	Baxendell	1879	10 C. 29,6 K.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
41	—	S Geminorum	7 35 32	+28 44.6	8.2 — 8.7	<13	294.2
42	—	T Geminorum	41 48	+24 2.7	8.1 — 8.7	<13	288.1
42a	—	S Puppis	48 6	—47 8.8	7¼	9	—
43	—	U Geminorum	47 41	+22 19.7	8.9 — 9.7	13.1	Irr.
43a	—	Puppis	55 0	—12 32	8½	<14	310
44	—	R Cancri	8 9 40	+12 6.5	6.2 — 8.3	<11.7	354.4
45	—	V Cancri	14 36	+17 40.9	6.8 — 7.2	<12	272
46	—	U Cancri	28 37	+19 19.5	8.2 — 10.4	<13	305.7
47	—	S Cancri	36 48	+19 29.0	8.2	9.8	9.5
48	—	S Hydræ	47 3	+ 3 32.4	7.5 — 8.5	<12.2	256.4
49	—	T Cancri	49 32	+20 19.7	8.2 — 8.5	9.3 — 10.5	484.2
50	—	T Hydræ	49 35	— 8 39.8	7.0 — 8.1	<12.5	289.4
50a	—	R Carinæ	9 29 6	—62 14.2	4.4	9.3	313
51	—	R Leonis min.	38 4	+35 5.2	6.1 — 7.5	<11.0	374.7
52	1752	R Leonis	40 50	+12 0.5	5.2 — 6.4	9.4 — 10.0	312.6
52a	—	l Carinæ	41 49	—61 55.9	3.7	5.2	31.2
52b	—	Leonis	53 8	+21 51.6	8½	8.6 <13	280 ?
52c	—	Antliæ	10 4 22	—37 7.1	6½	<8	—
52d	—	Carinæ	5 23	—60 56.3	6¼	9	—
52e	—	U Leonis	17 21	+14 38.1	9½	Inv.	—
52f	1869	Hydræ	31 22	—12 44.1	4½	6	—
53	1880	R Ursæ maj.	35 47	+69 25.9	6.0 — 8.1	12	303.4
54	—	η Argus	40 13	—59 1.6	>1	6.3	Irr.
54a	—	T Carinæ	50 18	—59 51.2	6.2	6.9	—
55	—	R Crateris	54 25	—17 39.2	>8	<9	—
56	—	S Leonis	11 4 23	+ 6 8.5	9.0 — 9.7	<13	187.6
57	—	T Leonis	32 2	+ 4 3.9	10 ?	<13	—
58	—	X Virginis	55 27	+ 9 46.1	7.8 ?	<10	—
59	—	R Comæ	57 51	+19 28.8	7.4 — 8.0	<13	368
60	—	T Virginis	12 8 12	— 5 20.4	8.0 — 8.8	<13	337
61	—	R Corvi	13 10	—18 33.5	6.8 — 7.3	<11.5	318.6
61a	—	— Virginis	27 26	— 3 43.8	8	14	210±
62	—	T Ursæ maj.	30 42	+60 10.6	7.0 — 8.3	12.2	255.6
63	2147	R Virginis	32 10	+ 7 40.6	6.5 — 7.5	10.0 — 10.9	145.7
63a	—	R Muscæ	34 28	—68 43.3	6.6	7.3	0.9
64	—	S Ursæ maj.	38 28	+61 46.7	7.7 — 8.2	10.2 — 11.1	224.8
65	—	U Virginis	44 46	+ 6 14.0	7.7 — 8.1	12.2 — 12.8	207.4
66	—	W Virginis	13 19 35	— 2 43.4	8.7 — 9.2	9.8 — 10.4	17.3
67	—	V Virginis	21 21	— 2 31.4	8.0 — 9.0	<13	251
68	2275	R Hydræ	22 53	—22 38.0	4.0 — 5.5	10 ?	409.3
69	2289	S Virginis	26 29	— 6 33.0	5.7 — 7.8	12.5	374.0
69a	—	Virginis	14 3 37	—12 42.7	9	14	—
69b	—	R Centauri	7 35	—59 19.8	6	10	—
70	—	T Bootis	8 14	+19 39.1	9.7 ?	<13	—
71	—	S Bootis	18 41	+54 22.7	8.1 — 8.5	13.2	272.4
72	—	R Camelopardi	27 8	+84 23.8	7.9 — 8.6	12 ?	266.2
73	2445	R Bootis	31 41	+27 16.9	5.9 — 7.5	11.3 — 12.2	223.0
73a	2459	Bootis	37 56	+27 3.6	5.2	6.1	370 ?
73b	—	Bootis	48 33	+18 12.1	9.1	12.0 — 13.6	173.8
74	2506	δ Libræ	54 18	— 8 1.2	4.9	6.1	2.3
74a	—	Libræ	15 3 37	—19 33.9	10	<13.5	700±
74b	—	R Triang. Austr.	8 37	—66 2.1	6.6	8.0	3.4
75	—	U Coronæ	13 6	+32 6.4	7.6	8.8	3.5
76	—	S Libræ	14 13	—19 56.1	8.0	12.5 ?	—
77	—	S Serpentis	15 48	+14 45.9	7.6 — 8.6	12.5 ?	361.0

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1884.
41	II.	Hind	1848	4 C.
42	II.	Hind	1848	8 C. 4 W.
42*a*	—	Gould	1874?	—
43	II.?	Hind	1855	18 C. 38,25 K. - Sk.
43*a*	II.	Pickering	1881	3 C.
44	II.	Schmidt	1829	5 C.
45	II.	Auwers	1870	9 C. 15 P. - Sk.
46	II.	Chacornac	1853	9 C. 16,15 K. - Sk. 3 W.
47	V.	Hind	1848	2 Hn.
48	II.	Hind	1848	5 C.
49	II.	Hind	1850	5 C. 18 P.
50	II.	Hind	1851	10 C.
50*a*	II.	Gould	1871	—
51	II.	Schönfeld	1863	7 C. 58 P. 20 Sr.
52	II.	Koch	1782	12 C. - Sk. 42 Sr. 12 W.
52*a*	—	Gould	1871	5 C.
52*b*	II.	Becker	1882	3 C.
52*c*	—	Gould	1872	—
52*d*	—	Gould	1871	—
52*e*	—	Peters	1876	—
52*f*	—	Gould	1871	—
53	II.	Pogson	1853	5 C. 22 Hn. 3,5 K. 32 Sr. 7 W. 15 Zk.
54	II.?	Burchell	1827	—
54*a*	—	Thome	1872	—
55	II.	Winnecke	1861	6 C. - Sk.
56	II.	Chacornac	1856	5 C. 3 W.
57	II.	Peters	1865	1 C.
58	II.	Peters	1871	5 C.
59	II.	Schönfeld	1856	4 C. 8,3 K. 19 P.
60	II.	Boguslawski	1849	5 C. 4 W.
61	II.	Karlinski	1867	10 C. - Sk.
61*a*	II.	Henry	—	9 C.
62	II.	Hencke	1856	11 C. - Sk. 26 Sr. 10 W.
63	II.	Harding	1800	8 C. 33 Sr. 10 W.
63*a*	IV.	Gould	1871	—
64	II.	Pogson	1853	11 C. 3,5 K. - Sk. 81 Sr. 10 W.
65	II.	Harding	1831	8 C. - Sk. 7 Sr.
66	II.?	Schönfeld	1866	11 C.
67	II.	Goldschmidt	1857	11 C. 9 P. 7 W.
68	II.	Miraldi	1704	4 C. - Sk. 26 Sr.
69	II.	Hind	1852	10 C. 16,0 K.
69*a*	II.	Palisa	1880	7 C.
69*b*	—	Gould	1871	—
70	I.?	Baxendell	1860	—
71	II.	At Bonn	1860	8 C. 11 Hn. - Sk. 3 W. 7 Zk.
72	II.	Hencke	1858	7 C. 10 P. - Sk. 4 W.
73	II.	At Bonn	1858	12 C. 6 Hn. - Sk. 46 Sr. 5 Zk.
73*a*	—	Schmidt	1867	9 C. 4 Zr.
73*b*	II.	Baxendell	1880	—
74	V.	Schmidt	1859	4 Zr.
74*a*	II.	Palisa	1878	5 C.
74*b*	IV.?	Gould	1871	—
75	V.	Winnecke	1869	8 Hn. 5 Zk.
76	II.	Borelly	1872	7 C. 8 P. - Sk.
77	II.	Harding	1828	10 C. 8 Hn. 7 Zk.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.			Dec. 1875.		Max.	Min.	Per.
			h.	m.	s.	°	'	m.	m.	d.
78	2553	S Coronæ	15	16	18	+31	49.1	6.1 — 7.8	11.9 — 12.5	361.0
78a	—	Libræ		34	46	—20	46.5	9	<14	—
79	2639	R Coronæ		43	25	+28	32.5	5.8	13.0	Irr
80	2647	R Serpentis		44	56	+15	30.8	5.6 — 7.6	<11	357.6
80a	—	V Coronæ		45	4	+39	57.0	7.7	12	360.0
81	—	R Libræ		46	32	—15	51.7	9.2 — 10.0	<18	723
82	2678	T Coronæ		54	16	+26	16.5	2.0	9.5	—
83	—	R Herculis	16	0	37	+18	42.5	8.0 — 9.0	<13	319.0
83a	—	W Scorpii		4	28	—19	48.6	10	<13	224.3
84	—	T Scorpii		9	36	—22	39.9	7	<10	—
85	—	R Scorpii		10	12	—22	38.2	9 ? — 10.5	<12.5	223
86	—	S Scorpii		10	13	—22	35.2	9.1 — 10.5	<12.5	176.9
86a	—	Ophiuchi		14	40	— 7	24.0	9.0	<13.5	326
87	—	U Scorpii		15	16	—17	35.3	9 ?	<12	—
87a	—	Ophiuchi		19	46	—12	8.5	7.5	10.5	365
88	—	U Herculis		20	16	+19	10.8	6.6 — 7.7	11.4 — 11.6	406.3
89	2772	g Herculis		24	32	+42	9.6	5	6.2	Irr.
90	—	T Ophiuchi		26	35	—15	51.8	10	<12.5	—
91	—	S Ophiuchi		27	4	—16	53.7	8.3 — 9.0	<12.5	233.8
91a	—	W Herculis		30	48	+37	35.6	8.0	<14.5	289
91b	—	Urs. Min.		31	40	+72	31.9	8.6	10.5	180 ?
91c	—	R Draconis		32	22	+67	0.7	7.2	13<	245.9
92	2828	S Herculis		46	13	+15	9.2	5.9 — 6.8	11.5 — 12.2	303
93	2830	Ophiuchi		52	30	—12	42.0	5.5	12.5	—
93a	—	V Herculis		53	41	+35	15.6	9.0	11.7	—
94	—	R Ophiuchi	17	0	36	—15	55.5	7.6 — 8.1	<12	302.4
95	2879	α Herculis		8	57	+14	32.1	3.1	3.9	Irr.
95a	2883	U Ophiuchi		10	12	+ 1	21.0	6.1	6.8	0.9
96	2890	u Herculis		12	42	+33	14.1	4.6	5.4	38.5
97	—	Serpentarii		23	9	—21	22.4	>1	?	—
98	2972	X Sagittarii		39	41	—27	46.8	4	6	7.0
99	3035	W Sagittarii		57	2	—29	35.1	5	6.5	7.6
100	—	T Herculis	18	4	22	+31	0.1	7.2 — 8.3	11.4 — 12.1	165.1
101	—	T Serpentis		22	43	+ 6	13.1	9.1 — 10.0	<12.8	342.3
102	—	V Sagittarii		24	4	—18	20.9	7.5 ?	9.5 ?	—
103	—	U Sagittarii		24	32	—19	12.7	7.0	8.3	6.7
104	—	T Aquilæ		30	45	+ 8	36.9	8.8	9.5	Irr.
105	3176	R Scuti		40	49	— 5	50.2	4.7 — 5.7	6.0 — 8.5	71.1
105a	—	κ Pavonis		44	3	—67	23.2	4.0	5.5	9.1
106	3193	β Lyræ		45	28	+33	18.0	3.4	4.5	12.9
107	3224	R Lyræ		51	32	+43	47.1	4.3	4.6	46.0
108	—	S Coron. Austr.		52	43	—37	7.2	9.8	11.5 ?	6.1
109	—	R Coron. Austr.		53	29	—37	7.2	10.5 — 11.5	<12.5	31
110	—	R Aquilæ	19	0	21	+ 8	2.6	6.4 — 7.4	10.9 — 11.2	345.1
111	—	T Sagittarii		9	1	—17	11.2	7.6 — 8.1	<11	381
112	—	R Sagittarii		9	21	—19	31.5	7.0 — 7.2	<12	270.0
113	—	S Sagittarii		12	7	—19	15.1	9.7 — 10.4	<12.7	230
114	3395	R Cygni		33	28	+49	55.1	5.9 — 8.0	13	425.8
115	—	11 Vulpeculæ		42	26	+27	0.5	3	?	—
116	—	S Vulpeculæ		43	16	+26	58.7	8.4 — 8.9	9.0 — 9.5	67.5
117	3434	χ Cygni		45	46	+32	36.0	4.0 — 6.0	12.8	[redacted]
118	3436	η Aquilæ		46	6	+ 0	41.2	3.5	4.7	7.2
119	—	S Cygni	20	2	53	+57	37.6	8.8 — 9.5	<13	322.8
120	—	R Capricorni		4	17	—14	38.2	8.8 — 9.7	<13	347
121	—	S Aquilæ		5	52	+15	14.9	8.9 — 9.9	10.7 — 11.8	147.2

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1884.
78	II.	Hencke	1860	5 C. 8 Hn. 2,1 K. - Sk. 45 Sr. 10 W
78*a*	—	Peters	1878	—
79	II.?	Pigott	1795	6 C. - Sk. 88 Sr. 10 W.
80	II.	Harding	1826	9 C. 5 P.
80*a*	II.	Dunér	1878	3 C. 15 D. - Sk. 8 W.
81	II.	Pogson	1858	8 C. 1,1 K.
82	I.	Birmingham	1866	21 B. 7 Hn. 3,1 K. 2 P. 4 Zk.
83	II.	At Bonn	1855	2 C. 10 Hn. 4 P. 6 Zk.
83*a*	II.	J. Palisa	1877	4 C. 6 P.
84	I.	Auwers	1860	3 C.
85	II.	Chacornac	1853	6 C. 11,1 K. 6 P.
86	II.	Chacornac	1854	6 C. 7,1 K. 6 P.
86*a*	II.	Schönfeld	1881	7 C.
87	I.?	Pogson	1863	—
87*a*	—	Dunér	1881	8 C. 11 D.
88	II.	Hencke	1860	3 C.
89	III.	Baxendell	1857	83 Sr.
90	II.	Pogson	1860	2 C.
91	II.	Pogson	1854	2 C. 1 Hn.
91*a*	—	Dunér	1880	14 C. 8 Hn. 9 P. 4 W. 5 Zk.
91*b*	II.	Pickering	1881	11 C. 1,1 K. - Sk.
91*c*	II.	Geelmuyden	1876	9 C. - Sk. 35 Sr. 12 W.
92	II.	At Bonn	1856	4 C. 10 Hn. - Sk. 7 Zk.
93	I.	Hind	1848	—
93*a*	II.	Baxendell	1880	9 C.
94	II.	Pogson	1853	8 C.
95	III.	W. Herschel	1795	3 Zr.
95*a*	V.	Sawyer	1881	5 Sr. 4 W.
96	III.	Schmidt	1869?	3 Zr.
97	I.	Fabricius	1604	2 Zr.
98	IV.	Schmidt	1866	29 Sr.
99	IV.	Schmidt	1866	85 Sr.
100	II.	At Bonn	1857	7 C. 11 Hn. - Sk. 7 Sr. 6 W. 6 Z
101	II.	Baxendell	1860	2 C.
102	II.	Quirling	1865	2 C.
103	IV.	Schmidt	1866	1 C.
104	II.	Winnecke	1860	1 C.
105	II.	Pigott	1795	89 Sr.
105*a*	IV.	Thome	1872	—
106	IV.	Goodricke	1784	10 Zr.
107	II.?	Baxendell	1856	62 Sr.
108	IV.?	Schmidt	1866	—
109	II.?	Schmidt	1866	—
110	II.	At Bonn	1856	1 C. - Sk.
111	II.	Pogson	1863	1 C. - Sk.
112	II.	Pogson	1858	5 P. - Sk.
113	II.	Pogson	1860	10 P. - Sk.
114	II.	Pogson	1852	8 C. 9 Hn. 12 P. - Sk. 5 Zk.
115	I.	Anthelm	1670	2 Zr.
116	II.	Hind	1861	4 C. 13 Hn. 5,0 K. 4 W. 6 Zk.
117	II.	Kirch	1686	2 C. 2 Hn. 56 P. - Sk. 41 Sr. 5 W. 2
118	IV.	Pigott	1784	79 Sr.
119	II.	At Bonn	1860	6 C. 7,17 K. 18 P.
120	II.	Hind	1848	- Sk.
121	II.	Baxendell	1863	2 C. 11 E. 2 P. 12,14 K.

TABLE I. — *Continued.*

No.	H. P.	Name.	R. A. 1875.			Dec. 1875.		Max.	Min.	Per.
			h.	m.	s.	°	′	m.	m.	d.
122	—	R Sagittæ	20	8	22	+16	21.0	8.5 — 8.7	9.8 — 10.4	70.4
123	—	R Delphini		8	53	+ 8	42.7	7.6 — 8.5	12.8	284.0
124	3547	P Cygni		13	11	+37	38.7	3 — 5	<6	—
125	—	U Cygni		15	44	+47	30.1	7.8 !	9.8 !	—
126	3557	R Cephei		34	29	+88	45.2	5 !	10 !	—
126a	—	— Cygni		37	17	+47	41.8	8	12	423.
127	—	S Delphini		37	19	+16	38.4	8.4 — 8.6	10.4 — 11.1	275.6
128	—	T Delphini		39	34	+15	56.7	8.2 — 8.9	<13	331.4
129	—	U Capricorni		41	11	—15	14.4	10.2 — 10.8	<13	203.5
130	3654	T Cygni		42	12	+33	55.0	5.5 ?	6 !	—
131	—	T Aquarii		43	20	— 5	36.5	6.7 — 7.0	12.4 — 12.7	203.2
132	—	R Vulpeculæ		58	49	+23	19.5	7.5 — 8.5	12.5 — 13.0	137.5
132a	—	Capricorni	21	0	19	—24	25.5	9½	14	—
132b	—	T Cephei		7	52	+67	58.9	5.6	9.5	382
133	—	T Capricorni		15	6	—15	41.4	8.9 — 9.7	<13	269.4
134	—	S Cephei		36	45	+78	3.6	7.4 — 8.5	11.5	485
134a	—	Nova Cygni		37	2	+42	18.2			—
135	3845	μ Cephei		39	41	+58	12.4	4 ?	5 ?	Irr.
136	—	T Pegasi	22	2	48	+11	55.7	8.8 — 9.3	<12.5	367.5
137	3981	δ Cephei		24	32	+57	46.6	3.7	4.9	5.4
137a	—	Lacertæ		37	43	+41	43.0	8.6	<13.5	315.
138	—	S Aquarii		50	25	—21	0.6	7.7 — 9.1	<11.5	279.4
139	4078	β Pegasi		57	45	+27	24.2	2.2	2.7	Irr.
140	—	R Pegasi	23	0	22	+ 9	52.1	6.9 — 7.7	12 ?	382.0
141	—	S Pegasi		14	14	+ 8	14.2	7.6	<12.2	—
142	4193	R Aquarii		37	21	—15	58.7	5.8 — 8.5	11 ?	388.0
143	4234	R Cassiopeiæ		52	4	+50	41.5	4.8 — 6.8	<12	425.9

REMARKS.

2. Max. 1883, December 1. Sr.

6. Light has remained nearly constant. Sr.

6a. Min. 1884, February 19, March 15, August 24, September 8. Hg.

12. Max. 1884, March 6 (approximate). Magn. 4.6. Sr.

16. Light has remained nearly constant. Sr.

17. Min. 1884, February 5, February 28, September 27. Hg. — 1884, November 29, 13h. 39m. G. M. T. Duration of observations, 4h. 35m. Sr.

25. Max. 1884, February 27. Min. 1884, January 18, March 18. Sr.

27a. Approximate period, 270 days. D.

29. Light apparently constant. Sr.

31a. Max. 1884, December 10 (approximate). Sr.

38b. Max. 1883, February 25. Magn. 8.6. K.

38c. Max. 1884, February 3, March 11. Min. 1884, January 5, February 24, April 9. Sr.

43. Max. 1883, October 25; 1884, January 28, October 24. Hg. — 1883, January 30 (approximate). Magn. about 9.5. 1884, January 26. Magn. 9.6. 1884, May 15 (approximate). 1884, October 22. Magn. 9.4. K.

46. Max. 1884, March 8. Magn. 8.5. K.

52. Max. 1884, February 5 (approximate). Magn. 6.7. Sr.

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1884.
122	II.?	Baxendell	1859	2 C. 22 E. 11 Hn. 1 P. 5 Zk.
123	II.	Hencke	1859	1 C. 8 Hn. - Sk. 4 Zk.
124	I.	Janson	1600	11 Zr.
125	II.	Knott	1871	3 C. 13,20 K. 16 P. - Sk.
126	II.?	Pogson	1856	1 C. 32 E. - Sk.
126a	II.	Birmingham	1881	- Hg. 11,8 K. 5 P.
127	II.	Baxendell	1860	2 C. 12 E. 8 P. - Sk.
128	II.	Baxendell	1863	4 C. 12 E. 18,17 K. - Sk. 4 W.
129	II.	Pogson	1858	13 P.
130	—	Schmidt	1864	7 E. 85 Sr.
131	II.	Goldschmidt	1861	3 C. 12 P. - Sk.
132	II.	At Bonn	1858	4 C. 12 E. 13,16 K. 2 P. 5 W.
132a	—	Peters	1867	12 C. 7 P.
132b	II.?	Ceraski	1878	12 E. - Hg. 29,25 K. - Sk. 4 W.
133	II.	Hind	1854	9 P.
134	II.	Hencke	1858	6 C. 12 E. - Sk.
134a	I.	Schmidt	1876	—
135	III.?	Hind	1848	2 B. 1 C. 12 E. 1 P. - Sk. 10 Zr.
136	II.	Hind	1863	2 C. 9 P.
137	IV.	Goodricke	1784	10 Zr.
137a	—	Deichmüller	1883	4 C. 7 E. - Hg. 6,16 K.
138	II.	Argelander	1853	4 P. - Sk.
139	III.	Schmidt	1847	9 B. 28 Sr.
140	II.	Hind	1848	1 C. 5 E. 1 P.
141	II.	Marth	1864?	4 C. 2 E.
142	II.	Harding	1811	4 C. 8 P. - Sk. 21 Sr.
143	II.	Pogson	1853	8 C. 1,0 K. 8 P. - Sk.

58. Max. 1884, August 29. Magn. 7.1. Sr.

62. Max. 1884, October 17. Magn. 7.8. Sr.

63. Max. 1884, April 11. Magn. 7.1. Sr.

64. Max. 1884, September 17 ± 1 day. Magn. 7.7. Sr.

73. Max. 1884, October 5. Magn. 7. 5. Sr.

78. Max. 1884, May 5. Magn. 7.3. Sr.

79. Slight fluctuations of light. Sr.

80a. Max. 1878, October 21.7. Period, 356.02 days. D.

85. Max. 1883, July 9. Magn. 10.1. K.

87a. V *Ophiuchi.* Approximate period 300 days. D.

89. Max. 1884, June 27, September 8, October 22. Min. 1884, May 30, August 4, September 23, November 14. Sr.

91c. Max. 1884, September 5. Magn. 7.5. Sr.

105. Max. 1884, June 7, August 3, October 17, November 30 (approximate). Min. 1884, July 12, September 15, November 11. Sr.

107. Two remarkable outbursts of light observed in 1884, from November 7 to 11, and from November 21 to 30. Sr.

117. Max. 1884, November 23. Magn. 5.4. Sr.

119. Max. 1884, early in December. K.

125. Max. 1884, February 1. Magn. 7.8. Period, 461 days. Min. 1883, June 14. Magn. 11.5. 1884, September 21. Magn. 11.1. K.

TABLE II.—Index to Designations.

Constellation.	R.	S.	T	U	V.	W.	Misc.
Andromeda	2	....	....	....	....	....	
Antlia	....	....	....	....	....	....	–,52c.
Aquarius	142	138	131	....	....	....	
Aquila	110	121	104	....	....	....	η,118.
Argo	....	....	....	....	....	....	η,54.
Aries	11	10	15	....	....	....	
Auriga	27	27a	....	....	....	....	ε,25.
Bootes	73	71	70	....	....	....	–,78a. –,73b.
Camelopardus	72	....	....	....	....	....	
Cancer	44	47	49	46	45	....	
Canis Minor	38	39	40	....	....	....	–,40a.
Capricornus	120	....	133	129	....	....	–,132a.
Carina	60a	....	54a	....	....	....	l,52a. –,52d.
Cassiopeia	143	7	1	....	....	....	B,4. a,6.
Centaurus	69b	....	....	....	....	....	
Cepheus	126	134	182b	6a	....	....	μ,135. δ,137.
Cetus	14	3	....	....	....	....	–,0a. –,8b. o,12.
Coma Beren.	59	....	....	....	....	....	
Corona Aus.	109	108	....	....	....	....	
Corona Bor.	79	78	82	75	80a	....	
Corvus	61	....	....	....	....	....	
Crater	55	....	....	....	....	....	
Cygnus	114	119	130	125	....	....	χ,117. P,124. –,126a. Nova, 134a.
Delphinus	123	127	128	....	....	....	
Doradus	....	....	....	....	....	....	–,22a.
Draco	91c	....	....	....	....	....	
Gemini	37	41	42	43	38b	....	η,31. ζ,36.
Hercules	83	92	100	88	93a	91a	g,89. a,95. u,96.
Hydra	68	48	50	....	....	....	–,52f.
Lacerta	....	....	....	....	....	....	–,137a.
Leo	52	50	57	52e	....	....	–,52b.
Leo Minor	51	....	....	....	....	....	
Lepus	26	....	....	....	....	....	
Libra	81	76	....	....	....	....	δ,74. –,74a. –,78a.
Lynx	85	....	....	....	....	....	
Lyra	107	....	....	....	....	....	β,106.
Monoceros	33	34	32	38c	....	....	–,31a.
Musca	63a	....	....	....	....	....	
Ophiuchus	94	91	90	95a	....	....	–,86a. –,87a. –,93. Nova, 97.
Orion	24	28	....	....	....	....	δ,29. –,29a. a,30.
Pavo	....	....	....	....	....	....	κ,105a.
Pegasus	140	141	136	....	....	....	β,139.
Perseus	18	13	....	....	....	....	ρ,16. β,17.
Pisces	9	8	5	....	....	....	–,8a.
Puppis	....	42a	....	....	....	....	–,38a. 43a.
Sagitta	122	....	....	....	....	....	
Sagittarius	112	113	111	103	102	99	X,98.
Scorpius	85	86	84	87	....	83a	
Sculptor	8c	....	....	....	....	....	
Scutum	105	....	....	....	....	....	
Serpens	80	77	101	....	....	....	
Serpentarius	....	....	....	....	....	....	See Ophiuchus.
Taurus	21	22	20	....	23	....	λ,19.
Triang. Aus.	74b	....	....	....	....	....	
Ursa Major	58	64	62	....	....	....	
Ursa Minor	....	....	....	....	....	....	–,91b.
Virgo	63	69	60	65	67	66	X,58. –,61a. –,69a.
Vulpecula	132	116	....	....	....	....	11,115.

128. Max. 1883, October 6. Magn. 9.6. 1884, September 15. Magn. 9.0. K.

132. Min. 1884, August 10. Magn. 12.7. Max. 1884, October 15. Magn. 7.6. K.

132*b*. Max. 1883, February 6. Magn. 6.3. 1884, March 7. Magn. 6.8. Min. 1883, August 23. Magn. 9.9. 1884, August 11. Magn. 9.7. K.

142. Max. 1883, December 25 (approximate). Magn. 6.3. Sr.

Table II. enables the number of any star in Table I. to be found from its usual designation. The constellations are arranged alphabetically in the first column, and the numbers are placed under the respective headings R, S, T, U, V, and W. Thus R, S, and T *Aquarii* are respectively Nos. 142, 138, and 131 in Table I. In the column headed "Misc." are given other designations; thus η *Aquilæ* is No. 118, and η *Argus* is No. 54. The number is preceded by a dash when no letter has been definitely assigned to the corresponding star; thus, –,52*c* in the second line indicates that a variable star in *Antlia*, No. 52*c* in Table I., has not been definitely designated by a letter.

The list of suspected variables, given in Table II. of the statement made in 1884, is not here repeated, since from the nature of the case such a list can only be provisional. It should be remarked that all the stars which were inserted on the authority of Dr. Peters are considered by him to be certainly variable. Dr. Dunér also furnishes the place of a variable star which he observed on 23 days in 1884; it was not given in either of the tables published last year. The place, reduced to 1875, is as follows: R. A. 14h. 24m. 41s., δ +39° 25′.2. The period is about 405 days. Four variable stars were detected by means of the observations made for the Cordoba Zone-Catalogue; see that work, p. xiv., and Dr. Gould's letter in the Astronomische Nachrichten, CXI. 63. The places for 1875 are as follows: R. A. 15h. 45m. 22s., Dec. —35° 55′.3; R. A. 22h. 10m. 53s., Dec. —30° 13′.6; R. A. 22h. 27m. 4s., Dec. —67° 55′.9; R. A. 23h. 49m. 58s., Dec. —50° 28′.9.

Table III. indicates the progress of observation of suspected variables given in Table II. of the statement made in 1884. The stars are designated in the first column by Mr. Chandler's provisional numbers, as in the previous statement. The second column gives the number of observations made by each observer, as in the last column of Table I. The third column indicates the result of these observations.

TABLE III. — Observations of Suspected Variables.

No.	Obs. 1884.	Results.	No.	Obs. 1884.	Results.
1	17 Hn.	Constant.	345	16 Hn. 14 Zk.	Constant.
9	11 P.		347	32 Hn. 29 Zk.	Constant.
47	2 B.		365	16 Hn. 16 Zk.	Constant.
73	7 P.		373	28 Hn. 25 Zk.	Constant.
81	1 B.		407	14 Hn. 14 Zk.	Doubtful.
87	9 P.		459	6 P.	Variable.
93	5 K.		471	4 P.	
111	2 B. 7 Hn. 4 Zk.	Constant.	509	1 B. 28 Hn. 20 Zk.	Constant.
113	5 Hn. 8 Zk	Constant.	547	2 B. 22 Hn. 12 Zk.	Constant.
139	5 P.		567	7 P.	
143	7 P.		601	5 P.	
145	8 Hn. 8 Zk.	Constant.	615	14 P.	
147	9 Hn. 8 Zk.	Constant.	625	13 Hn.	Constant.
205	21 P.	Prob. var.	635	2 K.	
294	11 P.	Variable.			

It is hoped that observers of variable stars will continue to furnish accounts of their work during each year as soon as possible after its close. It is desirable that these accounts should be received at the Harvard College Observatory as early as February 1 of the following year.

XVIII.

A PHOTOGRAPHIC STUDY OF THE NEBULA OF ORION.

By Edward C. Pickering.

Presented March 11, 1885

No portion of the heavens has been more carefully studied than that containing the Nebula of Orion. The monographs by Prof. G. P. Bond (Annals Harvard College Observatory, V.) and by Prof. E. S. Holden (Washington Astronomical Observations for 1878, Appendix I.) show the vast amount of material collected by eye observations. For a photographic study of the same region the following specimens are in the photographic collection of the Harvard College Observatory: —

A. Artotype enlargement of the first photograph of the nebula taken by Dr. Henry Draper, September 30, 1880. Exposure, 51 minutes.

B. Artotype enlargement of a photograph taken by Dr. Henry Draper, March 11, 1881. Exposure, 106 minutes.

C. The original negative taken by Dr. Henry Draper on March 14, 1882. Exposure, 136 minutes. This negative, except for a slight photographic blemish, is nearly identical with that from which D. was taken.

D. An enlarged glass positive of the second photograph taken by Dr. Henry Draper, March 14, 1882. Exposure, 137 minutes. This positive is a duplicate of that employed in making the paper prints, E. The two positives were taken, and any objects resembling stars, but not found on both, were assumed to be defects, and were painted out of the other positive by Dr. Draper.

E. Several artotype enlargements of the second photograph, taken March 14, 1882, by Dr. Henry Draper.

F. Carbon print of photograph taken by Mr. Common with his 3-foot reflector, January 30, 1883. Exposure, 39 minutes. Enlargement about 7 times.

G. Glass positive, — a direct copy of the negative taken by Mr. Common with his 3-foot reflector, February 26, 1883. Exposure, 60 minutes.

Admirable material is thus furnished for a comparison of the results of photographic and eye observations of this region. The photographs of the stars which are common to F and to the catalogue of Professor Bond (Annals, V. 270) were first compared by a method closely resembling that adopted by Argelander for the study of variable stars. Table I. gives the stars which were selected for standards, with which the others are to be compared. Each star in the photograph was then compared with two of these, — one a little brighter, the other a little fainter. The differences were estimated in grades. The sum of the two differences gave a measure of the interval between the two comparison stars. It frequently happened that no difference in brightness was perceptible between the star to be measured and one of the comparison stars. The number of measures of star intervals between the comparison stars is therefore less than the number of stars compared. In Table I. the successive columns give for each comparison star a designation, and the number and magnitude in the Bond catalogue. The next column gives the photographic magnitude, found by a process which will be detailed below. This is followed by the number of comparisons between each star and that following it, and the mean value of this difference in grades. The last column gives the assumed brightness in grades, and equals the number of grades by which each star is fainter than the first on the list.

TABLE I.

Desig.	Bond No.	Bond Magn.	Photog. Magn.	No. Comp.	Diff. Grades.	Grades.
a	570	9.4	9.4	1	1.0	0
b	505	11.3	9.5	3	3.0	1
c	523	10.1	9.8	2	2.5	4
d	479	10.0	10.1	5	2.4	7
e	449	10.5	10.4	8	3.0	9
f	427	10.7	10.9	8	3.0	12
g	506	11.3	11.3	5	2.0	15
h	458	11.2	11.6	7	4.2	17
i	373	12.0	12.3	3	3.0	21
j	409	13.9	13.0	9	2.9	24
k	378	14.8	13.8	6	3.0	27
l	490	14.2	14.8	5	2.8	30
m	737	15.0	15.6	....	....	33

The light of each star measured was next reduced to grades by the assumed light in grades of the comparison stars. Two values were found, — one derived from the brighter, the other from the fainter comparison star. In 50 cases the results agreed exactly, in 11 cases

they differed by one grade, and in one case only by two grades. The relation between the grades and the scale of magnitude of the Bond Catalogue was next found by grouping the stars by half-magnitudes. The middle points of each group are given in the first column of Table II., the number of stars in the group in the second column, and the mean of the corresponding values in grades in the third. Points were then constructed with the first and third columns as ordinates, and a smooth curve drawn through them. The comparative values of the

TABLE II.

Bond Magn.	No. Stars.	Gr.	Bond Magn.	No. Stars.	Gr.
9.5	2	0.5	13.0	7	22.7
10.0	10	6.1	13.5	4	27.5
10.5	11	9.4	14.0	14	27.2
11.0	11	11.9	14.5	....	....
11.5	12	16.8	15.0	9	30.6
12.0	3	18.7	16.5	1	37.0
12.5	9	24.8			

grades and magnitudes derived from this curve are given in Table III. Applying the results of this table to the last column of Table I. gives the fourth column of that table. The results for all the stars in the Bond Catalogue differing less than 1000″ in right ascension and declination from *θ′ Orionis*, the brightest star in the nebula, are given

TABLE III.

Gr.	Magn.	Gr.	Magn.	Gr.	Magn.
0	9.4	12	10.9	24	13.0
1	9.5	13	11.0	25	13.2
2	9.6	14	11.2	26	13.5
3	9.7	15	11.3	27	13.8
4	9.8	16	11.5	28	14.1
5	9.9	17	11.6	29	14.5
6	10.0	18	11.8	30	14.8
7	10.1	19	12.0	31	15.1
8	10.3	20	12.1	32	15.3
9	10.4	21	12.3	33	15.6
10	10.6	22	12.5	34	15.8
11	10.7	23	12.7	35	16.1

in Table IV. The first four columns give the number, difference in right ascension and declination from *θ′ Orionis*, and magnitude according to the Bond Catalogue. The fifth column gives the magnitude found as described above from F, the photograph of Mr. Common.

TABLE IV.

Bond No.	Δα.	Δδ.	Bond Magn.	Common Magn.	Draper Magn.	Resid. Common.	Resid. Draper.
308	−979.2	+ 13.9	9.9	10.0	....	+ 1	....
311	−928.2	+681.9	10.7	*a*	....	....	....
314	−923.1	−307.8	11.4	11.6	....	+ 2	....
315	−921.3	−825.4	10.2	10.3	....	+ 1	....
323	−886.4	−816.6	10.7	10.7	....	0	....
329	−867.6	−626.9	14.2	*d*	....	....	....
332	−863.9	−646.8	14.8	*d*	....	....	....
335	−851.6	−239.0	10.9	10.6	....	− 3	....
339	−839.7	−446.6	14.8	14.8	....	0	....
346	−833.4	−947.1	10.7	10.9	....	+ 2	....
347	−829.4	−462.6	14.8	*c*	....	....	....
363	−772.4	+ 66.3	10.7	10.3	....	− 4	....
370	−746.6	− 66.8	13.3	14.1	....	+ 8	....
373	−732.9	− 70.5	12.0	12.3	....	+ 3	....
377	−724.1	−525.0	11.3	11.5	....	+ 2	....
378	−720.3	−107.1	14.8	13.8	....	−10	....
382	−703.5	− 45.0	11.7	11.2	....	− 5	....
387	−687.0	−252.2	10.4	10.2	....	− 2	....
390	−644.9	+ 15.4	12.5	12.5	....	0	....
402	−635.1	−314.2	12.3	11.3	....	−10	....
409	−608.9	−539.2	13.9	13.0	....	− 9	....
413	−593.4	−776.6	15.0	14.8	....	− 2	....
419	−574.7	−701.7	14.2	*d*	....	....	....
423	−558.3	+790.8	10.8	*a*	....	....	....
427	−546.6	− 71.1	10.7	10.9	11.6	+ 2	+ 9
430	−542.2	−206.8	11.7	11.5	11.8	− 2	+ 1
435	−530.1	+ 30.9	13.1	12.0	....	−11	....
438	−525.9	+963.4	9.4	*a*	....	....	....
443	−519.6	+649.4	13.1	*c*	....	....	....
449	−495.5	+290.3	10.5	10.4	10.8	− 1	+ 3
458	−464.7	−107.8	11.2	11.6	12.0	+ 4	+ 8
464	−442.5	−946.4	14.2	14.8	....	+ 6	....
467	−431.0	−658.6	8.7	8.8	9.4	+ 1	+ 7
471	−420.0	−836.5	14.8	14.8	....	0	....
479	−400.4	+272.3	10.0	10.1	10.5	+ 1	+ 5
490	−380.7	− 49.3	14.2	14.8	....	+ 6	....
497	−356.1	−592.2	9.9	10.3	10.7	+ 4	+ 8
505	−309.6	−424.7	9.6	9.6	9.6	0	0
506	−306.0	+ 5.6	11.3	11.3	11.4	0	+ 1
508	−300.9	+704.3	12.3	14.5	....	+22	....
510	−290.9	−505.9	13.1	13.5	....	+ 4	....
516	−276.0	− 29.5	13.5	13.8	....	+ 3	....
523	−242.3	−116.0	10.1	10.2	10.0	+ 1	− 1
524	−241.4	+ 16.8	12.5	13.4	....	+ 9	....
532	−218.6	+449.6	14.2	*d*	....	....	....
543	−196.1	+909.4	10.3	*a*	....	....	....
545	−195.5	−401.3	13.1	13.8	....	+ 7	....
551	−175.1	+510.8	10.1	11.0	11.4	+ 9	+13
552	−169.2	−398.3	14.9	*d*	....	....	....
554	−163.1	+666.0	9.0	9.4	9.4	+ 4	+ 4
558	−158.9	−118.6	10.7	11.7	11.2	+10	+ 5
563	−120.0	+990.6	14.4	*a*	....	....	....
566	−104.1	−406.3	13.3	13.0	....	− 3	....
567	−102.8	− 8.3	13.9	*b*	....	....	....
570	− 91.8	−273.2	9.4	9.4	9.3	0	− 1

TABLE IV. — *Continued.*

Bond No.	Δα.	Δδ.	Bond Magn.	Common Magn.	Draper Magn.	Resid. Common.	Resid. Draper.
573	—87.8	—179.0	13.9	12.5	....	— 6	....
575	—84.8	— 22.8	11.9	*b*	....	....	....
580	—77.6	+885.8	12.3	14.5	11.9	+22	— 4
581	—76.1	—159.1	14.2	*b*	....	....	....
583	—74.0	—914.	11.5	12.0	....	+ 5	....
587	—61.5	—806.5	13.9	13.2	....	— 7	....
589	—57.2	— 20.4	12.7	*b*	....	....	....
595	—46.9	— 15.0	13.9	*b*	....	....	....
598	—38.6	—455.1	12.3	12.0	12.0	— 3	— 3
599	—36.5	—974.1	11.8	11.2	....	— 6	....
601	—36.	— 31.	15.6	*b*	....	....	....
602	—33.0	— 67.5	14.3	*b*	....	....	....
605	—27.8	—953.3	13.9	13.2	....	— 7	....
608	—23.7	— 18.0	14.8	*b*	....	....	....
612	—16.4	+ 24.6	13.5	*b*	....	....	....
615	—12.0	+500.5	14.2	*d*	....	....	....
617	—10.7	+ 12.9	....	*b*	....	....	....
618	—10.4	+ 24.6	13.1	*b*	....	....	....
619	—10.0	+ 8.7	....	*b*	....	....	....
620	— 9.0	—053.3	13.1	12.5	....	— 6	....
621	— 8.	— 36.	15.6	*b*	....	....	....
622	— 7.5	— 27.8	12.7	*b*	....	....	....
624	— 5.0	+ 16.1	....	*b*	....	....	....
625	— 4.	— 28.	15.6	*b*	....	....	....
628	0.0	0.0	....	*b*	....	....	....
631	+ 3.	— 42.	14.8	*b*	....	....	....
633	+ 3.5	— 2.1	....	*b*	....	....	....
635	+ 8.3	+ 98.3	10.5	10.9	10.7	+ 4	+ 2
636	+ 8.4	— 8.7	13.3	*b*	....	....	....
639	+11.0	—951.5	11.1	10.6	....	— 5	....
640	+11.5	+ 6.8	....	*b*	....	....	....
641	+11.9	+111.2	14.8	*d*	....	....	....
642	+13.	+ 48.	15.6	*b*	....	....	....
647	+22.6	+ 38.0	12.1	*b*	10.3	....	—18
648	+24.2	— 8.7	14.3	*b*	....	....	....
650	+28.5	+408.8	13.1	15.2	12.0	+21	—11
651	+29.4	+ 47.8	13.1	*b*	....	....	....
652	+30.2	+171.6	13.9	13.4	....	— 5	....
653	+30.8	+429.7	13.9	13.5	12.2	— 4	—17
654	+33.2	+ 10.0	12.8	*b*	....	....	....
657	+39.6	+165.2	13.1	12.0	11.6	—11	—15
663	+55.5	+147.1	11.7	14.8	11.2	+31	— 5
666	+59.7	—195.8	13.9	13.8	....	— 1	....
667	+60.5	+848.9	9.4	*a*	....	....	....
669	+63.3	+100.0	9.8	10.4	10.2	+ 6	+ 4
670	+64.2	+673.2	10.8	10.8	10.9	0	+ 1
671	+69.6	— 24.4	11.5	*b*	....	....	....
674	+73.6	+976.5	14.2	*a*	....	....	....
675	+74.5	— 93.4	15.2	*b*	....	....	....
676	+78.5	— 27.6	13.1	*b*	....	....	....
677	+78.6	—201.4	14.8	14.8	....	0	....
678	+79.2	+852.2	13.9	*a*	....	....	....
680	+82.2	—675.3	13.9	11.6	....	—23	....
681	+90.3	+173.2	14.8	13.2	....	—16	....
684	+96.8	+744.8	14.5	*a*	....	....	....

TABLE IV. — *Continued.*

Bond No.	Δα.	Δδ.	Bond Magn.	Common Magn.	Draper Magn.	Resid. Common.	Resid. Draper.
685	+ 97.7	— 95.0	8.3	8.8	....	+ 5	....
686	+100.	— 39.	15.6	*b*	....	....	....
688	+108.	— 18.	15.6	*b*	....	....	....
690	+119.4	—443.7	10.3	10.9	11.2	+ 6	+ 9
693	+131.7	+751.6	13.9	*a*	....	....	....
695	+132.8	+818.1	12.5	*a*	....	....	....
696	+136.2	+886.3	11.5	*a*	....	....	....
700	+143.4	+492.7	11.5	11.0	11.0	— 5	— 5
701	+143.7	—417.2	14.8	13.8	....	—10	....
703	+145.4	+736.4	13.9	15.1	....	+12	....
705	+147.2	+611.2	11.5	11.0	11.2	— 5	— 3
707	+151.2	—253.5	11.2	11.2	11.1	0	— 1
708	+151.4	— 98.5	9.6	9.1	....	— 5	....
709	+152.9	—136.4	12.3	13.4	11.4	+11	— 9
722	+179.7	—710.4	13.3	13.2	....	— 1	....
724	+183.3	—176.0	10.5	10.2	9.8	— 3	— 7
732	+209.7	—570.4	11.5	11.2	11.5	— 3	0
734	+217.7	+443.8	9.0	8.8	....	— 2	....
737	+220.2	+266.1	15.0	15.6	....	+ 6	....
740	+225.5	+841.3	13.1	*a*	....	....	....
741	+225.9	—110.5	10.0	9.6	9.2	— 4	— 8
746	+233.1	—583.8	10.8	10.0	10.0	— 8	— 8
747	+236.4	—333.4	15.0	14.5	....	— 5	....
750	+248.4	—467.1	10.8	11.0	11.3	+ 2	+ 5
755	+277.7	—348.3	14.8	14.5	....	— 3	....
757	+280.5	+666.1	10.0	10.0	10.0	0	0
759	+285.2	+108.7	15.6	14.5	....	—11	....
762	+ 308.1	—848.7	14.8	15.3	....	+ 5	....
767	+317.0	—193.9	13.9	13.8	....	— 1	....
772	+334.5	+869.2	13.9	*a*	....	....	....
776	+363.	+380.	16.4	15.6	....	— 8	....
778	+366.7	—216.0	13.1	12.1	11.7	—10	—14
779	+370.	+864.	15.6	*a*	....	....	....
781	+373.8	+195.5	10.8	10.6	11.0	— 2	+ 2
783	+386.9	—746.6	13.9	15.1	....	+12	....
784	+388.4	—286.0	10.8	10.6	10.9	— 2	+ 1
785	+389.7	+587.2	10.8	10.6	11.0	— 2	+ 2
786	+389.7	+684.8	13.9	*c*	....	....	....
787	+389.7	+849.3	13.3	*a*	....	....	....
789	+395.9	—245.7	14.8	15.3	....	+ 5	....
793	+414.6	—516.7	11.7	11.5	11.8	— 2	+ 1
794	+416.0	+971.9	12.5	*a*	....	....	....
795	+416.9	—776.5	12.5	12.1	....	— 4	....
797	+427.4	+172.7	15.0	*d*	....	....	....
801	+445.	—282.	13.1	*d*	....	....	....
805	+457.2	+331.9	13.9	*d*	....	....	....
806	+459.8	+780.1	11.7	*a*	....	....	....
808	+464.7	+891.2	11.9	11.8	....	— 1	....
820	+510.8	+978.9	14.2	*a*	....	....	....
822	+514.8	—306.0	10.7	10.0	10.6	— 7	— 1
824	+518.0	+922.2	12.1	*a*	....	....	....
825	+518.1	—716.6	14.2	15.1	....	+ 9	....
826	+521.0	+419.6	14.8	*c*	....	....	....
832	+537.3	—322.4	13.9	14.1	....	+ 2	....
840	+563.	—171.	15.6	*d*	....	....	....

TABLE IV. — *Continued.*

Bond No.	$\Delta\alpha$.	$\Delta\delta$.	Bond Magn.	Common Magn.	Draper Magn.	Resid. Common.	Resid. Draper.
843	+578.1	—853.6	8.6	8.8	....	— 3	....
846	+603.3	—119.1	15.0	*d*	....	....	....
847	+619.4	+634.9	13.1	*d*	....	....	....
848	+631.2	+ 60.2	9.9	9.6	9.6	— 3	— 3
855	+654.0	—989.3	11.0	10.7	....	— 3	....
859	+661.1	—577.5	14.8	*d*	....	....	....
863	+680.9	+357.8	12.5	15.1	....	+26	....
865	+683.1	+957.0	13.9	*a*	....	....	....
873	+707.0	+981.7	11.9	*a*	....	....	....
875	+709.4	+839.4	14.8	*a*	....	....	....
889	+801.6	—258.2	11.3	11.5	....	+ 2	....
893	+815.0	+228.3	13.1	15.1	....	+20	....
899	+861.3	+744.6	14.2	*a*	....	....	....
904	+884.1	—184.0	14.2	*d*	....	....	....
905	+892.8	—918.3	7.8	8.0	....	+ 2	....
908	+901.2	+714.9	13.2	*a*	....	....	....

The light of the brightest stars is derived from Table V., as will be described below. The letter *a* is substituted for the magnitude in the case of stars outside the limits of the photograph; *b* is used to designate stars in the central nebulosity, which are therefore not easily distinguished; *c* indicates stars visible on G, but not on F; and *d*, those not contained on either F or G. The sixth column gives the magnitude derived from the photograph of Dr. Draper. The light of each star in a copy of E was found by Argelander's method, and also by arranging the stars in a sequence. The mean of these magnitudes is that here employed. The last two columns give the residual, expressed in tenths of a magnitude, found by subtracting the Bond magnitude from the photographic magnitudes given in the two previous columns.

A list of the stars visible in other copies of E is given by Professor Holden in Table A, on page 228 of his Memoir. Three of these stars, Nos. 685, 708, and 734, are too bright for satisfactory measurement in E, and two others, 435 and 863, are not visible either in E, D, or C. No. 497 is apparently omitted by mistake in Professor Holden's list. For five stars, 580, 650, 653, 663, and 709, the results derived from E and F are discordant. The first three of these are the faintest stars measured on E, and the last two are so surrounded by nebulosity that the measure is difficult. Were the first three stars as faint as F would indicate, it would be impossible to see them on E. They are certainly visible on D, and 653 on C also. No. 663 is brighter than 681 in E, as bright in D, and not seen in C; in F and G it is much fainter than 681.

TABLE V.

Bond No.	Bond Magn.	A.	B.	E.	E′.	F.	F′.
905	7.8	....	....	....	....	8.0	....
843	8.6	....	....	....	....	8.3	....
467	8.7	....	....	9.4	9.3	8.8	....
685	8.3	8.7	8.8	....	....	8.8	....
734	9.0	....	9.2	....	....	8.8	....
708	9.6	9.0	9.0	....	....	9.1	....
570	9.4	9.4	9.5	9.4	9.2	9.4	9.4
554	9.0	....	....	9.3	9.6	9.4	....
741	10.0	9.3	9.4	9.2	....	9.7	9.5
848	9.9	....	9.7	9.7	9.6	9.7	9.6
505	9.6	....	9.7	9.6	9.6	9.7	9.5
757	10.0	....	....	10.0	10.1	10.2	9.7
724	10.5	9.7	9.7	9.8	9.8	10.5	9.8
746	10.8	....	....	10.0	9.9	....	10.0
523	10.1	10.0	10.3	10.0	10.0	10.5	9.8
669	9.4	9.9	10.0	10.1	10.3	....	10.4
635	10.5	10.1	10.6	....	10.7	....	10.9

The brightest stars in the nebula are compared in Table V. The first and second columns give the Bond number and magnitude. The columns headed A, B, E, and F give the magnitudes derived from those photographs respectively by the method of sequences. The results derived from E and F by the method of Argelander are given in the columns headed E′ and F′. The mean values of E and E′, and of F and F′, are given in Table IV.

One of the most important applications of the determination of photographic magnitudes is to the measurement of the colors of the stars. The rays affecting the photographic plate have in general a less wave-length than those to which the eye is most sensitive. It therefore follows that a reddish star, that is, one in which the rays of great wave-length predominate, will appear relatively too faint in the photograph. The residuals in the last columns of Table IV. will then be positive. A bluish star is similarly indicated by a large negative residual. These residuals form a convenient measure of the color of the stars. In most stars the difference in color is due to slight differences in the relative intensities of the blue and red rays. Until the law defining the relation of the intensity to the wave-length is known, a single number serves to describe the principal cause of the color. Of course in the case of stars in which a large part of the light is concentrated in bands or lines, the residuals will not be directly comparable with those of other stars. Even here, however, this test may be advantageously employed to compare stars of the same class, as, for instance, those of the third type of Secchi.

TABLE VI.

Red Stars.						Blue Stars.					
Bond No.	Bond Magn.	A.	B.	B—A.	Resid.	Bond No.	Bond Magn.	A.	B.	B—A.	Resid.
508	12.3	14.5	15.1	+.6	+2.5	378	14.8	13.8	13.8	0	—1.0
558	10.7	11.7	11.5	—.2	+0.9	402	12.3	11.8	11.4	+.1	—0.9
580	12.3	14.5	14.3	—.2	+2.1	435	13.1	12.0	12.0	0	—1.1
650	13.1	15.2	15.1	—.1	+2.1	573	13.9	12.5	12.3	—.2	—1.5
663	11.7	14.8	14.8	0	+3.1	657	13.1	12.0	12.1	+.1	—1.1
703	13.9	15.1	15.1	0	+1.2	680	13.9	11.6	12.0	+.4	—2.1
709	12.3	13.4	13.4	0	+1.1	681	14.8	13.2	13.2	0	—1.6
783	13.9	15.1	15.1	0	+1.2	701	14.8	13.8	13.5	—.3	—1.2
863	12.5	15.1	15.1	0	+2.6	759	15.6	14.5	14.8	+.3	—1.0
893	13.1	15.1	15.1	0	+2.0	778	13.1	12.1	11.9	—.2	—1.1

The first part of Table VI. contains the stars in which the residual equals or exceeds one magnitude. The first three columns give the Bond number and magnitude and the photographic magnitude, taken from the first, fourth, and fifth columns of Table IV. The photographic magnitude was determined a second time to see if the large residual was due to error. The results are given in the fourth column of Table VI. The difference in the two measures is given in the next column, and in the last column the residual found by subtracting the second column from the mean of the third and fourth columns. The second part of Table VI. gives the corresponding values for the blue stars in which the residual has a negative value exceeding one magnitude.

The first part of Table VII. contains the stars given in the Bond Catalogue not contained in the photograph, and accordingly marked *d* in Table IV. As the faintest stars visible in the photograph have a photographic magnitude of about 15.0, it follows that a slight redness of the stars in Table VII. would account for their absence in the photograph. The stars marked *c* in Table IV. are Bond 367, 443, 786, and 826; although not visible in F, they were detected in G.

The second part of Table VII. contains the stars which are visible in both the photographs F and G, but are not given in the Bond Catalogue. The successive columns give a current number, the approximate difference in right ascension and declination from *θ' Orionis*, and the photographic magnitude.

Many more objects which cannot be distinguished from stars are visible on either F or G, but not on both. After completing this list, it was compared with the map of the Earl of Rosse (Phil. Trans., 1868, Pl. III.). Stars appear on this map which are moderately near Nos. 4 and 11, but none are near any of the other stars in the second part

TABLE VII.

Bond No.	Bond Magn.	No.	Δα	Δδ	Magn.
329	14.2	1	−464	−424	14.8
352	14.8	2	−370	−122	13.8
419	14.2	3	−143	−611	13.5
532	14.2	4	−130	−515	13.5
552	14.9	5	−62	−514	13.8
615	14.2	6	−17	−1011	14.5
641	14.5	7	−9	−592	13.8
797	15.0	8	+34	−990	14.8
805	13.9	9	+42	−854	15.1
840	15.6	10	+77	−480	12.7
847	13.1	11	+316	−639	13.0
904	14.2				

of Table VII. None of them are given in the list prepared by Lord Rosse of the stars not contained in the catalogue of Struve (Phil. Trans., 1868, p. 59). A comparison with the map of Mr. Common (Monthly Notices, XLIII. 256) showed that Nos. 10 and 11 were already given there. Mr. Common's stars *nf.* 690 and *np.* 750 are not visible on G. although the first of them is well shown on F. The stars near Bond 685 and 741 were not measured on account of the nebulous light with which they are surrounded. Their presence in G is somewhat doubtful. Until the remaining stars are actually seen, we may infer that they are too faint to be visible to the eye, and that our only evidence of their existence is by means of the photographic plate. These stars are also probably of a bluish color. As the number of stars is nearly the same in the two parts of Table VI., we may infer that for white stars the limiting magnitude for the photograph does not differ much from that for the eye.

The agreement of the results given on page 408 is hardly a fair test of the errors of measurement. A better indication is afforded by the repetition of the measurement of the red and blue stars in Table V. The average difference in the results is .14 of a magnitude, which indicates a probable error of each of about .08. The two measures of E by Argelander's method and by sequences give for the 35 stars compared by both methods an average deviation of .20, or a probable error of .12. Forty stars are common to E and F. Omitting the five which are stated on page 413 to be discordant, the average difference in the two magnitudes of the remaining thirty-five is .27. The probable error of each, if they are equal, is .16.

XIII.

EARLY EXPERIMENTS IN TELEGRAPHING SOUND.

By Edward C. Pickering.

Communicated May 26, 1885.

In 1870, when Professor of Physics at the Massachusetts Institute of Technology, I wished to show to an audience the experiment of transmitting sound by electricity. The only means of doing this, of which I was then aware, was by the sound produced when a piece of soft iron is suddenly magnetized or demagnetized. The sound thus produced is extremely feeble, and I proposed to replace it by the following device. Loud sounds may be produced by the vibrations of a plate, and a strong vibratory force may be applied to such a plate by means of an electro-magnet. The first receiver consisted of a powerful electro-magnet attached to the bottom of a wooden box, whose cover was replaced by a tin plate, to the centre of which a soft iron armature was attached. The dimensions were such that the armature was near the magnet, but not in contact with it. The plate appears also to have been used without the armature. It is not certain but that this form of apparatus may have been tried first, and the armature added to increase the energy of the vibration, and consequently the loudness of the sound. A tin box was also employed, the bottom of which replaced the plate and armature, and the box served to reinforce the sound. The transmitter was composed of a sonometer, around the wire of which a short wire was wound, dipping into mercury. An electric current was passed through both wires, the mercury cup, and the magnet. When the principal wire of the sonometer was set in vibration by a violin bow, or otherwise, the current was broken at each vibration at the surface of the mercury. When the circuit was made, the magnet drew the plate down, and when it was broken, the elasticity of the plate drew it back. A loud sound was thus produced, whose pitch could be varied by changing the length or tension of the wire of the sonometer. On December 13, 1869, I gave the first of eighteen lectures on Sound, forming one of the

Lowell Free Courses given that year at the Institute of Technology. It is probable that this experiment was prepared for and shown in the lecture of this course which was delivered on January 5, 1870, and related to sympathetic vibrations.

On August 23, 1870, at the meeting of the American Association at Troy, Professor R. H. Van der Weyde, of New York, presented paper No. 141 to Section A, in the hall of the Troy Female Seminary, Professor John M. Ordway acting as chairman. This paper was entitled, "Further Improvements in the Method of transmitting, audibly, Musical Melodies by the Electric Telegraph Wire." In the discussion which followed the presentation of this paper, I described my experiment, and pointed out that my difficulty was mainly with the transmitter, Professor Van der Weyde's with the receiver; also that, if he could combine his transmitter with my receiver, I thought he might obtain valuable results. I have since been informed that he adopted this suggestion, and ascribes to it the use of the metallic diaphragm which he afterwards employed. The "Troy Press" of the following morning, August 24, 1870, contained the following report of my remarks. On account of its importance, I may be pardoned for giving it verbatim.

"Professor Pickering described a simple means he had employed for rendering these vibrations audible. It consisted of a simple electro-magnet placed close to the bottom of a large tin box, whose resonance rendered the sound very intense. His remarks were greeted with marked approbation. Another member said Professor Pickering's method was beautiful in the extreme, because it did away with the armature."

Professor Charles R. Cross, then Assistant Professor of Physics in the Institute of Technology, was invited to give a lecture on Sound to the pupils of the New England Conservatory of Music, in February, 1872. He desired to show the experiment of telegraphing sound. Accordingly, Professor Cross, Mr. Waldo O. Ross (who was present at the experiment in 1870 also), and I spent an evening at the Institute, and repeated the experiment of 1870 under more favorable conditions. The great difficulty in the early experiment was with the transmitter. A tuning-fork was accordingly substituted for the sonometer, which enabled the circuit to be broken with greater certainty and regularity. It was, however, open to the objection, that sounds of one pitch only could be transmitted. The details of this experiment are known with much greater certainty than those of the first experiment. The battery consisted of six small Grove cells. An "Albert Biscuit"

box, 20 centimeters long by 12 wide and 12 deep, was first used as a receiver. Afterwards, a large tin packing-box, 80 cm. long, 50 cm. wide, and 50 cm. deep, was substituted for it, and gave a very loud sound. Most of this apparatus is still preserved at the Institute of Technology, and was used in the legal examination quoted below. In the last part of December, 1873, and the first week of January, 1874, a number of other experiments were tried by Professor Cross, and another exhibition of the instrument was made by him on January 7, 1874, at the first lecture of his Lowell Free Course of that year. Several forks were tried, and the receiver transmitted the characteristic sound of each, but one only was used publicly. The sound persisted when the magnet touched the box, but was then feeble.

Most of the above facts were testified to by Professor Cross in a legal examination held on June 18, 1879. It was agreed that the same deposition should be used in the two cases, Harmonic Telegraph Company et al. *vs.* The New England Telephone Company, and Harmonic Telegraph Company et al. *vs.* Charles Williams, Jr. Present, Causten Browne, Esq., of counsel for complainants, Chauncey Smith and J. J. Storrow, Esqs., of counsel for defendants, and W. P. Preble, Jr., Examiner. Professor Cross also testified as follows: —

"*Ans.* On the 10th of June, 1879, I tried a number of experiments with this receiver, which was set up in a manner similar to its arrangement in the exhibition already mentioned, it being in the office of Mr. J. J. Storrow, Union Building, State Street, Boston. It, together with a Blake transmitter, was connected in the primary circuit of a battery; the transmitter being placed in Room 42, Union Building, and connected with the receiver by wires stretching across the open court separating these two rooms, and about one hundred feet in length. A second circuit, containing two ordinary hand telephones, was stretched between the two rooms, so that any messages sent by the apparatus which I am describing could easily be verified.

"The magnet in these experiments was placed near to the outside of one end of the tin box, which rested upon its side, and when faint sounds were transmitted the head was placed inside the box, with the ear opposite the poles of the magnet, in order to detect the sounds. I first asked Mr. Watson, who was assisting me, to play a small mouth harmonica before the mouthpiece of the Blake transmitter. I was expecting to hear the sound given by that instrument, and so was somewhat surprised to hear loud and clear notes resembling those of a music-box. On inquiring of Mr. Watson what instrument he had been using, I was informed that he had been transmitting the sound produced

by a child's music-box which he happened to have there. Afterwards the mouth harmonica was substituted, and its characteristic notes clearly perceived. In both of these cases the characteristic pitch and quality of the tones in the musical instruments used could be heard at the distance of eighteen inches or more from the receiver. Mr. Watson was then asked to speak into the transmitter, and, while the ear was still at a distance of about fifteen inches from the receiver, a sound was heard which was recognized as articulation, although the words could not be distinguished at that distance. I then placed my ear close to that portion of the box which was opposite the poles of the magnet, and asked Mr. Watson to speak into the transmitter. I then without difficulty was able to hear a number of sentences, which were entirely unexpected to me.

"*Int.* 9. Please state whether on this occasion you also tried to transmit articulate speech, using as a receiver a common hand Bell telephone connected upon the circuit, with the mouthpiece and ordinary diaphragms removed, and the instrument held against one end of the tin box, so that the box should take the place of the diaphragm. And if so, with what result?

"*Ans.* I did. With it we succeeded in transmitting articulate speech from one station to the other with very great ease.

"*Int.* 10. Will the Institute of Technology give or sell the defendants the receiving apparatus you constructed, to be filed as an exhibit in this case? and if not, will you please produce duplicates of said tin box and horseshoe electro-magnet, and a wooden box like the one used in your experiment?

"*Ans.* These early experiments have so much interest attached to them, that it seems desirable to retain the original apparatus in the possession of the Department of Physics. I will produce duplicates."

Copies of the apparatus were also presented, in May, 1885, to the defendants in the suit in equity, American Bell Telephone Company et al. *vs.* The Western Pennsylvania Telegraph and Telephone Company et al., Circuit Court of the United States, Western District of Pennsylvania, on motion for preliminary injunction. At this time, also, it was shown that articulate speech could be transmitted by the apparatus, by attaching a mouthpiece to one prong of the tuning-fork and placing a piece of carbon in the mercury under the style. Affidavits were also produced from Mr. G. W. Blodgett and Mr. L. W. Wood, stating that I showed the apparatus described above to their class in 1872. An affidavit of Mr. A. D. Blodgett shows that it was also shown to his class by Professor Cross in 1874.

The following extract from a letter to Professor Bell may also be of interest, as showing that I advised him to increase the sound of his telephone by an instrument resembling the Blake transmitter, in 1877, or two years before the invention of that instrument. This letter was written during the night following Professor Bell's communication on the telephone to the American Academy, on May 4, 1877. The telephone then exhibited gave a very feeble sound, and, according to my letter-book, I wrote on "the problem of introducing more kinetic energy into your telephonic circuit, — in other words, on making a telephonic relay. An idea occurred to me which I hope may give the desired result, and I shall be very glad if you have the means of giving it a trial. The problem is to utilize a local current, so that in a given circuit it shall be proportional to the current induced by the magnets. Now this may be done by attaching to the plate of the receiving telephone A, a fine wire, dipping in water, and nearly touching a wire connected with a second telephone and battery. The resistance of this circuit will be mainly that of the water between B and C. Now, as A vibrates, the interval B C will alter, and with it the total resistance, and consequently the current. Moreover, a feeble exciting current may regulate a powerful local battery." A sketch of the apparatus showed that the current from the battery passed through the vibrating diaphragm of a receiving telephone, to which was attached a wire B, dipping in water, and nearly in contact with a second wire C. The variations of the current thus magnified were then passed through a second telephone. A modification of the apparatus was also described, by which the current could be reversed, like the primary current. Had this instrument been tried, a loud-sounding telephone might have been obtained earlier. A carbon button should have been substituted for the liquid resistance, as the varying resistance of carbon was then well known.

No secret was ever made of these experiments, which were described and exhibited publicly and privately whenever this was desired. A patent for the apparatus was not taken out, from a belief that a scientific man should place no restrictions upon his work which would tend to prevent the repetition of an experiment of scientific interest. A full description should have been published. This was at first delayed from the pressure of other work and lack of appreciation of the importance of the results. Afterwards I was unwilling to enter into a controversy, or to obstruct my friends, who were struggling to obtain proper recognition of the great results they had obtained in the same field. Now that some of them, at least, have been amply rewarded, a

full statement seems necessary as a contribution to the history of the telephone.

It will be seen from the above statement, that in 1870, several years before the telephones now in use were invented, a receiver was devised, constructed, and tried, which consisted of a flexible iron diaphragm, supported at the edges and replacing the armature of an electro-magnet. Musical sounds were telegraphed successfully, and the apparatus was described at a scientific meeting, as the newspaper report shows. In 1872 and later, the experiment was repeated under various conditions. In 1879 it was shown that it was capable of serving as a telephone, and of rendering articulate speech audible at a distance. It appears to differ in no way in principle from the receiver now used. On the other hand, it should be stated that all my experiments were made, or were intended to be made, with a discontinuous current, and, although the instrument is capable of showing the variations of a continuous current, I did not have this application in mind when I constructed it.

08

ATMOSPHERIC REFRACTION.

BY

EDWARD C. PICKERING.

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INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATION FROM THE RUMFORD FUND.

XIV.

ATMOSPHERIC REFRACTION.

BY EDWARD C. PICKERING.

Communicated May 26, 1885.

PART I.

DURING the summer of 1876 several thousand observations of the altitudes of the White Mountains were made by the writer.* The method of zenith distances was employed, the instrument used being the micrometer level. Great difference of opinion exists among geodetists regarding the value of the zenith distance of a distant object as a means of determining its height, on account of the uncertainty in the atmospheric refraction. A large number of measures of this quantity were accordingly made in subsequent years, to show to what extent the observations of 1876 were likely to be affected by this error. Four micrometer levels have been employed, which will be designated as A, B, C, and D.

The first of these, A, was shown to this Academy on January 11, 1876,† and was exhibited at the International Exposition of 1876 in Philadelphia. It consisted of a telescope conveniently mounted, carrying a level, and having in the eyepiece a spider-line micrometer.

Instrument B, which was that used in 1876, was made out of an architect's level. It consisted of a telescope having an aperture of 3 cm. and a focal length of 25 cm., with an erecting eyepiece magnifying eighteen diameters. A delicate spirit-level was attached directly to the telescope, which rested in two wyes, 19 cm. apart. One end of the bar carrying these wyes rested upon a bearing, and the other upon the pointed end of a micrometer screw. The bearing and the nut of the screw were carried upon a bar which rotated around a vertical axis. A horizontal circle divided into degrees was added, for

* Appalachia, I. 138.

† Proc. Amer. Acad., XI. 256.

convenience in identifying the objects observed. The pitch of the screw was about 0.1 cm., and its head was divided into hundredths. The value of one division in seconds of arc was found by placing the instrument upon the cube of the telescope of the Meridian Circle of the Harvard College Observatory. A series of readings was then taken, setting the screw in various positions, and inclining the cube until the bubble showed that the small telescope was horizontal. The position of the bubble was then read, and the inclination determined by the large circle of the Meridian Circle. The value of one division was thus found, on June 7, 1877, to be $13''.95$. As the instrument is used only for small altitudes, the angles are practically measured by an accurate tangent screw on a circle about 38 cm. in diameter.

Instrument C is larger than B, and the cross-hairs in the latter are replaced by a filar-micrometer. It therefore combines the advantages of A and B. Its telescope has an aperture of 4 cm., a focal length of 45 cm., and a magnifying power of sixteen diameters.

Instrument D closely resembles B. Light is secured by the use of an inverting eyepiece. The aperture of the telescope is 2.5 cm., its focal length 21 cm., and its magnifying power ten. The value of one division of the screw was found as described above, and gave the result $13''.95$, or the same as B.

An important improvement was devised by Mr. J. R. Edmands, and introduced in this instrument, in March, 1882. The level was attached to the bar carrying the wyes, instead of to the telescope. As the instrument was originally used, the observations were rendered differential, the quantity measured being the difference in apparent altitude of the various objects. The only quantity liable to vary in using the instrument is the angle between the true level line and the axis of the instrument when the bubble of the level is the centre. This angle may be divided into two parts: first, the collimation error due to the deviation of the axis from the line passing through the centre of the pivots by which the telescope is supported in its wyes; and, secondly, the level error, by which this last line deviates from the horizontal when the bubble is in the centre. Each of these constants may be determined by the device proposed by Mr. Edmands. The level is attached to the bar carrying the wyes, instead of to the telescope. If now the telescope is turned around its axis 180°, the mean of the readings in the two positions will eliminate the collimation. The value of this constant is equal to one half the difference in these readings. The level error is similarly eliminated or determined by turning the telescope end for end around a vertical axis. Rotating the telescope around a horizontal

axis perpendicular to its own axis, gives both constants together, without distinguishing between them. The direction of the three axes around which the telescope should be rotated to give the collimation, level, or both combined, may be remembered as the axis of the telescope, the vertical cross-hair, and the horizontal cross-hair, respectively. With this modification, the micrometer level compares favorably with the vertical circle as regards economy in time and expense, portability, and accuracy when small angles are to be measured.

Before this modification, the difficulty in determining the combined level and collimation error was avoided by observing the difference in height of a distant and near mountain which are nearly in line. These were commonly so selected as to be in the same field of view. It was therefore only necessary to observe the difference in altitude by the micrometer screw under various atmospheric conditions.

The first series of measurements were made on Mt. Wachusett, in Princeton, Mass., on June 22, 1877. Accompanied by Col. C. W. Folsom, I reached the summit at about noon. The observations were begun at once with instrument B, and were continued with short intermissions until nearly eight o'clock, when darkness interfered. The following morning was exceptionally clear, and observations were begun at a quarter after four, and continued until two in the afternoon, when we left the summit.

Mt. Monadnock, Cheshire Co., N. H., was visited, July 3, 1877, by Mr. J. R. Edmands, Professor C. E. Fay, and myself. We reached the summit at about three in the afternoon, and began observing with instrument B. The air was so hazy that the Sandwich range was barely visible, and Ascutney, the Southern Kearsarge, and Gunstock were the most distant points observed. The observations were abandoned at about five o'clock on account of the increasing haze. On July 15, 1877, Mr. Edmands and I visited Mt. Washington, in the White Mountains, where we remained until July 24. The almost continuous cloud and haze prevented much work, except on July 18, when a few preliminary observations were made with instrument C; and on July 21 and 22, when observations were obtained with D. Mt. Kearsarge, Merrimack Co., N. H., was occupied at the same time by Mr. J. B. Henck, Jr., with instrument A, in order that simultaneous observations might be obtained at the ends of the line from Mt. Washington to Kearsarge.

Observations were undertaken in August and September, 1877, at Jefferson Hill, N. H., by Mr. Henck, and at East Jefferson, by Mr. Edmands, with instruments B and D, respectively. Again long

periods of storm and haze prevented the accomplishment of satisfactory work.

Some observations were obtained by Mr. Edmands from Mt. Starr King, Coos Co., N. H., in October, 1878. In November and December of the same year, Mr. Edmands resided at Arlington Heights, Mass., for the purpose of obtaining similar observations.

Plans were also made for occupying simultaneously for a week Mts. Moosilauk, Kearsarge (S.), and Wachusett, which lie nearly in a straight line. Observations could thus be obtained at the ends and near the middle of a line nearly a hundred miles in length.

Much time was lost, owing to bad weather, when stations were occupied which were especially adapted to the determination of the atmospheric refraction. That a very distant point might be seen, lofty and comparatively inaccessible stations were required. Moreover, in order that observations might be made under the extremes of temperature and other atmospheric conditions, it was necessary that the work should extend over a considerable period of time. A series of measurements was accordingly undertaken at the Harvard College Observatory, where the absence of very distant points was compensated by the ease with which observations could be made at all times and seasons without materially interfering with other work. The observations were begun on January 3, 1882, and continued until January 26. During this series the temperature fell as low as —15° F. The position of the level was then changed, as described on page 269. A second series extended from May 6 to August 7, 1882, and included observations in which the temperature was +91° F. All the observations were made with instrument D, by Mr. Edmands and myself.

Only the observations made at Wachusett and at the Observatory will be considered in the present paper.

I. Wachusett.

The micrometer level was mounted on the sill of the north window of room No. 19, in the second story of the hotel, on the summit of Wachusett. This point is approximately 15m. north, 3m. west, and 4m. above the United States Coast Survey bolt in the top of the mountain. The effect of the unsteadiness in the support is almost entirely eliminated by the differential character of the observation. The observations were made by directing the telescope towards the mountain to be observed, and turning the screw so that the level-tube was nearly horizontal. The position of the screw, of the two ends of

the bubble, and the time, were then recorded. The telescope was next directed alternately to the distant and near mountains, five times to each, and the screw readings recorded. The level was then again observed. These measures constituted one set. Sometimes a third mountain was also observed, and sometimes the level was omitted. Care was taken never to move the telescope horizontally during a set, and when several points were observed, they were always measured in the same order.

Table I. gives in the first column the Greenwich mean time, expressed in hours and tenths, the beginning of the day being taken at Greenwich noon. The second column gives a designation for the object pointed at; and the third gives the corresponding mean of the readings of the micrometer screw. L is used to denote that the telescope was level, and the reading in the third column is here corrected for the position of the bubble. The objects belonging to the same set are indicated by placing the time opposite the first only.

The various designations in the second column have the following meanings: —

B. Barrett Hill. Top of trees on hill nearly in line with Moosilauk and Kearsarge (S.). E. Mt. Equinox. G. Gunstock. Gp. Gap Mt. Gr. Greylock. J. Joe English Hill. K. Kearsarge (S.). The object observed was the base of the house, a short distance from the summit. This is apparently the house which has been destroyed, and not that now standing a few feet only from the summit. Ki. Killington Peak. L. Level, as stated above. M. Moosilauk. Mn. Monadnock. P. Passaconnaway. Pi. Piscataquog, the hill nearly in line with Whiteface. W. Whiteface. 1. Summit of Kearsarge (S.). 2. Top of house on Kearsarge (S.). 3. Summit of a cloud-dome, presumably over Mt. Washington. 4. Northwestern of the Uncanoonucs. 5. Southeastern of the Uncanoonucs. 6. Cupola of Barnard House, Arlington Heights. 7. Roof of barn placed in right-hand part of field to determine inclination of wire. 8. Roof of barn in left-hand part of field. The two sets in which this object was observed showed that the inclination of the wire equals 0.6 division for $1^\circ.1$, or about $8''$ per degree, including error in level.

TABLE I.—JOURNAL.

G. M. T.	Object.	Setting.	G. M. T.	Object.	Setting.
h.			h.		
4.8	L.	142.86	12.1	G.	249.78
"	Mn.	72.40	"	J.	254.40
4.9	L.	140.77	12.2	G.	255.06
"	1.	27.80	"	J.	259.94
5.0	L.	138.97	12.3	W.	50.38
"	K.	227.91	"	S.	68.2
5.9	L.	150.04	"	Pi.	66.95
"	Mn.	80.72	12.3	G.	251.46
6.0	L.	148.81	"	J.	256.04
"	K.	237.22	12.5	G.	52.78
6.1	L.	146.97	"	J.	57.54
"	G.	259.16	12.5	W.	57.22
"	J.	261.74	"	Pi.	72.06
6.2	L.	150.45	"	G.	51.42
"	Ki.	230.78	"	J.	55.94
"	Gp.	256.16	21.0	L.	158.44
6.3	Ki.	228.96	"	G.	62.42
"	Gp.	255.01	"	J.	69.30
6.4	G.	256.02	21.1	L.	159.03
"	J.	258.74	"	P.	63.76
7.1	G.	255.70	"	W.	62.28
"	J.	258.88	"	Pi.	81.42
7.2	G.	256.14	21.2	G.	61.84
"	J.	259.18	"	J.	68.58
7.2	K.	231.06	21.6	G.	60.94
"	B.	244.44	"	J.	67.58
7.3	K.	231.18	21.7	L.	158.08
"	B.	248.82	"	P.	262.54
10.1	L.	154.98	"	W.	260.72
"	Mn.	85.00	"	Pi.	279.88
10.2	L.	150.97	21.8	L.	157.92
"	K.	264.74	"	M.	256.14
"	B.	276.48	"	K.	265.50
10.3	1.	261.46	"	B.	279.08
"	2.	262.70	21.8	L.	157.71
"	K.	263.72	"	Mn.	87.90
"	B.	272.70	21.9	L.	159.24
10.4	G.	256.04	"	Mn.	88.34
"	J.	260.10	22.0	L.	157.91
10.4	K.	233.98	"	Ki.	225.34
"	B.	247.70	"	Gp.	256.06
10.8	G.	261.08	22.2	L.	158.97
"	J.	265.22	"	M.	257.57
10.8	G.	260.02	"	K.	264.94
"	J.	263.74	"	B.	278.02
11.6	G.	253.82	22.3	L.	158.30
"	J.	258.26	"	P.	262.36
11.7	K.	257.02	"	W.	260.52
"	B.	268.34	"	Pi.	279.88
11.8	W.	255.76	22.8	L.	157.84
"	Pi.	269.92	"	G.	260.96
11.9	G.	255.18	"	J.	267.84
"	J.	259.34	23.0	L.	151.68
12.1	G.	249.08	"	Gr.	192.38
"	J.	253.80	23.1	L.	154.36

TABLE I. — *Continued.*

G. M. T.	Object.	Setting.	G. M. T.	Object.	Setting.
h.			h.		
23.1	E.	175.70	4.2	Pi.	281.28
23.2	L.	157.55	4.3	L.	154.19
"	P.	261.88	"	G.	264.52
"	W.	259.76	"	J.	268.42
"	Pi.	278.74	4.4	L.	154.12
23.3	L.	158.24	"	G.	265.36
"	M.	259.57	"	J.	268.06
"	K.	265.18	4.5	L.	155.36
"	B.	278.46	"	M.	286.40
23.4	L.	156.76	4.6	L.	153.44
"	G.	261.52	"	J.	267.20
"	J.	267.20	"	4.	264.75
23.6	L.	157.27	"	5.	265.90
"	G.	262.54	4.8	L.	152.96
"	J.	268.08	"	G.	264.32
0.7	L.	156.97	"	J.	267.48
"	G.	265.22	4.9	L.	152.16
"	J.	269.22	"	G.	263.40
0.9	L.	157.87	"	J.	266.84
"	Mn.	86.96	5.4	L.	179.24
1.0	L.	157.92	"	6.	366.44
"	Mn.	87.16	5.6	L.	144.92
1.0	L.	156.92	"	G.	255.54
"	P.	269.12	"	J.	259.76
"	W.	266.08	6 2	L.	148.97
"	Pi.	281.24	"	G.	260.52
1.1	L.	156.29	"	J.	263.64
"	G.	265.14	6.2	L.	151.08
"	J.	268.74	"	Mn.	81.20
1.2	L.	155.81	6.3	L.	148.97
"	G.	265.74	"	K.	263.28
"	J.	269.62	"	B.	273.94
1.4	L.	155.45	6.4	L.	148.87
"	J.	269.04	"	P.	264.78
"	4.	266.49	"	W.	262.02
"	5.	267.23	"	Pi.	274.90
2.6	L.	153.88	6.4	L.	149.60
"	G.	264.22	"	Ki.	228.44
"	J.	267.98	"	Gp.	254.22
2.8	L.	153.35	6.5	L.	148.87
"	G.	263.81	"	G.	259.60
"	J.	266.78	"	J.	263.26
4.2	L.	155.34	6.6	7.	29.84
"	G.	265.44	"	8.	29.24
"	J.	268.80	"	L.	149.56
4.2	L.	155.18	"	G.	261.38
"	P.	269.26	"	J.	264.12
"	W.	267.64			

Besides the above settings, the altitudes and azimuths of ab(thirty mountains were taken on June 22 between $6^h.4$ and $7^h.1$, a of about forty more on June 23, between $1^h.4$ and $2^h.6$. The to number of pointings of the telescope was 816, and of level readir

262, although the entire stay upon the mountain little exceeded twenty-four hours.

The most complete series of measurements relates to the comparative altitudes of Gunstock and Joe English Hill. Table II. gives in successive columns, first, the Greenwich time in hours and tenths; secondly, the height of the barometer reduced to 32° F. by means of the reading of the attached thermometer; and, thirdly, the temperature of the outer air in Fahrenheit degrees. These quantities are taken from curves drawn to represent the observations which were made at intervals during the day. The fourth column gives the zenith distance of the sun in degrees uncorrected for refraction. The next column gives the apparent height of Gunstock above Joe English in seconds of arc, being the observed micrometric distance multiplied by $13''.95$, the value of one division in seconds. From the assumed heights of the two hills and the apparent difference in altitude, the coefficient of refraction has been computed by Mr. Edmands, and is given in the last column but one of the table. A smooth curve was constructed with these values, and the last column gives the residual found by subtracting the result given by the curve from that given by observation. Any error in the assumed heights of the three points would alter the coefficient of refraction by an amount constant throughout the table, but would not affect the amount of its variation.

Several important conclusions may be drawn from Table II. The angular interval between the two hills gradually increased, and with it the coefficient of refraction gradually diminished, during the first afternoon. The changes were small during the last half-hour, notwithstanding the cooling of the air in the upper regions of the atmosphere, and other important meteorological changes caused by sunset. The following morning the refraction was much less, increasing during the morning, and becoming somewhat irregular during the day. These changes are much greater than the errors of observation, as is shown by the accordance of successive sets, especially on the afternoon of June 22.

Table III. gives the comparison of the observations of Passaconaway, Whiteface, and Piscataquog. The first column gives the time, and the second the angular interval between the first and third of these mountains expressed in seconds of arc. The next column gives the corresponding coefficient of refraction, and the fourth gives the residual found by subtracting from this coefficient the corresponding ordinate of the smooth curve deduced from the observations given in Table II. The last three columns give the corresponding quantities for the line Whiteface and Piscataquog.

TABLE II. — Gunstock and Joe English.

G. M. T.	Barom.	Therm. F.	Z. D.	G. — J.	Coef. Refrac.	O. — C.
h.	in.	°	°	"		
6.1	27.62	62	25	36	.0708	+.0001
6.4	27.62	62	28	38	.0707	+.0001
7.1	27.64	61	35	44	.0702	—.0001
7.2	27.64	61	36	42	.0704	+.0001
10.4	27.69	57	70	57	.0693	—.0003
10.8	27.71	56	75	58	.0693	—.0002
10.8	27.71	56	75	52	.0697	+.0003
11.6	27.73	54	83	62	.0690	—.0001
11.9	27.74	54	87	58	.0692	+.0002
12.1	27.74	54	89	66	.0688	—.0001
12.1	27.74	54	89	64	.0688	—.0001
12.2	27.74	54	90	68	.0685	—.0003
12.3	27.74	54	91	64	.0688	.0000
12.5	27.75	53	93	66	.0686	—.0001
12.5	27.75	53	93	63	.0689	+.0002
21.0	27.87	46	94	96	.0665	—.0001
21.2	27.87	47	92	94	.0667	+.0001
21.6	27.88	48	88	93	.0667	+.0001
22.3	27.90	50	80	96	.0665	—.0003
23.4	27.91	53	68	79	.0677	.0000
23.6	27.92	53	67	63	.0678	.0000
0.7	27.94	55	55	56	.0694	+.0001
1.1	27.94	56	50	50	.0698	+.0003
1.2	27.94	56	48	54	.0695	—.0001
2.6	27.95	57	34	52	.0696	—.0003
2.7	27.95	57	32	41	.0704	+.0004
4.2	27.93	59	19	47	.0700	—.0001
4.3	27.93	59	19	54	.0695	—.0006
4.4	27.93	59	19	38	.0707	+.0006
4.8	27.92	59	19	44	.0702	+.0001
4.9	27.92	59	19	48	.0700	—.0001
5.6	27.90	59	21	59	.0692	—.0009
6.2	27.89	60	26	44	.0703	+.0002
6.5	27.88	60	29	51	.0697	—.0004
6.6	27.88	60	30	38	.0707	+.0006

TABLE III. — Passaconaway, Whiteface, and Piscataquog.

G. M. T.	P. — Pl.	Ref.	O. — C.	W. — Pl.	Ref.	O. — C.
h.	"			"		
11.6	198	....	....	224	.0670	—.0021
12.3	....	....	....	231	.0667	—.0021
12.5	....	....	....	207	.0677	—.0010
21.1	245	.0669	+.0003	267	.0652	—.0014
21.6	242	.0671	+.0005	267	.0652	—.0014
22.5	244	.0670	+.0002	270	.0651	—.0017
23.2	235	.0674	.0000	279	.0647	—.0027
1.0	169	.0700	+.0005	211	.0675	—.0020
4.2	168	.0701	.0000	190	.0684	—.0017
6.4	141	.0712	+.0010	180	.0688	—.0018

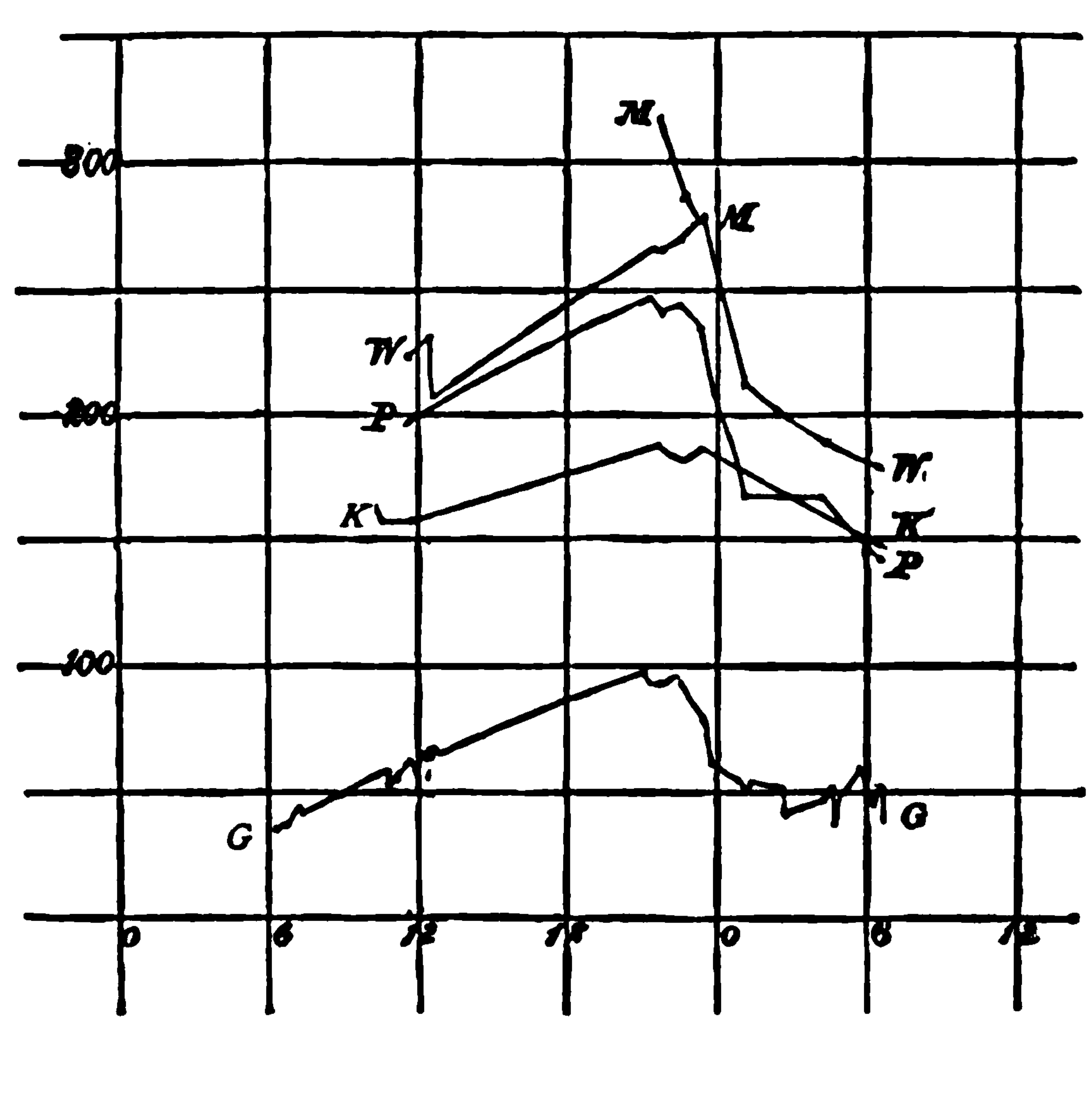
TABLE IV. — MOOSILAUK, KEARSARGE, AND BARRETT HILL.

G. M. T.	M. — B.	Ref.	O. — C.	K. — B.
h.	"			"
10.2	....	....	....	164
10.3	....	....	....	157
11.7	....	....	....	158
21.6	319	.0697	+.0030	189
22.5	288	.0710	+.0043	182
23.3	278	.0713	+.0037	186
6.3	....	....	....	149

Table IV. gives the corresponding results for Moosilauk, Kearsarge, and Barrett Hill. The last two columns are omitted, since the height of Barrett Hill is not known with sufficient precision to compute them.

The height of the station occupied on Wachusett is 620 meters. The heights and distances of the various points considered in Tables II. and III. are as follows: — Gunstock, height, 730 m., distance, 121.5 km. Joe English Hill, height, 401 m., distance, 54.1 km. Passaconnaway, height, 1280 m., distance, 167.5 km. Whiteface, height, 1221 m., distance, 164.7. Piscataquog, height, 368 m., distance, 48 km. Moosilauk, height, 1466 m., distance, 170.3 km. Kearsarge (S.), height, 897 m., distance, 99 km.

The general agreement of these results is shown in the following diagram, where abscissas represent times, and ordinates the angular distance given in Tables II., III., and IV. The letters G, K, P, W,



and M respectively indicate Gunstock, Kearsarge (S.), Passaconnaway, Whiteface, and Moosilauk, as in Table I.

II. Harvard College Observatory.

The station occupied was on the southern corner of the western balcony of the Sears Tower of the Harvard College Observatory. Its height is about 30 meters above mean tide level. Three points were selected very nearly in line which are designated below as *A*, *B*, and *C*. *A* is the summit of Fenno's Peak, the second highest of the Blue Hills of Milton, and nearly the most distant terrestrial object visible from the Observatory. Its height is about 160 m., and its distance assumed to be 18.865 km. *B* is the top of the chimney of the house No. 20 Terrace Avenue, Roxbury District, Boston. Its distance is assumed to be 7.042 km. *C* is the top of the roof at the base of the lightning-rod of the gasholder on Everett Place, Cambridge. Its assumed distance is 0.700 km. *B* is about 6′ below *A*, and *C* about 17′ below *B*. Each set consisted of nine settings in the order *A B C A B C A B C*, the readings being made to tenths of one division of the micrometer screw. The mean of the three readings of *A* was then subtracted from the corresponding means of *B* and *C*. The various sets are enumerated in Table V. The successive columns give a number for reference, the date, and the hour and tenth in Greenwich mean time. The day is here assumed to begin at Greenwich noon, or at about seven o'clock in the morning of Cambridge mean time. The only occasion on which the day changes during a series of observations is between Nos. 120 and 121. The barometric pressure in inches after correction for the temperature of the mercury, and the temperature in degrees Fahrenheit, are given in the next columns. These are followed by the intervals *AC* and *BC* expressed in seconds of arc. One division of the micrometer is assumed to be equal to 14″.2, in accordance with a determination made at that time. As the observations are differential, the uncertainty in the value of this constant will be inappreciable in the result. The following column gives the zenith distance of the sun in degrees. The next two columns give the residuals in seconds, found by subtracting from the observed value of *AC* its mean value, 1392″, and also the same residuals after applying the correction for temperature and pressure to be described below. The last column but one gives the probable error of a single setting, expressed in seconds of arc, derived from the accordance of the individual readings. The last column gives the initial of the observer. E. indicates the observations of Mr. Edmands, and P. my own observations.

TABLE V.—JOURNAL.

No.	G. M. T.		Bar.	Th.	*A C.*	*B C.*	Sun.	Res.	Res.	P. E.	Obs.
	1882.	h.	in.	°	"	"	°	"	"	"	
1	Jan. 3	3.5	30.01	+12	1406	1016	68	+14	— 5	3.3	P.
2	"	3.5	30.01	+12	1404	1008	68	+12	— 6	0.9	E.
3	"	8.3	29.98	+22	1404	1013	82	+12	— 9	3.2	P.
4	"	9.2	29.98	+20	1407	1014	88	+15	— 6	1.1	"
5	"	9.3	29.98	+20	1412	1016	89	+20	— 1	2.0	"
6	"	9.4	29.98	+19	1412	1013	90	+20	0	1.7	"
7	"	9.5	29.98	+19	1415	1010	91	+23	+ 3	1.7	"
8	"	9.6	29.98	+19	1420	1015	92	+28	+ 8	1.6	"
9	"	9.7	29.98	+18	1422	1016	93	+30	+10	2.3	"
10	"	9.8	29.98	+18	1423	1008	94	+31	+11	2.3	"
11	"	9.9	29.98	+17	1425	1020	95	+33	+14	5.3	"
12	"	10.0	29.98	+17	1450	....	96	+58	+39	7.5	"
13	Jan. 4	1.0	30.27	+ 3	1406	1002	77	+14	+ 1	3.3	"
14	"	2.0	30.30	+ 3	1405	1002	76	+18	0	1.8	E.
15	"	5.3	30.37	+ 6	1399	1006	67	+ 7	— 6	4.9	P.
16	"	6.1	30.39	+ 8	1404	1006	69	+12	— 2	3.0	"
17	Jan. 5	1.1	30.64	0	....	1040	82	....	....	4.0	"
18	"	5.2	30.68	+ 6	1423	1015	67	+31	+20	2.6	"
19	"	8.2	30.64	+20	....	1008	78	....	....	2.2	"
20	Jan. 7	0.9	29.85	+33	1405	1035	77	+13	—13	...	"
21	"	0.9	29.85	+33	1410	1007	83	+18	— 8	2.7	"
22	"	1.5	29.87	+34	1407	1008	78	+15	—11	2.1	"
23	"	2.7	29.89	+36	1401	1005	70	+ 9	+16	2.4	"
24	Jan. 22	3.2	29.46	+38	1403	1010	65	+11	+15	2.4	"
25	"	4.4	29.46	+38	1391	1000	62	— 1	+ 3	0.9	"
26	Jan. 23	3.6	30.01	+ 4	1398	1004	64	+ 6	— 9	2.2	E.
27	"	5.6	30.01	+ 5	1389	1000	64	— 3	+16	2.1	"
28	"	7.5	30.07	+ 4	1392	999	75	0	—14	1.8	"
29	"	9.2	30.11	+ 3	1394	1000	84	+ 2	—11	1.8	P.
30	"	9.6	30.13	+ 3	1398	1002	88	+ 6	— 7	5.2	E.
31	"	9.7	30.13	+ 3	1394	1003	89	+ 2	—11	2.0	P.
32	"	9.7	30.14	+ 2	1397	1008	89	+ 5	— 8	2.9	"
33	"	9.9	30.14	+ 2	1405	1006	91	+13	0	1.6	E.
34	"	10.0	30.15	+ 2	1395	1000	92	+ 3	—10	2.2	P.
35	"	10.1	30.15	+ 1	1414	1018	93	+22	— 3	2.0	E.
36	"	10.2	30.16	+ 1	1387	996	94	— 5	—18	4.9	P.
37	Jan. 24	0.4	30.59	—15	1407	1005	84	+15	+11	4.6	"
38	"	0.6	30.59	—14	1414	1010	82	+22	+17	0.9	"
39	"	1.0	30.59	—13	1410	1004	78	+18	+13	2.6	"
40	"	1.1	30.60	—13	1412	1013	77	+20	+15	2.7	E.
41	"	3.2	30.66	— 8	1409	1008	64	+17	+11	2.7	P.
42	"	6.3	30.64	+ 2	1389	999	66	— 3	—13	3.7	"
43	"	7.2	30.66	+ 3	1398	1004	72	+ 6	— 5	2.6	"
44	"	9.2	30.67	+ 4	1402	1009	84	+10	— 1	1.4	"
45	"	10.0	30.70	+ 3	1413	1018	92	+21	+10	3.3	"
46	Jan. 25	0.3	30.62	0	1430	1012	84	+38	+29	1.9	E.
47	"	0.6	30.62	+ 1	1440	1010	82	+48	+38	5.6	"
48	"	1.7	30.61	+ 4	1424	1001	73	+32	+21	1.9	P.
49	"	8.9	30.42	+26	1405	1000	81	+13	— 7	2.2	E.
50	"	9.9	30.41	+25	1405	997	91	+13	— 7	2.1	"
51	"	10.1	30.41	+25	1415	991	93	+23	+ 3	2.1	"
52	Jan. 26	7.5	29.79	+42	1411	1015	74	+19	—10	1.8	P.
53	May 4	1.7	29.71	+59	1386	1001	48	— 6	+ 6	2.4	"
54	"	2.9	29.70	+62	1382	994	86	—10	+ 1	0.9	"

TABLE V. — *Continued.*

No.	G. M. T.		[redacted]	Th.	*A C.*	*B C.*	Sun.	Res.	Res.	P.R.	Obs.
	1882.	h.	in.	°	″	″	°	″	″	″	
55	May 4	3.8	29.68	+65	1384	1002	29	— 8	+ 2	4.4	P.
56	"	5.0	29 65	+67	1383	996	27	— 9	0	9.3	"
57	"	5.2	29.65	+67	1397	1012	27	+ 5	+14	2.7	E.
58	"	5.8	29 62	+69	1378	991	29	—14	— 6	3.0	P
59	"	8.0	29.57	+68	1369	982	49	—23	—14	7.4	"
60	"	8.6	29 58	+66	1383	996	54	— 9	0	2.2	"
61	May 5	0.8	29.78	+45	1386	995	57	— 6	+11	2.4	"
62	"	3.6	29.81	+47	1388	1005	31	— 4	+13	6.4	"
63	"	5.5	29.79	+48	1376	1000	28	—16	+ 1	5.8	"
64	"	8.7	29 77	+49	1370	988	56	—22	— 6	5.1	"
65	May 6	2.6	30 04	+47	1381	984	39	—11	+ 7	1.0	"
66	May 31	1.6	29.79	+66	1373	996	45	—10	— 9	5.8	"
67	"	1.7	29 79	+66	1381	991	44	—11	— 1	7.0	"
68	"	4.1	29 75	+74	1391	1008	22	— 1	+ 7	3.6	"
69	June 2	2.0	29.76	+58	1372	995	41	—20	— 7	2.2	"
70	"	3.4	29.78	+62	1384	1009	27	— 8	+ 5	2.2	"
71	"	4.7	29.79	+ 65	1379	1008	20	—13	— 2	1.0	"
72	"	10.6	29.77	+69	1373	995	72	—19	—10	1.8	"
73	"	12 4	29.79	+64	1418	1025	92	+26	+11	3.1	"
74	June 5	8.0	29.55	+68	1370	1000	44	—22	—14	3.0	"
75	"	9.0	29.55	+66	1382	994	55	—10	— 1	1.4	"
76	"	12.1	29.59	+63	1390	997	69	— 2	+10	3.0	"
77	"	12.4	29.60	+62	1409	1015	92	+17	+18	4.8	"
78	June 24	4.6	29.88	+86	1377	991	20	—15	—11	2 6	"
79	"	5 4	29.86	+88	1367	973	21	—25	+18	2.4	"
80	"	6.4	29.88	+90	1364	977	29	—38	+ 9	3.7	"
81	"	7.4	29.83	+91	1368	982	39	—24	+13	2.4	"
82	"	8.4	29.81	+91	1361	975	50	—31	+ 5	2.1	"
83	"	9.4	29.81	+91	1372	986	61	—20	+16	3.6	"
84	"	10.3	29.81	+89	1380	986	70	—12	— 9	3.3	"
85	July 11	7.2	29 62	+91	1363	987	37	—23	+12	2.1	"
86	"	9.1	29 61	+89	1385	991	58	— 7	— 5	3.1	"
87	"	9.4	29.60	+87	1384	993	61	— 8	— 6	1.6	"
88	Aug. 6	2 5	29.97	+80	1387	994	38	— 5	+ 1	1.9	E.
89	"	3.9	29.94	+86	1382	995	27	—10	— 5	1.8	"
90	"	5.7	29.93	+91	1386	994	29	— 6	— 3	2.8	P.
91	"	5.8	29.91	+91	1372	980	29	—20	0	3.1	E.
92	"	6.7	29.91	+91	1381	982	36	—11	— 8	3.4	P.
93	"	7 9	29.90	+88	1382	989	48	—10	— 6	1.4	"
94	"	8.7	29.89	+87	1384	988	57	— 8	— 4	2.2	"
95	"	9.7	29.88	+88	1378	988	71	—14	—10	3.1	"
96	"	10.7	29.88	+86	1389	999	79	— 8	+ 1	1.9	"
97	"	11.4	29.88	+84	1398	1006	85	+ 6	+11	3 1	E.
98	"	11.6	29.88	+84	1401	1012	87	+ 9	+14	1.8	"
99	"	11 6	29 88	+83	1396	1009	87	+ 4	+ 9	2.1	"
100	"	11.7	29 88	+83	1397	1005	88	+ 5	+10	3.0	"
101	"	11.9	29.88	+83	1402	1007	90	+10	+15	3.0	P.
102	"	12 0	29.88	+82	1395	998	91	+ 3	+ 8	3.2	"
103	"	12.2	29.88	+82	1395	995	93	+ 3	+ 8	1.4	"
104	"	12.3	29.88	+81	1390	990	94	— 2	+ 4	1.1	"
105	"	12.3	29.88	+81	1384	997	94	— 6	— 2	1.4	"
106	"	12.4	29.88	+80	1364	....	95	—28	+12	6.8	"
107	"	21.4	29.85	+71	....	997	93	....	....	0.3	E.
108	"	21.6	29.85	+71	....	995	91	....	....	1.5	"

TABLE V. — *Continued.*

No.	G. M. T.		Bar.	Th.	*A C.*	*B C.*	Sun.	Res.	Res.	P. E.	Obs.
	1882.	h.	in.	°	"	"	°	"	"	"	
109	Aug. 6	21 7	29.85	+71	....	993	90	....	....	2.3	E.
110	"	21 8	29.85	+71	....	994	80	....	....	1.7	"
111	"	22.2	29.85	+71	....	1001	85	....	....	2.3	"
112	"	22.3	29.85	+71	....	1000	[illegible]	....	....	1.7	"
113	"	22.4	29.84	+71	....	998	88	....	....	2.8	"
114	"	22.4	29.84	+71	...	996	83	....	....	2.2	"
115	"	23.1	29.84	+71	1394	997	75	+ 2	+11	2.8	"
116	"	23.2	29 78	+71	1396	1002	74	+ 4	+13	2.3	"
117	"	23.4	29.79	+72	1395	999	72	+ 8	+12	4.3	"
118	"	23.6	29 80	+72	1384	993	70	− 8	+ 1	3.0	"
119	"	23.7	29 80	+73	1388	993	68	− 9	− 1	1.8	"
120	"	23.8	29.81	+73	1387	997	67	− 5	+ 3	2.1	"
121	Aug. 7	0.0	29.81	+74	1389	997	65	− 3	+ 5	2.1	"
122	"	0.1	29.82	+74	1404	994	64	+12	+20	1.6	"
123	"	0.7	29.82	+76	1384	987	57	− 8	− 1	4.6	P.
124	"	1.7	29.80	+79	1387	987	47	− 5	+ 1	3.1	"
125	"	2.7	29.80	+83	1384	988	37	− 8	− 3	3.9	"
126	"	3 7	29.80	+85	1382	985	29	−10	− 6	1.2	"
127	"	4.7	29 78	+88	1385	987	26	− 7	− 4	1 7	"
128	"	5.7	29.76	+88	1379	981	29	−13	−10	1 9	"
129	"	6.7	29.73	+90	1362	975	37	−30	+ 6	2 0	"
130	"	6 8	29 72	+90	1368	974	38	−24	+12	1 6	"
131	"	7.4	29 72	+90	1365	974	44	−27	+ 9	3.2	"
132	"	7.7	29.71	+89	1367	985	47	−25	+12	2.8	"
133	"	9.5	29.70	+85	1392	1006	66	0	+ 4	0.7	E.
134	"	9 6	29 69	+84	1384	1002	67	− 8	− 4	2.2	"
135	"	9 7	29.69	+84	1384	1003	68	− 8	− 4	1.4	"
136	"	9.9	29 69	+83	1385	1005	70	− 7	− 3	1.8	"
137	"	10.1	29 68	+83	1385	1005	73	− 7	− 3	2.3	"
138	"	10.3	29 68	+82	1381	1002	75	−11	− 6	2 1	"
139	"	10.4	29 68	+82	1392	1009	76	0	+ 5	1.6	"
140	"	10.5	29 67	+81	1391	1003	77	− 1	+ 4	1 9	"
141	"	11 6	29.67	+81	1395	1001	87	+ 8	+ 8	1.6	P.
142	"	11 7	29 68	+80	1390	994	88	− 2	+ 3	2.6	"
143	"	11 8	29.70	+80	1388	994	89	− 4	+ 1	1.7	"
144	"	11 9	29.71	+79	1397	995	90	+ 5	+11	1.6	"
145	"	12.0	29.71	+79	1393	991	91	+ 1	+ 7	3.0	"

Additional evidence regarding the conditions under which these observations were made is furnished by the following notes.

1. Sun shining, snow on ground. Images somewhat unsteady.

12. *B* invisible, *A* and *C* difficult, owing to darkness.

15. Wind very high.

16. Image of *A* unsteady.

17. Hazy to south, but sun shining brightly.

20. The readings for *A* are 74.8, 79.7, and 98.8; for *B*, 03.6, 07.8, and 26.3; for *C*, 76.9, 81.2, and 96.2. The cause of this curious change is not certain. It was obvious to the eye, the cross-wires

appearing to withdraw from A after they had been set upon it. Ice had filled the holes in which the points of the tripod rested, and probably caused this motion. It may, however, have been due to a real change in the refraction, or to a change in the instrument, which had just been removed from a warmer place. In the subsequent sets the positions of A, B, and C remained constant near 91, 19, and 90.

24. Air unusually clear and steady. Wind high. Sun shining.
25. Cloudy.
26. Sun shining, wind high.
36. C difficult on account of darkness; settings discordant.
37. Air clear, wind light.
46. Hazy. A difficult.
53. Cloudy and a little hazy, but images good.
65. Images very bad.
74. Broken clouds, sun shining on C and Observatory.
75. Broken clouds, sun shining on A.
86. Heavy thunder-storm to the southwest.
87. A seen with difficulty owing to intervening rain.
107. A hidden in mist.

Fifty-two sets were taken during the winter, and gave the mean value of $AC = 1408''$, and $BC = 1008''$. The mean height of the barometer was 30.21 in., and of the thermometer 11°. Ninety-two sets were taken in warm weather, and gave $AC = 1384''$, and $BC = 995''$. Barometer 29.78 in., thermometer 76°.

The mean value of AC for the whole series was $1392''$, and that of $BC = 1000''$. The extreme range of the barometer was from 30.70 to 29.45 in., or 1.25 in. The thermometer varied from +91° to −16°, or 107°. The various values of AC differ from their mean of $1392''$ on the average by $16''.8$. But nearly all of the residuals of the winter observations are positive, those in summer negative. If the residuals of each series are taken from the mean of that series, the average value is reduced to $8''.5$. This is nearly the same as $8.''3$, the mean of the residuals after correcting for temperature and pressure. The probable error of a single setting is $2''.7$, both in the winter and summer observations. Since three pointings are made on each object, the errors of observation are much less than the deviations due to refraction. Moreover, some observations are included in which one or more of the objects were seen with great difficulty, on account of haze, unsteadiness of the air, or increasing twilight. Probably the error due to phase caused by varying illu-

mination, personal equation, and other causes, greatly exceeds this, especially on an object like a hill-top, on which there is no signal to be observed.

To study the nature of the variations they have been grouped in Table VI., according to the zenith distance of the sun, the height of the barometer, the height of thermometer, and the logarithm of the correction for temperature and pressure according to Bessel's formula for refraction. In each portion of the table the first column gives the approximate mean value of the argument in each group; the second, the number of sets; the third, the mean value of *A C.*

TABLE VI. — Grouping of Observations.

Sun's Zenith Distance.			Barometer.		
Z. D.	No.	*A C.*	Reading.	No.	*A C.*
°		″	in.		″
81	1	1405.0	29.40	1	1403.0
82	..	.....	29.50	2	1380.5
83	..	.....	29.60	10	1384.6
84	2	1398.0	29.70	22	1383.9
85	1	1398.0	29.80	30	1389.8
86	..	.....	29.90	20	1401.9
87	8	1397.3	30.00	16	1409.7
88	4	1398.0	30.10	6	1396.7
89	5	1396.2	30.20	8	1398.7
90	8	1403.7	30.30	2	1405.5
91	5	1402.6	30.40	5	1405.6
92	4	1409.2	30.50	..	.....
93	4	1411.5	30.60	8	1403.2
94	4	1396.0	30.70	5	1409.0
95	2	1394.5			
96	1	1450.0			

Thermometer.			Bessel's Refractions.		
Reading.	No.	*A C.*	Log.	No.	*A C.*
°		″			″
−10	5	1410.4	−.030	35	1378.9
0	20	1403.5	−.020	30	1389.0
+10	5	1407.2	−.010	15	1386.3
+20	10	1411.9	.000	8	1377.8
+30	6	1408.2	+.010	6	1395.5
+40	4	1401.5	+.020	8	1407.8
+50	5	1380.2	+.030	18	1416.5
+60	8	1390.0	+.040	3	1399.7
+70	20	1384.8	+.050	15	1400.9
+80	28	1389.4	+.060	7	1413.7
+90	24	1374.5	+.070	5	1410.4

The first part of this table shows that no appreciable effect is produced on the refraction by the varying altitude of the sun. This conclusion was also derived from the observations on Wachusett. The group corresponding to a zenith distance of 96° consists of a single set, No. 12. The separate settings are discordant, the observation was very uncertain on account of the darkness, and *B* had become invisible. A variation in the refraction is obvious when either the barometer or thermometer varies. Low temperatures are, however, accompanied by high pressures, as is shown in the mean values given on page 282. The effect of these two causes cannot readily be distinguished. Probably both act, since both affect the density of the air. The grouping according to both temperature and pressure shows that this correction is a real one, and should be applied, although this is not customary in geodetic work.

TABLE VII. — CONSTANTS OF INSTRUMENT.

Date.	G. M. T.	Level.	Coll.	Alt.	Res.	Obj.
1882.	h.	"	"	"	"	
August 6	2.7	+19	+128	− 320	−3	*C*
"	3.4	+12	+126	− 322	−5	*C*
"	7.3	+29	+125	− 313	+4	*C*
"	11.4	+12	+133	+1083	...	*A*
"	"	+14	+129	+ 688	...	■
"	"	+14	+128	− 318	−1	*C'*
"	21.3	+ 1	+136	+ 681	...	*B*
"	"	0	+136	− 314	+8	*C'*
"	22.2	+ 2	+131	+ 687	...	*B*
"	"	+ 4	+131	− 311	+6	*C'*
"	23.2	+ 7	+132	+1078	...	*A*
"	"	+ 9	+132	+ 680	...	■
"	"	+ 7	+132	− 317	0	*C*
"	"	+ 8	+132	− 315	+2	*C'*
"	23.7	+11	+129	+1076	...	*A*
"	"	+ 6	+133	+ 680	...	*B*
"	"	+ 5	+133	− 316	+2	*C*
August 7	0.4	+ 9	+133	+1068	...	*A*
"	"	+ 7	+135	+ 680	...	*B*
"	"	+ 7	+135	− 323	−6	*C'*
"	10.1	+ 8	+129	+1068	...	*A*
"	"	+ 5	+130	+ 682	...	*B*
"	"	+ 3	+127	− 322	−5	*C'*

A series of measurements was made on August 6 and 7 to determine the steadiness of the constants of the instrument. In Table VII. the successive columns give the date, the Greenwich mean time, the level, the collimation, and the absolute altitude of the point observed,

in seconds of arc. The mean altitude of the nearer point, C, was found by these observations to be $-317''$, and the next column gives the residual from this mean. The last column gives the object observed, C' being employed to indicate a second point on C, which with certain illuminations was more readily seen. C' was found to be $277''$ below C, and the measures of its altitude have been corrected by that amount.

The average value of the residuals in the last column but one is $3.''4$, which shows the degree of accuracy that may be attained with this instrument in determining absolute altitudes. The accuracy, portability, and cheapness of the micrometer level ought to render it useful for many purposes. Valuable work could be done with it from any station commanding a distant view. Observations of distant points in different azimuths, under varying meteorological conditions, are still much to be desired.

From the differential character of the observations described above, it has not been necessary to consider the curvature of the earth, or its variation in different azimuths, in the present discussion.

INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATION FROM THE RUMFORD FUND.

XV.

ATMOSPHERIC REFRACTION.

By Edward C. Pickering.

Commmunicated December 9, 1885.

PART II.

The observations described in Part I. relate exclusively to the refraction of the portion of the air between two objects on the surface of the earth. In astronomical observations we have to consider the effect of the entire column of air traversed by the light from an object outside the earth's atmosphere until it reaches the observer. The variation of this quantity, and the effect of local causes upon it, is an important source of error in many astronomical observations. For instance, the systematic differences in the declinations of the southern stars, as determined at different observatories, may be due to different refractions near the northern and southern horizons. The study of this matter has usually been left to the large alt-azimuths and transit-circles to be found in an astronomical observatory. From the fixed position of these instruments it is not easy to vary the conditions as much as might be desired. We are therefore ignorant of the variations of the refraction in different azimuths, or the effect upon it of the proximity of large masses of water, of forests, or of snow-covered mountains. Even its variations in different parts of the world are but little known, and it is usual to employ the refraction tables of Bessel, or those of the Pulkova Observatory, under the most varied conditions of climate or local surroundings. The micrometer level seems to be especially adapted to measuring the atmospheric refraction, and it is hoped that the observations described below will show that it is quite practicable for a traveller to determine this quantity at any point where the results are likely to be of interest. Not the least interesting of the results which may be thus obtained is the determination of the law regulating the refraction at great elevations.

The use of the micrometer level is limited to altitudes of three or four degrees; but within these limits the refraction and its uncertain variations are so large that angular measurements of great accuracy are not required. The only instruments needed are the micrometer level, a chronometer with some means of determining its error and rate, a barometer, and a thermometer.

The observations described below consisted in a series of determinations of the corresponding altitudes and times at which the sun or a star gradually approached the horizon. A complete observation consisted in observing the temperature and pressure of the air, and determining the error of the chronometer by comparing it with a standard clock whose error was known. The micrometer level was then placed in position on the west balcony of the dome of the Harvard College Observatory, and its collimation and level constants determined as described on page 269. The telescope was next turned nearly in the direction in which the sun or star would set, and several readings of the level taken in various azimuths. A series of measures was then made of the apparent altitude of the object as it approached the horizon, and the corresponding times. Finally, the preliminary measures, or such a portion of them as seemed to be essential, were repeated. When the sun was observed, the settings were first made on its lower limb until it disappeared below the horizon, and then the upper limb was measured until it also disappeared. For night observations, a fine needle was inserted in the field, and this formed a dark bar, which was visible against the sky without the necessity of a special field illumination.

A summary of the measures is contained in Table VIII., which gives, in successive columns, a number for reference, the date, the approximate Greenwich mean time, and the object observed. These are followed by the number of settings made, the corrected atmospheric pressure in inches, and the temperature in Fahrenheit degrees.

Series 2 to 8 inclusive were made by Mr. D. B. Pratt, the others by myself. Of the settings, 329 were made on *a Bootis*, 294 on the upper limb of the sun, and 122 on the lower limb, or 745 in all.

The value of the instrumental constants employed are given in Table IX. The successive columns give a number for reference, the date, and the Greenwich mean time. The next four columns give the apparent elevation of the object observed with the instrument placed in its four different positions, O_p, O_v, E_p, and E_v. The mean of these four readings gives the apparent height of the object. The excess of either of the four readings over the mean gives the correction to

TABLE VIII.—SUMMARY OF SERIES.

No.	Date.	G. M. T.	Object.	No. Obs.	Barometer.	Ex. Therm.
	1885.	h.			in.	°
1	Aug. 8	12.7	Sun	99	30.072	69.2
2	" 11	11.8	"	38	29.955	76.1
3	" 14	11.7	"	34	29.550	78.2
4	" 15	11.7	"	26	29.867	66.6
5	" 22	11.6	"	60	29.836	76.6
6	Sept. 2	11.2	"	76	30.034	55.0
7	" 2	15.3	α Bootis	39	30.034	55.0
8	" 8	11.2	Sun	83	30.012	59.9
9	" 20	14.2	α Bootis	51	30.222	48.0
10	" 21	14.1	"	45	29.991	52.1
11	Oct. 10	12.8	"	50	30.075	52.4
12	" 17	12.3	"	55	29.988	50.8
13	" 25	11.8	"	89	30.149	44.2

be applied to measures made in the corresponding position of the instrument. The position of the instrument actually employed was O_p, except in the observations to which Nos. 20 to 31 relate, when it was O_v. The corresponding excess is given in the next column. The last two columns give the values of the collimation and level error, in seconds of arc.

During September the level was used in some geodetic observations among the mountains of New Hampshire and Vermont. Nos. 20 to 25 were taken from the top of Mt. Moosilauk, and Nos. 26 to 31 from Mt. Mansfield. Although the instrument was carried in a wagon over rough mountain roads, the effect on its constants seemed to be inappreciable. The level error, as shown in the last column, did not appear to change perceptibly during the entire series. The collimation at first underwent a singular change which may have been due to a looseness of the screws holding the reticule. No change appears to have taken place during a single series of observations, since the collimation was always substantially the same before and after it, that is, in the pairs of measures made upon the same date. The effect is therefore eliminated in the final results. No change appears to have taken place after September 3.

Nos. 11, 15, 16, 19, 32, 33, 34, 35, 36, and 37 relate to the needle used to observe *α Bootis;* in the other cases the intersection of the cross-wires was observed. The interval between them was about 85 divisions, which affects the collimation, but not the level, to this extent.

In No. 10 the reading of E_v has been assumed to be in error by one turn of the screw; otherwise, the observed value would be —46.7.

TABLE IX. — CONSTANTS OF INSTRUMENT.

No.	Date.	G.M.T.	*Op.*	*Ov.*	*Ep.*	*Ev.*	Ex.	Col.	Level.
		h.						"	"
1	Aug. 8	11.6	— 16.1	— 21.7	+ 22.5	+ 15.1	+ 2.7	+ 45	0
2	" 8	12.7	— 17.1	— 18.8	+ 20.0	+ 15.7	+ 0.8	+ 21	0
3	" 11	11.5	— 18.0	— 19.0	+ 19.3	+ 17.0	+ 0.8	+ 11	— 3
4	" 11	11.8	— 18.0	— 19.6	+ 19.7	+ 18.0	+ 0.8	+ 11	0
5	" 14	11.4	— 21.0	— 13.4	+ 14.2	+ 22.1	— 3.3	— 54	+ 7
6	" 14	11.7	— 21.4	— 14.0	+ 13.0	+ 21.5	— 3.9	— 56	— 3
7	" 15	11.7	— 28.4	— 12.5	+ 8.6	+ 25.0	— 9.8	— 116	—25
8	" 22	11.1	— 42.9	— 19.0	+ 19.7	+ 42.6	—11.9	— 168	+ 1
9	" 22	11.6	— 41.9	— 19.2	+ 19.8	+ 42.8	—11.0	— 160	+ 6
10	" 29	11.2	— 47.5	— 92.2	+ 93.1	+ 58.3	+24.0	+ 294	+24
11	" 31	11.4	— 21.5	—161.2	+160.6	+ 23.0	+70.1	+ 967	+ 3
12	Sept. 1	11.0	— 35.5	— 10.0	+ 9.7	+ 33.4	—13.8	— 172	— 9
13	" 2	10.8	— 36.0	— 10.7	+ 9.6	+ 33.8	—13.5	— 173	—11
14	" 2	11.2	— 36.7	— 7.2	+ 7.8	+ 36.4	—14.7	— 193	+ 1
15	" 2	14.7	+123.0	— 17.4	+ 15.7	—123.5	+69.6	+ 976	— 9
16	" 2	15.3	+123.0	— 17.2	+ 30.6	—131.5	+71.3	+1054	+14
17	" 3	10.7	— 28.2	— 17.4	+ 16.5	+ 26.5	— 6.0	— 78	— 9
18	" 3	11.2	— 26.2	— 16.7	+ 17.2	+ 28.4	— 4.1	— 73	+10
19	" 3	16.1	+130.1	— 26.3	+ 27.0	—122.0	+80.4	+1066	+31
20	" 6	8.6	—148.3	—149.3	+153.9	+150.1	+ 2.1	+ 17	+22
21	" 6	6.0	—150.1	—153.0	+154.5	+150.6	+ 1.9	+ 24	+ 7
22	" 7	1.6	+ 75.1	+ 70.7	— 70.6	— 74.8	+ 2.3	— 81	+ 1
23	" 7	4.8	—176.5	—180.5	+181.4	+177.9	+ 2.0	— 27	+ 9
24	" 7	7.9	—177.4	—180.5	+180.6	+177.5	+ 1.6	+ 22	0
25	" 7	11.8	—112.8	—116.2	+116.9	+112.0	+ 1.4	+ 29	0
26	" 10	4.0	—215.9	—211.4	+212.4	+216.3	— 1.9	+ 29	+ 6
27	" 10	6.6	—177.9	—181.6	+181.6	+178.3	+ 1.9	— 25	+ 1
28	" 12	8.0	+ 79.0	+ 74.8	— 78.2	— 79.8	+ 2.4	+ 39	+ 1
29	" 12	10.8	+ 79.2	+ 74.0	— 75.1	— 78.5	+ 2.5	— 31	— 1
30	" 15	1.9	+ 77.3	+ 74.3	— 73.7	— 78.0	+ 1.5	+ 25	0
31	" 16	8.4	+ 76.6	+ 74.6	— 73.4	— 76.6	+ 1.8	+ 18	+ 4
32	" 20	14.2	—108.0	—282.3	+283.8	+112.3	+88.6	+1206	+20
33	" 21	14.1	+ 54.3	—117.6	+121.8	— 51.1	+87.7	+1201	+24
34	" 27	12.4	—112.0	—288.2	+283.9	+113.6	+85.9	+1190	+ 7
35	Oct. 10	12.8	+140.5	— 29.4	+ 33.2	—138.4	+86.4	+1192	+22
36	" 14	11.4	+176.6	+ 5.2	— 6.0	—174.4	+83.6	+1186	+ 6
37	" 17	12.3	+125.6	— 45.3	+ 47.4	—123.7	+86.4	+1193	+14
38	" 25	11.8	—112.0	—284.5	+282.5	+115.1	+86.5	+1186	+ 4

These results have next been compared with Bessel's refractions by the aid of Table X. This gives the mean refraction for altitudes of every 100″ from the horizon to 5°, for a temperature of 48°.8 and a barometric pressure of 29.6 inches. The altitude corresponding to any refraction given in the table is found by adding the argument at the top of the column to that given in the first column, all the quantities being expressed in seconds of arc. Thus the refraction 1815″ corresponds to the altitude 1400″, 618″ to 16900″, etc.

After applying the corrections for temperature and pressure of the air to each observation, the residuals have been found by subtracting

TABLE X. — MEAN REFRACTIONS.

Alt.	00	100	200	300	400	500	600	700	800	900
00	2094	2078	2052	2031	2010	1990	1969	1949	1929	1909
1000	1890	1871	1852	1834	1815	1797	1779	1762	1744	1727
2000	1710	1693	1677	1660	1643	1627	1612	1597	1581	1566
3000	1550	1535	1521	1507	1493	1479	1465	1452	1439	1426
4000	1413	1400	1387	1375	1363	1351	1339	1327	1316	1305
5000	1295	1284	1273	1262	1251	1241	1231	1221	1211	1202
6000	1192	1183	1174	1165	1156	1147	1138	1130	1122	1114
7000	1106	1097	1089	1081	1073	1065	1058	1051	1043	1036
8000	1029	1022	1015	1008	1001	994	987	980	974	968
9000	961	955	948	942	935	929	923	917	911	905
10000	900	894	888	882	877	871	866	861	855	850
11000	845	840	834	829	824	819	814	810	805	800
12000	795	791	786	782	777	773	768	764	760	756
13000	752	748	744	740	736	733	729	725	721	717
14000	714	711	707	703	699	695	692	688	685	682
15000	678	675	672	669	665	662	659	656	653	650
16000	646	643	640	637	634	631	627	624	621	618
17000	615	612	609	606	603	600	597	594	592	589

TABLE XI. — NUMBER OF OBSERVATIONS.

Alt.	1.	2.	3.	4.	5.	6.	8.	9.	10.	11.	12.	13.
° ′												
0 20	..	..	..	..	..	1	..	..	..	..	..	..
30	..	..	..	..	..	1	5	..	..	..	..	..
40	8	..	..	..	..	4	8	2	2	..	..	..
50	18	6	7	4	..	5	8	3	2	..	..	..
1 0	16	6	6	4	1	7	6	2	3	..	..	..
10	8	1	6	4	8	4	4	8	2	..	..	2
20	8	..	..	8	5	4	4	2	2	..	..	4
30	8	..	..	4	2	3	4	3	3	..	..	5
40	8	3	..	5	3	8	4	2	2	4	..	5
50	5	6	3	2	3	8	4	2	2	4	..	6
2 0	..	4	4	..	8	3	4	3	8	5	2	5
10	..	4	3	..	4	3	4	2	1	5	2	6
20	..	2	1	..	3	3	4	2	8	5	5	6
30	..	3	..	..	3	3	4	2	2	3	4	6
40	..	2	..	..	4	3	4	1	3	1	4	6
50	8	1	..	..	8	8	4	3	2	..	8	6
3 0	8	..	..	..	2	8	4	2	2	1	4	6
10	4	..	..	..	3	4	4	2	2	5	4	5
20	..	..	1	..	3	3	3	2	2	4	4	6
30	..	..	2	..	3	3	1	1	2	4	5	5
40	..	..	1	..	3	4	..	2	2	4	3	5
50	..	..	..	..	2	3	..	2	2	2	2	8
4 0	..	..	..	..	2	3	..	2	1	2	3	2
10	..	..	..	..	..	..	..	2	..	1	2	..
20	..	..	..	..	..	..	..	1	..	..	2	..
30	..	..	..	..	..	..	..	2	..	..	1	..
40	..	..	..	..	..	..	..	1	..	..	3	..
50	..	..	..	..	..	..	..	..	..	..	1	..

from them the refraction as given by Bessel. These residuals are arranged in groups in Tables XI. and XII. Each group extends over 10′, its central point being given in the first columns. Table XI. gives the number of residuals contained in each group, and Table XII. their mean value. The corresponding number of the series is given at the top of each column. The measures of the upper and lower limbs of the sun are combined, as there seems to be no systematic difference between them. Series 7 is omitted, since there is an error in the number of turns of the micrometer screw, or in the number of minutes in the observed times.

The results of these two tables are combined in Table XIII. The successive columns give the altitude, the corresponding refraction according to Bessel, the total number of observations of the sun, and of *a Bootis*. The next two columns are derived from Table XII., and give the means of the residuals contained in that table relating to the sun, and the means of those relating to *a Bootis*.

TABLE XII.—MEAN RESIDUALS.

Alt.	1.	2.	3.	4.	5.	6.	8.	9.	10.	11.	12.	13.
° ′												
0 20	..	..	..	..	..	—173	..	..	..	..	..	..
30	..	..	..	..	..	—131	— 92	..	..	..	..	..
40	—121	..	..	..	..	—129	—117	+40	—29	..	..	..
50	—136	— 96	—65	—75	..	—137	—119	+16	—24	..	..	..
1 0	—142	—119	—93	—78	—75	—134	—110	—26	—26	..	..	..
10	—139	—146	—78	—52	—89	—119	—104	—23	—21	..	..	—28
20	—114	..	..	—56	—85	—104	— 98	+ 1	—58	..	..	—46
30	—123	..	..	—64	—74	— 51	— 80	0	—29	..	..	—26
40	—128	— 40	..	—70	—64	—107	— 79	+ 8	—58	—16	..	—29
50	—146	— 72	—74	—91	—59	—117	— 88	+22	—28	—13	..	—35
2 0	..	— 41	—86	..	—59	—110	— 92	+15	—25	0	—44	—33
10	..	— 63	—64	..	— 54	— 66	— 82	+20	—14	— 6	—58	—27
20	..	— 38	—53	..	—43	— 25	— 78	+15	—20	—16	—45	—29
30	..	— 29	..	..	—36	—111	— 61	+37	+ 4	— 5	—40	—28
40	..	— 66	..	..	—49	— 85	— 59	+26	— 9	+ 2	—39	—21
50	— 88	— 89	..	..	—20	— 81	— 60	+20	— 4	..	—32	— 6
3 0	— 90	..	..	..	—37	— 78	— 61	+29	— 2	— 9	—32	+ 2
10	— 94	..	..	..	—29	— 82	— 70	+65	— 2	+ 6	—34	+ 5
20	..	..	—59	..	—20	— 66	— 60	+26	+12	+11	—20	+10
30	..	..	—32	..	—20	— 51	— 61	+50	+ 9	— 6	—21	+12
40	..	..	—62	..	—14	— 60	..	+66	+26	+14	— 8	+24
50	..	..	..	..	—10	— 68	..	+34	+12	+ 8	+11	+18
4 0	..	..	..	..	—12	— 67	..	+52	— 2	+16	+ 9	+ 8
10	..	..	..	..	..	..	..	+62	..	+19	—48	..
20	..	..	..	..	..	..	..	+59	..	..	+14	..
30	..	..	..	..	..	..	..	+60	..	..	+14	..
40	..	..	..	..	..	..	..	+56	..	..	+33	..
50	..	..	..	..	..	..	..	..	..	..	+22	..

TABLE XIII.

Alt.	Bessel Ref.	No.		Mean.	
		S.	a.	S.	a.
° ′					
0 20	1852	1	..	—178	..
30	1744	6	..	—111	..
40	1643	20	4	—122	+ 6
50	1550	48	5	—105	— 4
1 0	1465	46	5	—107	—26
10	1387	35	7	—104	—24
20	1316	24	8	— 91	—34
30	1251	21	11	— 78	—18
40	1192	26	18	— 83	—24
50	1138	26	14	— 92	—14
2 0	1089	18	18	— 78	—17
10	1043	18	16	— 66	—17
20	1001	18	21	— 47	—19
30	961	13	17	— 59	— 6
40	923	13	15	— 65	— 8
50	888	19	14	— 58	— 6
3 0	855	17	15	— 66	— 2
10	824	15	18	— 69	+ 8
20	795	10	18	— 51	+ 8
30	768	9	17	— 41	+ 9
40	744	8	16	— 45	+25
50	721	5	11	— 39	+17
4 0	699	5	10	— 40	+17
10	678	..	5	..	+11
20	659	..	8	..	+36
30	640	..	3	..	+37
40	621	..	4	..	+44
50	603	..	1	..	+22

The fact noticed by Argelander, that the refraction derived from the setting sun is less than that of a star is well shown in this table. The difference amounts to one or two minutes of arc.

In this investigation the value of one division of the screw in seconds must be known with accuracy. It was therefore redetermined August 6, 1885, with the same result, 13″.95, as that originally found.

This paper is intended to show that the micrometer level is capable of giving useful results where a larger instrument has generally been considered necessary. Its portability, and the rapidity with which observations may be made by it, adapt it especially to the wants of travellers, and would permit the accumulation of valuable information regarding the atmospheric conditions of comparatively inaccessible points. If required, much greater accuracy could doubtless be attained than is indicated by the stellar observations described above. The instrument was mounted on a wooden balcony, and the times were only taken to whole seconds. Instead of moving the telescope each

time, it might be better to have a series of lines in the field, and observe the transits over each, as in a meridian instrument. The advantages of the two forms of instrument employed, attaching the level to the telescope or to the wyes, will vary with the surrounding conditions. The principal objection to the second method is the time required to determine the constants. This may be done almost equally well when the level is attached directly to the telescope, by taking reciprocal readings from two points one or two hundred yards apart. The variations of the instrumental constants will also doubtless be less with this form of instrument. In either case, if many observations are to be taken from a given station, it is advisable to determine once for all the absolute altitude of some convenient object, and refer everything to that, like the meridian mark of a transit instrument.

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XVI.

A NEW FORM OF POLARIMETER.

BY EDWARD C. PICKERING.

Presented May 26, 1885.

Two instruments have been proposed for measuring the relative intensity of the components of a ray of polarized light. First, the Arago polarimeter, in which a number of glass plates are placed in front of a Savart polariscope, and may be inclined at any desired angle. Bands are produced by the Savart plate, which are made to disappear by inclining the plates so as to neutralize the polarization of the beam. The inclination of the plates is a measure of the intensity of the polarization. Secondly, a polarimeter was devised by the writer,* in which the light is admitted through a rectangular aperture, two juxtaposed images of which are formed by a double-image prism. If the light is polarized, the two rectangles will be of unequal brightness. Interposing a Nicol prism, the two images may always be rendered equal by rotating the Nicol. The intensity of the polarization is measured by the cosine of the angle of rotation. The principal objection to the Arago polarimeter is, that the law connecting the inclination of the plates and the amount of polarization is extremely complex, and is best determined experimentally in each case. This instrument is also ill adapted to measure a ray which is almost perfectly polarized, but when the polarization is slight, the well-known sensitiveness of the Savart bands renders the accidental errors small. The other form of polarimeter is open to the objection that it is not very sensitive when the polarization is slight. On the other hand, the amount of polarization is directly connected with the angle of rotation of the prism by a simple formula, and when a ray is strongly polarized it gives excellent results.

The advantages of both instruments appear to be combined in the following modification of the second instrument. The greater sensi-

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* Proc. Amer. Acad., IX. 1.

tiveness of the Arago polarimeter seems to be due to the fact, that, instead of comparing the brightness of the field on two sides of the line separating them, the comparison may be made on both sides of each band, or throughout nearly the whole field. Accordingly, the rectangular aperture was replaced by a series of metallic bars separated by intervals exactly equal to their width. The double-image prism is then placed at such a distance that the separation of the images shall equal the width of the bars, so that the two images of the intervals shall be exactly in contact. If this condition is fulfilled, the bands will disappear, and the field will be perfectly uniform when the Nicol is turned so that the two images have equal intensity. A slight motion of the Nicol will render the bands alternately light and dark, and the exact point of disappearance may be determined with much precision. The sensitiveness appears to be even greater than that of the Savart plate, since the change in brightness is abrupt at the edges of the band, instead of varying continuously from the centre of the band to the centre of the interspace. The bands were made by cementing a piece of tin-foil to a plate of glass, ruling a series of parallel lines upon it, and removing the foil from the space between the second and third, and the fourth and fifth lines, &c. It was necessary that these intervals should be made less than the interval between the first and second, and the third and fourth lines, &c., by the width of the line cut in the foil; otherwise the bands would be narrower than the interspaces by this amount. The bands might also be sawed out of a plate of brass, and finished with a file. Since the separation of the images produced by Iceland spar varies with the angular direction of the ray traversing it, the bands will not disappear in all parts of the field at once. This, however, is rather an advantage, since the eye is extremely sensitive to a certain phase, in which the bands reappear equally in the two sides of the field. A pointer serves to guide the eye to the centre of the field. Should it be desirable to employ a large field, the rays may be rendered parallel by a convex lens at a distance from the eye equal to its focal distance. There are some advantages in substituting a simple plate of Iceland spar for the double-image prism. The rays compared then come in the same direction, instead of being inclined by an angle equal to that of the images of the prism. The two images do not lie in the same plane. This may be remedied by inclining the bars so that the right-hand edges shall always be in front or behind the left-hand edges. If, however, the bands are narrow, and are placed at the distance of distinct vision, this error is unimportant, and the bands may be made to disappear almost entirely.

Removing the Nicol prism, this polarimeter becomes a very sensitive polariscope. The plane of a polarized ray may be found by turning it until the bands disappear, when the plane of the prism will be inclined 45° to the plane of polarization.

In the paper referred to above, it was shown that the polarization

$$P = \frac{A - B}{A + B} = \cos 2\,v,$$

in which A and B represent the intensity of the two components of the polarized beam, and v is the angle between the position in which the two images are equal and that in which one of them disappears. Of course the bands must first be placed parallel, or perpendicular to the plane of polarization of the ray. It is best to set the Nicol prism in the four positions in which the bands disappear. Subtracting the first reading from the second, and the third from the fourth, and adding the differences, gives $4\,v$. The observations are thus rendered symmetrical, the uncertain point of disappearance of the images is eliminated, and the errors of observation are reduced one half as compared with a single setting.

Table I. serves to reduce the observations. The argument gives the sum of the two differences found by subtracting the first reading from the second, and the third from the fourth. The tabular number gives the corresponding polarization. If the sum of the differences exceeds 180°, it must be subtracted from 360°. This may be avoided by beginning with the second reading.

TABLE I. — Reduction Table.

$2v$	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0°	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
3	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9	99.9
4	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
5	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
6	99.9	99.9	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8
7	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8
8	99.8	99.8	99.7	99.7	99.7	99.7	99.7	99.7	99.7	99.7
9	99.7	99.7	99.7	99.7	99.7	99.7	99.6	99.6	99.6	99.6
10	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6
11	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5
12	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4
13	99.4	99.4	99.3	99.3	99.3	99.3	99.3	99.3	99.3	99.3
14	99.2	99.2	99.2	99.2	99.2	99.2	99.2	99.2	99.2	99.2

TABLE I. — *Continued.*

2v	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
15°	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.0	99.0
16	99.0	99.0	99.0	99.0	99.0	99.0	99.0	99.0	98.9	98.9
17	98.9	98.9	98.9	98.9	98.8	98.8	98.8	98.8	98.8	98.8
18	98.8	98.8	98.7	98.7	98.7	98.7	98.7	98.7	98.7	98.6
19	98.6	98.6	98.6	98.6	98.6	98.6	98.5	98.5	98.5	98.5
20	98.5	98.5	98.4	98.4	98.4	98.4	98.4	98.4	98.4	98.3
21	98.3	98.3	98.3	98.3	98.3	98.2	98.2	98.2	98.2	98.2
22	98.2	98.1	98.1	98.1	98.1	98.1	98.1	98.0	98.0	98.0
23	98.0	98.0	98.0	97.9	97.9	97.9	97.9	97.9	97.8	97.8
24	97.8	97.8	97.8	97.8	97.7	97.7	97.7	97.7	97.7	97.6
25	97.6	97.6	97.6	97.6	97.6	97.5	97.5	97.5	97.5	97.5
26	97.4	97.4	97.4	97.4	97.4	97.3	97.3	97.3	97.3	97.3
27	97.2	97.2	97.2	97.2	97.2	97.1	97.1	97.1	97.1	97.0
28	97.0	97.0	97.0	97.0	96.9	96.9	96.9	96.9	96.9	96.8
29	96.8	96.8	96.8	96.7	96.7	96.7	96.7	96.7	96.6	96.6
30	96.6	96.6	96.5	96.5	96.5	96.5	96.5	96.4	96.4	96.4
31	96.4	96.3	96.3	96.3	96.3	96.2	96.2	96.2	96.2	96.2
32	96.1	96.1	96.1	96.1	96.0	96.0	96.0	96.0	95.9	95.9
33	95.9	95.9	95.8	95.8	95.8	95.8	95.7	95.7	95.7	95.7
34	95.6	95.6	95.6	95.6	95.5	95.5	95.5	95.4	95.4	95.4
35	95.4	95.3	95.3	95.3	95.3	95.2	95.2	95.2	95.2	95.1
36	95.1	95.1	95.1	95.0	95.0	95.0	94.9	94.9	94.9	94.9
37	94.8	94.8	94.8	94.8	94.7	94.7	94.7	94.6	94.6	94.6
38	94.6	94.5	94.5	94.5	94.4	94.4	94.4	94.4	94.3	94.3
39	94.3	94.2	94.2	94.2	94.2	94.1	94.1	94.1	94.0	94.0
40	94.0	93.9	93.9	93.9	93.8	93.8	93.8	93.8	93.7	93.7
41	93.7	93.6	93.6	93.6	93.5	93.5	93.5	93.4	93.4	93.4
42	93.4	93.3	93.3	93.3	93.2	93.2	93.2	93.1	93.1	93.1
43	93.0	93.0	93.0	92.9	92.9	92.9	92.8	92.8	92.8	92.8
44	92.7	92.7	92.7	92.6	92.6	92.6	92.5	92.5	92.5	92.4
45	92.4	92.4	92.3	92.3	92.3	92.2	92.2	92.2	92.1	92.1
46	92.0	92.0	92.0	91.9	91.9	91.9	91.8	91.8	91.8	91.7
47	91.7	91.7	91.6	91.6	91.6	91.5	91.5	91.5	91.4	91.4
48	91.4	91.3	91.3	91.2	91.2	91.2	91.1	91.1	91.1	91.0
49	91.0	91.0	90.9	90.9	90.8	90.8	90.8	90.7	90.7	90.7
50	90.6	90.6	90.6	90.5	90.5	90.4	90.4	90.4	90.3	90.3
51	90.3	90.2	90.2	90.1	90.1	90.1	90.0	90.0	90.0	89.9
52	89.9	89.8	89.8	89.8	89.7	89.7	89.6	89.6	89.6	89.5
53	89.5	89.4	89.4	89.4	89.3	89.3	89.3	89.2	89.2	89.1
54	89.1	89.1	89.0	89.0	88.9	88.9	88.9	88.8	88.8	88.7
55	88.7	88.7	88.6	88.6	88.5	88.5	88.5	88.4	88.4	88.3
56	88.3	88.2	88.2	88.2	88.1	88.1	88.0	88.0	88.0	87.9
57	87.9	87.8	87.8	87.8	87.7	87.7	87.6	87.6	87.5	87.5
58	87.5	87.4	87.4	87.3	87.3	87.2	87.2	87.2	87.1	87.1
59	87.0	87.0	87.0	86.9	86.9	86.8	86.8	86.7	86.7	86.6
60	86.6	86.6	86.5	86.5	86.4	86.4	86.3	86.3	86.2	86.2
61	86.2	86.1	86.1	86.0	86.0	85.9	85.9	85.9	85.8	85.8
62	85.7	85.7	85.6	85.6	85.5	85.5	85.4	85.4	85.4	85.3
63	85.3	85.2	85.2	85.1	85.1	85.0	85.0	84.9	84.9	84.8
64	84.8	84.8	84.7	84.7	84.6	84.6	84.5	84.5	84.4	84.4
65	84.3	84.3	84.2	84.2	84.2	84.1	84.1	84.0	84.0	83.9
66	83.9	83.8	83.8	83.7	83.7	83.6	83.6	83.5	83.5	83.4
67	83.4	83.3	83.3	83.2	83.2	83.1	83.1	83.0	83.0	83.0
68	82.9	82.9	82.8	82.8	82.7	82.7	82.6	82.6	82.5	82.5
69	82.4	82.4	82.3	82.3	82.2	82.2	82.1	82.1	82.0	82.0

TABLE I. — *Continued.*

2v	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
° 70	81.9	81.9	81.8	81.8	81.7	81.7	81.6	81.6	81.5	81.5
71	81.4	81.4	81.3	81.3	81.2	81.2	81.1	81.1	81.0	81.0
72	80.9	80.9	80.8	80.7	80.7	80.6	80.6	80.5	80.5	80.4
73	80.4	80.3	80.3	80.2	80.2	80.1	80.1	80.0	80.0	79.9
74	79.9	79.8	79.8	79.7	79.6	79.6	79.5	79.5	79.4	79.4
75	79.3	79.3	79.2	79.2	79.1	79.1	79.0	79.0	78.9	78.9
76	78.8	78.7	78.7	78.6	78.6	78.5	78.5	78.4	78.4	78.3
77	78.3	78.2	78.2	78.1	78.0	78.0	77.9	77.9	77.8	77.8
78	77.7	77.7	77.6	77.5	77.5	77.4	77.4	77.3	77.3	77.2
79	77.2	77.1	77.0	77.0	76.9	76.9	76.8	76.8	76.7	76.7
80	76.6	76.5	76.5	76.4	76.4	76.3	76.3	76.2	76.2	76.1
81	76.0	76.0	75.9	75.9	75.8	75.8	75.7	75.6	75.6	75.5
82	75.5	75.4	75.4	75.3	75.2	75.2	75.1	75.1	75.0	75.0
83	74.9	74.8	74.8	74.7	74.7	74.6	74.6	74.5	74.4	74.4
84	74.3	74.3	74.2	74.1	74.1	74.0	74.0	73.9	73.8	73.8
85	73.7	73.7	73.6	73.6	73.5	73.4	73.4	73.3	73.2	73.2
86	73.1	73.1	73.0	73.0	72.9	72.8	72.8	72.7	72.7	72.6
87	72.5	72.5	72.4	72.4	72.3	72.2	72.2	72.1	72.1	72.0
88	71.9	71.9	71.8	71.8	71.7	71.6	71.6	71.5	71.4	71.4
89	71.3	71.3	71.2	71.1	71.1	71.0	71.0	70.9	70.8	70.8
90	70.7	70.6	70.6	70.5	70.5	70.4	70.3	70.3	70.2	70.2
91	70.1	70.0	70.0	69.9	69.8	69.8	69.7	69.7	69.6	69.5
92	69.5	69.4	69.3	69.3	69.2	69.1	69.1	69.0	69.0	68.9
93	68.8	68.8	68.7	68.6	68.6	68.5	68.4	68.4	68.3	68.3
94	68.2	68.1	68.1	68.0	67.9	67.9	67.8	67.8	67.7	67.6
95	67.6	67.5	67.4	67.4	67.3	67.2	67.2	67.1	67.0	67.0
96	66.9	66.8	66.8	66.7	66.6	66.6	66.5	66.5	66.4	66.3
97	66.3	66.2	66.1	66.1	66.0	65.9	65.9	65.8	65.7	65.7
98	65.6	65.5	65.5	65.4	65.3	65.3	65.2	65.1	65.1	65.0
99	64.9	64.9	64.8	64.7	64.7	64.6	64.6	64.5	64.4	64.3
100	64.3	64.2	64.1	64.1	64.0	63.9	63.9	63.8	63.7	63.7
101	63.6	63.5	63.5	63.4	63.3	63.3	63.2	63.1	63.1	63.0
102	62.9	62.9	62.8	62.7	62.7	62.6	62.5	62.5	62.4	62.3
103	62.2	62.2	62.1	62.0	62.0	61.9	61.8	61.8	61.7	61.6
104	61.6	61.5	61.4	61.4	61.3	61.2	61.2	61.1	61.0	60.9
105	60.9	60.8	60.7	60.7	60.6	60.5	60.5	60.4	60.3	60.2
106	60.2	60.1	60.0	60.0	59.9	59.8	59.8	59.7	59.6	59.6
107	59.5	59.4	59.3	59.3	59.2	59.1	59.1	59.0	58.9	58.8
108	58.8	58.7	58.6	58.6	58.5	58.4	58.4	58.3	58.2	58.1
109	58.1	58.0	57.9	57.9	57.8	57.7	57.7	57.6	57.5	57.4
110	57.4	57.3	57.2	57.1	57.1	57.0	56.9	56.9	56.8	56.7
111	56.6	56.6	56.5	56.4	56.4	56.3	56.2	56.1	56.1	56.0
112	55.9	55.8	55.8	55.7	55.6	55.6	55.5	55.4	55.3	55.3
113	55.2	55.1	55.0	55.0	54.9	54.8	54.8	54.7	54.6	54.5
114	54.5	54.4	54.3	54.2	54.2	54.1	54.0	54.0	53.9	53.8
115	53.7	53.7	53.6	53.5	53.4	53.4	53.3	53.2	53.2	53.1
116	53.0	52.9	52.8	52.8	52.7	52.6	52.6	52.5	52.4	52.3
117	52.2	52.2	52.1	52.0	52.0	51.9	51.8	51.7	51.6	51.6
118	51.5	51.4	51.4	51.3	51.2	51.1	51.0	51.0	50.9	50.8
119	50.8	50.7	50.6	50.5	50.4	50.4	50.3	50.2	50.2	50.1
120	50.0	49.9	49.8	49.8	49.7	49.6	49.5	49.5	49.4	49.3
121	49.2	49.2	49.1	49.0	48.9	48.9	48.8	48.7	48.6	48.6
122	48.5	48.4	48.3	48.2	48.2	48.1	48.0	47.9	47.9	47.8
123	47.7	47.6	47.6	47.5	47.4	47.3	47.2	47.2	47.1	47.0
124	47.0	46.9	46.8	46.7	46.6	46.6	46.5	46.4	46.3	46.2

TABLE I. — *Continued.*

$2v$	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
125°	46.2	46.1	46.0	45.9	45.9	45.8	45.7	45.6	45.6	45.5
126	45.4	45.3	45.2	45.2	45.1	45.0	44.9	44.9	44.8	44.7
127	44.6	44.5	44.5	44.4	44.3	44.2	44.2	44.1	44.0	43.9
128	43.8	43.8	43.7	43.6	43.5	43.4	43.4	43.3	43.2	43.1
129	43.0	43.0	42.9	42.8	42.7	42.7	42.6	42.5	42.4	42.3
130	42.3	42.2	42.1	42.0	41.9	41.9	41.8	41.7	41.6	41.6
131	41.5	41.4	41.3	41.2	41.2	41.1	41.0	40.9	40.8	40.8
132	40.7	40.6	40.5	40.4	40.4	40.3	40.2	40.1	40.0	40.0
133	39.9	39.8	39.7	39.6	39.6	39.5	39.4	39.3	39.2	39.2
134	39.1	39.0	38.9	38.8	38.8	38.7	38.6	38.5	38.4	38.4
135	38.3	38.2	38.1	38.0	37.9	37.9	37.8	37.7	37.6	37.5
136	37.5	37.4	37.3	37.2	37.1	37.1	37.0	36.9	36.8	36.7
137	36.6	36.6	36.5	36.4	36.3	36.2	36.2	36.1	36.0	35.9
138	35.8	35.8	35.7	35.6	35.5	35.4	35.3	35.3	35.2	35.1
139	35.0	34.9	34.9	34.8	34.7	34.6	34.5	34.4	34.4	34.3
140	34.2	34.1	34.0	34.0	33.9	33.8	33.7	33.6	33.5	33.5
141	33.4	33.3	33.2	33.1	33.0	33.0	32.9	32.8	32.7	32.6
142	32.6	32.5	32.4	32.3	32.2	32.1	32.1	32.0	31.9	31.8
143	31.7	31.6	31.6	31.5	31.4	31.3	31.2	31.1	31.1	31.0
144	30.9	30.8	30.7	30.7	30.6	30.4	30.4	30.3	30.2	30.2
145	30.1	30.0	29.9	29.8	29.7	29.6	29.6	29.5	29.4	29.3
146	29.2	29.2	29.1	29.0	28.9	28.8	28.7	28.6	28.6	28.5
147	28.4	28.3	28.2	28.1	28.1	28.0	27.9	27.8	27.7	27.6
148	27.6	27.5	27.4	27.3	27.2	27.1	27.1	27.0	26.9	26.8
149	26.7	26.6	26.6	26.5	26.4	26.3	26.2	26.1	26.0	26.0
150	25.9	25.8	25.7	25.6	25.5	25.4	25.4	25.3	25.2	25.1
151	25.0	25.0	24.9	24.8	24.7	24.6	24.5	24.4	24.3	24.2
152	24.2	24.1	24.0	23.9	23.8	23.8	23.7	23.6	23.5	23.4
153	23.3	23.3	23.2	23.1	23.0	22.9	22.8	22.7	22.7	22.6
154	22.5	22.4	22.3	22.3	22.2	22.1	22.0	21.9	21.8	21.7
155	21.6	21.5	21.5	21.4	21.3	21.2	21.1	21.0	21.0	20.9
156	20.8	20.7	20.6	20.5	20.4	20.4	20.3	20.2	20.1	20.0
157	19.9	19.9	19.8	19.7	19.6	19.5	19.4	19.3	19.2	19.2
158	19.1	19.0	18.9	18.8	18.7	18.7	18.6	18.5	18.4	18.3
159	18.2	18.1	18.0	18.0	17.9	17.8	17.7	17.6	17.5	17.4
160	17.4	17.3	17.2	17.1	17.0	16.9	16.8	16.8	16.7	16.6
161	16.5	16.4	16.3	16.2	16.2	16.1	16.0	15.9	15.8	15.7
162	15.6	15.6	15.5	15.4	15.3	15.2	15.1	15.0	15.0	14.9
163	14.8	14.7	14.6	14.5	14.4	14.3	14.3	14.2	14.1	14.0
164	13.9	13.8	13.7	13.7	13.6	13.5	13.4	13.3	13.2	13.1
165	13.1	13.0	12.9	12.8	12.7	12.6	12.5	12.4	12.4	12.3
166	12.2	12.1	12.0	11.9	11.8	11.8	11.7	11.6	11.5	11.4
167	11.3	11.2	11.2	11.1	11.0	10.9	10.8	10.7	10.6	10.5
168	10.4	10.4	10.3	10.2	10.1	10.0	9.9	9.8	9.8	9.7
169	9.6	9.5	9.4	9.3	9.2	9.1	9.1	9.0	8.9	8.8
170	8.7	8.6	8.5	8.5	8.4	8.3	8.2	8.1	8.0	7.9
171	7.8	7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	7.1
172	7.0	6.9	6.8	6.7	6.6	6.5	6.5	6.4	6.3	6.2
173	6.1	6.0	5.9	5.8	5.8	5.7	5.6	5.5	5.4	5.3
174	5.2	5.1	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.5
175	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.8	3.7	3.6
176	3.5	3.4	3.3	3.2	3.1	3.1	3.0	2.9	2.8	2.7
177	2.6	2.5	2.4	2.4	2.3	2.2	2.1	2.0	1.9	1.8
178	1.7	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0	1.0
179	0.9	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.2	0.1

TABLE II. — OBSERVATIONS.

No.	Date.	G. M. T.	Dist.	Table of Quantities.	No.	Date.	G. M. T.	Dist.	Table of Quantities.
	1884.	h. m.	°			1885.	h m.	°	
1	Aug. 1	11 58	90	21.7	45	July 15	13 58	90	18.3
2	"	59	90	24.2	46	"	14 3	90	12.6
3	"	12 2	90	25.1	47	"	5	90	9.1
4	"	4	90	30.7	48	July 21	16 13	90	39.2
5	"	9	90	36.0	49	"	16	90	36.5
6	"	16	90	43.6	50	"	18	90	39.8
7	"	17	90	44.4	51	July 27	16 5	90	50.2
8	"	20	90	44.6	52	"	18	90	49.2
9	"	25	90	45.9	53	"	23	90	49.4
10	"	29	90	40.4	54	July 28	15 40	90	51.4
11	"	34	90	48 2	55	"	41	90	48.9
12	Aug. 2	9 40	90	45.6	56	"	43	90	49.9
13	"	42	90	45.3	57	"	55	15	21.1
14	"	44	75	36.6	58	"	58	30	6.8
15	"	47	60	37.4	59	"	16 1	45	10 1
16	"	50	45	9.6	60	"	4	60	22.6
17	"	52	30	6.4	61	"	9	75	42.1
18	"	54	15	0.4	62	"	11	90	50.1
19	"	56	60	15.1	63	"	14	105	40.6
20	"	10 1	90	29.6	64	"	17	120	21.6
21	"	3	90	29.2	65	"	34	105	38.8
22	Sept. 13	10 48	90	33.6	66	"	40	90	50.1
23	"	49	90	34.0	67	"	42	75	43.0
24	"	51	90	35.3	68	"	45	60	27.1
25	"	52	90	38.3	69	"	48	45	15.6
26	"	54	90	39.1	70	"	50	30	10.4
27	"	11 0	90	47.9	71	"	53	15	18.2
28	"	1	90	48.8	72	Sept. 21	16 5	90	58.8
29	"	3	90	49.5	73	"	7	90	60.7
30	"	10	90	58.2	74	"	9	90	59.1
31	"	12	90	59.3	75	"	14	15	16.6
32	"	14	90	58.8	76	"	17	30	2.3
33	"	16	90	58.3	77	"	20	45	12.6
34	Nov. 18	17 40	90	40.5	78	"	23	60	31.6
					79	"	27	75	50.8
	1885.				80	"	30	90	57.0
35	June 6	14 22	90	37.9	81	"	32	105	54.3
36	"	24	90	41.4	82	"	35	120	38.8
37	"	25	90	36.5	83	"	42	105	53.2
38	June 14	10 24	90	44.2	84	"	44	90	60.5
39	"	55	90	50.7	85	"	46	75	49.0
40	"	58	90	52.6	86	"	48	60	29.8
41	"	11 0	90	52.2	87	"	51	45	11.8
42	July 1	4 32	90	45.8	88	"	53	30	1.5
43	"	35	90	45.3	89	"	55	15	16.1
44	"	36	90	45.3					

The use of this polarimeter is shown in Table II., which gives some illustrations of its application to the study of atmospheric polarization. All these observations are made in the vertical plane passing through the sun. The successive columns give a number for reference, the

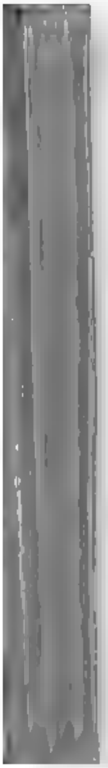
date, the Greenwich mean time, the angular distance of the point observed from the sun, and the resulting polarization. The first 44 sets were made by myself, the others by Miss N. A. Farrar.

Nos. 1 to 11 were made on the zenith after sunset. The later observations were difficult on account of the increasing darkness. On reducing the observations, an unexpected result was attained. The last column shows that the polarization increases regularly, becoming twice as great at the end as at the beginning of the series. The probable explanation of this phenomenon is, that after the sun has set the light reaching us from the zenith comes from the upper portions of the atmosphere. As this is presumably clearer, and more free from dust and haze, we should anticipate a more complete polarization. It affords a simple means of determining the effect of ascending to various heights above the surface of the earth. From the distance of the sun below the horizon these heights are readily calculated. Nos. 22 to 33 were made in the same way, and afford a confirmation of these results. The measures on August 1 are probably somewhat too small, owing to an inclination of the bars to the plane of polarization. On August 2, the sky was clear, but hazy around the sun. The observations were abandoned owing to the approach of cirrus clouds, which probably caused the diminution in the amount of the polarization. On September 13, the air was very clear. No satisfactory explanation appears for the small amount of polarization on July 15. The record states that clouds interfered with set 45, but that it was clear when sets 46 and 47 were taken. The bars may not have been set parallel to the plane of polarization, as it was the first time the observer had used the instrument.

The law determining the amount of polarization at different distances from the sun is indicated by the observations made on July 28 and on September 21. On the first date, the remark occurs, "Sky hazy," and on the second, "Sky entirely clear." The greater polarization on the latter date is doubtless due to this cause. Both sets show the maximum polarization at 90°. They also indicate a greater polarization at 15° than at 30°. This may be due to Babinet's neutral point; but in that case we should anticipate a negative polarization at 15°. Probably these sets are uncertain, on account of the proximity of the sun.

One of the most important applications of this polarimeter will be as a measure of the haziness of the air. As the polarization becomes insensible when the sky is entirely overcast, we apparently have a very delicate test for the amount of haze. Simultaneous observations of a

distant mountain would be particularly valuable in this connection, as a means of comparing the polarization of the sky with that of the light received from the mountain. The relative intensity of the light from the mountain and the adjacent sky should also be compared, since this quantity would also vary with the clearness of the air. All of these quantities are readily measured with this instrument. To compare the light of the sky with that received from the mountain, it is only necessary to conceal the sky line behind one of the bars, and thus compare the relative intensities of the two rays. As the vertical component of one ray will be compared with the horizontal component of the other ray, the relative intensities will be modified by the amount of the polarization. The observation should therefore be repeated, placing the sky line of the mountain behind one of the adjacent bars. The polarization of the light received from the sky and the mountain should also be determined, and we can then determine the relative intensities of the four components. These measurements will be made more readily if the polarimeter is inserted in the eyepiece of a telescope of low power, placing the bars in the focus and the double-image prism between them and the field lens. The Nicol may be placed between the field and eye lenses, or between the eye lens and the eye.



XIX.

OBSERVATIONS OF VARIABLE STARS IN 1885.

By Edward C. Pickering.

Communicated February 10, 1886.

The present publication is the third in a series of annual statements relating to variable stars, which was begun in 1884. The value of these statements mainly depends upon their completeness, and it is therefore to be hoped that all observers of variable stars will send accounts of their work to the Observatory of Harvard College as soon as possible after the close of each year. The principal facts desired are the designations of the stars observed, and the number of nights on which each of them was examined. But brief statements of the results obtained, such as the times of maxima and minima, are also desirable, when they can be readily furnished. Information with regard to the instruments employed and the method of observation will likewise be very serviceable.

The evident need of a new catalogue of variable stars has occasioned some preparations for such a list to be made, and the question of publishing it in the *Astronomische Nachrichten* has been discussed by correspondence with Dr. Krüger, the editor of that periodical. It is, however, desirable that the catalogue, when published, should have a definite and authoritative character, so that it appears best to avoid premature action with regard to it. Meanwhile, it has been decided to make no addition to the list of known variables published in the article entitled "Recent Observations of Variable Stars," although some stars not belonging to that list are now certainly known to be variable. In the present publication such stars are therefore still mentioned in connection with suspected variables. Several stars included in the original list are not there designated by the letters which have at times been applied to them, and it has been decided not to make any change in this respect for the present, in order to avoid the possibility of confusion. Observers are therefore requested to give the right ascensions and declinations of all stars mentioned in their statements, unless they can be designated by the numbers given in the first columns of Tables I. and II. in "Recent Observations of Variable Stars."

Photometric observations of all the stars with which variables are known to have been compared have been undertaken at the Observatory of Harvard College, to which all observers are requested to send lists of the comparison stars employed by them, or references to the published places of such stars. The lists of Argelander, Schönfeld, and Oudemans form the foundation of the work, but information of many comparison stars not included in those lists has been received, and these stars are also to be observed.

It is hoped that it will ultimately be practicable to give the number of observations of each variable star which have been made each year since its discovery. Any information of this kind, hitherto unpublished, will therefore be gratefully received.

It is much to be desired that observers stationed in the southern hemisphere should pay some attention to the variable stars which their situation gives them special facilities for observing, and should send some notice of their work for appearance in this series of publications. It will be seen on examination that no variables in large southern declinations are known to have been recently observed.

The present report relates principally to observations made during 1885; but earlier observations, notice of which has been received since the last report was published, are also mentioned in it. The observers whose work is here stated are named below in alphabetical order, with the abbreviations employed to designate them in the subsequent tabular statements.

B. These observations were made by Mr. T. W. Backhouse, of Sunderland, England, where most of the comparisons were made. A refracting telescope by Cooke, aperture 4¼ inches, magnifying powers 38 and 75, was often used; other observations were made with the finder, power 9, and the rest with a field-glass and similar instruments of low power, or with the naked eye. The methods of comparison were chiefly three: that of Argelander, that in which the relative brightness of the star observed is indicated by a fraction of the apparent difference between two comparison stars, and that of verbal description, in which, however, the words employed are regarded as having numerical values. A copy of the observations has been sent to the Observatory of Harvard College.

D. These observations were made by Dr. N. C. Dunér, at the Observatory of Lund, Sweden, according to the method of Argelander.

Ee. These observations were made by Mr. John H. Eadie, at Bayonne, New Jersey. The telescope employed was made by John Byrne; its aperture is 3¼ inches, and the lowest magnifying power

about 50. Argelander's method of comparison is used. A copy of the observations has been furnished to the Harvard College Observatory. Mr. Eadie acts in co-operation with Mr. Parkhurst in a manner which is explained in the remarks upon Mr. Parkhurst's observations. This system of co-operation appears to be highly efficient and economical. It deserves extension and general introduction among observers not too far separated for ready communication with each other.

En. These observations were made by the Rev. T. E. Espin, of Walsingham, Darlington, England. The instruments employed were a binocular glass and reflecting telescopes of 9 and 17¼ inches in aperture, the last by Calver.

G. These observations were made by Mr. J. E. Gore, of Ballysodare, Ireland, by Argelander's method. The instrument was a binocular field-glass having object-glasses of 2 inches in aperture, and a magnifying power of about six diameters.

Hg. These observations were made at Dorpat, by Dr. E. Hartwig.

Hn. These observations were made by the Rev. J. Hagen, S. J., at the College of the Sacred Heart, Prairie du Chien, Wisconsin. The instrument is a telescope by Merz; its aperture is 3 inches. The observations were made by the division into tenths of the interval between two comparison stars. A copy of the observations has been furnished to the Harvard College Observatory. Mr. Zaiser has taken part in the work as an assistant.

K. These observations were made by Mr. George Knott, at Knowles Lodge, Cuckfield, Hayward's Heath, England. The telescope employed was made by Alvan Clark and Sons; its aperture is 7⅓ inches, and that of the finder 2 inches. The variable is compared with stars differing little from it in brightness; the magnitudes of the comparison stars, and sometimes the magnitude of the variable, are determined by the method of limiting apertures.

M. These observations, which relate exclusively to suspected variables, were made with the meridian photometer of the Harvard College Observatory. The observers using this instrument are E. C. Pickering and O. C. Wendell. The magnitudes of the stars observed with it are referred to a series of one hundred circumpolar stars, the brightness of which was determined by observations described in Volume XIV. of the Annals of the Observatory.

P. These observations were made by Mr. H. M. Parkhurst, at Brooklyn, N. Y. The instrument is a telescope made by Fitz; its aperture is 9 inches, and the magnifying powers employed are 56 and 150. Many of the observations are made by Argelander's method,

and others by photometric apparatus devised by Mr. Parkhurst, and partially described in the article entitled "Recent Observations of Variable Stars," which was mentioned above. A copy of the observations has been furnished to the Harvard College Observatory. The co-operation of Mr. Eadie with Mr. Parkhurst has already been mentioned. The former observer undertakes the observation of certain stars while they are sufficiently bright to be well seen in his telescope, and when they become too faint for further observations he notifies Mr. Parkhurst to begin observing them. When they are again sufficiently bright to be observed by Mr. Eadie, he is informed of the fact by Mr. Parkhurst. This system permits each observer to employ his time to the best advantage. It is hoped that some astronomer having a large telescope at command will undertake observations of the stars studied by Messrs. Parkhurst and Eadie at times when they are too faint to be followed by either observer.

Sk. Professor Safarik, of Prague, Austria, has furnished a statement of observations made by him in 1885, and also of those made in 1883 and 1884, which were incompletely noticed in the circular published in 1885. The observations were made by Argelander's method. The instrument used previous to March, 1885, was a Newtonian reflector 6¼ inches in aperture, with a mirror of silvered glass. The ordinary magnifying power was 32. The finder had an aperture of 3 inches, and a magnifying power of 12. Subsequent observations were made with a refractor by Schröder, with an aperture of 4.5 inches, a magnifying power of 23, and a field of 1° 30′. Between 1880, January 1, and 1884, December 31, Professor Safarik has made a little more than 5700 observations of about 100 stars. During the year 1885 he made 1647 observations of 89 stars. On all suitable occasions he estimates the colors of the stars observed upon Schmidt's scale. The number of observations made in the separate months of each year are given in the statements just described.

Sr. These observations were made, according to Argelander's method, by Mr. E. F. Sawyer, at Cambridgeport, Massachusetts, by means of an opera-glass for the brighter stars, and of a field-glass for the others.

Zr. The observations of Assistant Zaiser have already been mentioned under the heading Hn.

Table I. indicates the progress of observation for stars included in Table I. of previous reports. Other stars, whether known or suspected to be variable, are included in Table II. All the columns of Table I. except the last are repeated from the statement of the previous year.

The first column of the left-hand page gives a provisional number for designating the star. This number is taken from Schönfeld's Catalogue when the star occurs there; in other cases, a letter is added to the number. The second column contains numbers from the Photometric Catalogue called Harvard Photometry, and published in Volume XIV. of the Annals of the Harvard College Observatory. The following columns contain the usual designation of the star, its right ascension and declination for 1875, magnitude at maximum and minimum, and period in days.

The first column of the right-hand page repeats the number to be used for the provisional designation of the star. The second gives the class to which the star belongs, upon the system of classification employed in the Proceedings of the American Academy of Arts and Sciences, XVI. 257. Upon this system, Class I. includes temporary stars; Class II., stars undergoing large variations in periods of several months; Class III., irregularly variable stars, undergoing but slight changes in brightness; Class IV., variable stars of short period, like *β Lyræ* or *δ Cephei;* Class V., Algol stars, or those which at regular intervals undergo sudden diminutions of light, lasting for a few hours only. The third column gives the name of the discoverer, and the fourth column the date.

The last column contains the number of nights on which each star was observed by the astronomer whose designation is attached to the number. The abbreviations employed have been explained above. The designation Sk. is preceded by three numbers, which relate respectively to observations made in 1883, 1884, and 1885. A dash replaces a number when some observations are known to have been made, but their number has not been ascertained.

Table I. is followed by a series of remarks containing observed dates of maximum and minimum, and other information derived from the observers with regard to particular stars.

Table II. indicates the progress of observation of stars suspected or known to be variable, but not included in Table I. for reasons previously explained. The stars are designated in the first column by Mr. Chandler's provisional numbers, as in the previous statement. The number is replaced by a dash when the star has not yet been entered in Mr. Chandler's catalogue. The second and third columns give the right ascensions and declinations of the stars for 1875. The fourth column gives the number of observations made by each observer, as in the last column of Table I. The abbreviations are likewise the same. The letters in the last column refer to the remarks on page 334.

TABLE I. — VARIABLE STARS.

No.	H.P.	Name.	R. A. 1875.	[redacted] 1875.	[redacted]	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
0a	—	Ceti	0 15 26	−20 45.1	5.2	7.0	—
1	51	T Cassiopeiæ	16 29	+55 5.9	6.5 — 7.0	11 — 11.2	436
2	54	R Andromedæ	17 28	+37 58.0	5.6 — 8.6	<12.8	404.7
3	—	S Ceti	17 42	−10 1.3	7.0 — 8.0	<10.7	323.6
4	—	B Cassiopeiæ	17 52	+63 27.2	>1	?	—
5	—	T Piscium	25 31	+13 54.6	9.5 — 10.2	10.5 — 11.0	Irr.
6	94	α Cassiopeiæ	33 25	+55 51.1	2.2	2.8	Irr.
6a	—	U Cephei	51 18	+81 12.1	7.0	9.5	2.5
7	—	S Cassiopeiæ	1 10 30	+71 57.2	6.7 — 8.5	<13	615
8	—	S Piscium	11 2	+ 8 16.3	8.8 — 9.3	<13	406.6
8a	—	Piscium	16 22	+12 12.7	10	14	—
8b	—	Ceti	19 31	− 4 36.6	6.5	7.8	—
8c	—	R Sculptoris	21 13	−33 11.5	5½	7½	207
9	—	R Piscium	24 12	+ 2 14.1	7.4 — 8.3	<12.5	345
10	—	S Arietis	57 55	+11 55.5	9.1 — 9.8	<13	288.8
11	—	R Arietis	2 9 1	+24 28.4	7.6 — 8.5	11.9 — 12.7	186.2
12	370	o Ceti	13 1	− 3 32.7	1.7 — 5.0	8 — 9	331.3
13	—	S Persei	13 54	+58 0.8	8.5 ?	<9.7	—
14	—	R Ceti	19 30	− 0 44.6	7.9 — 8.7	<12.8	167.1
15	—	T Arietis	41 22	+16 59.8	7.9 — 8.2	9.4 — 9.7	324
16	489	ρ Persei	57 10	+38 21.3	3.4	4.2	Irr
17	406	β Persei	3 0 2	+40 28.4	2.2	3.7	2.9
18	—	R Persei	22 6	+35 14.3	8.1 — 9.2	12.5	208.8
19	657	λ Tauri	53 45	+12 8.2	3.4	4.2	4.0
20	—	T Tauri	4 14 43	+19 14.3	9.2 — 11.5	12.8 — <	Irr
21	—	R Tauri	21 27	+ 9 52.9	7.4 — 9.0	<13	325.6
22	—	S Tauri	22 22	+ 9 40.1	9.9	<13	378
22a	—	Doradus	35 19	−62 19.4	5½	6¾	—
23	—	V Tauri	44 48	+17 19.6	8.3 — 9.0	<12.8	168.6
24	—	R Orionis	52 13	+ 7 56.3	8.7 — 8.9	<13	378.8
25	877	ε Aurigæ	53 0	+43 38.2	3.0	4.5	Irr.
26	880	R Leporis	53 55	−14 59.7	6 — 7	8.5 ?	437.8
27	—	R Aurigæ	5 7 12	+53 26.6	6.5 — 7.4	12.5 — 12.7	465
27a	—	S Aurigæ	18 52	+34 2.8	9.4	<13	—
28	—	S Orionis	22 50	− 4 47.5	8.3 ?	<12.3	—
29	1005	δ Orionis	25 37	− 0 23.6	2.2 ?	2.7	Irr.
29a	—	Orionis	29 42	− 5 33.5	10	13	—
30	1091	α Orionis	48 24	+ 7 23.3	1	1.4	Irr.
31	1160	η Geminorum	6 7 20	+22 22.4	3.2	3.7 — 4.2	229.1
31a	—	Monocerotis	16 26	− 2 8.1	7	<10	—
32	1205	T Monocerotis	18 20	+ 7 9.1	6.2	7.6	26.8
33	—	R Monocerotis	32 21	+ 8 50.7	9.5	11.5	Irr.
34	1256	S Monocerotis	34 6	+10 0.5	4.9	5.4	3.4
35	—	R Lyncis	50 59	+55 30.2	9 ?	<12.3	—
36	1334	ζ Geminorum	56 41	+20 45.1	3.7	4.5	10.2
37	—	R Geminorum	59 49	+22 53.8	6.6 — 7.3	<12.3	371.0
38	—	R Canis min.	7 1 50	+10 13.1	7.2 — 7.9	9.5 — 10.0	335.0
38a	—	Puppis	9 43	−44 26.2	3¼	<6	135
38b	—	V Geminorum	16 10	+13 21.8	8.5	12 — 13½	275
38c	1417	U Monocerotis	24 50	− 9 31.0	6.0	7.2	46.0
39	—	S Canis min.	25 56	+ 8 35.0	7.2 — 8.0	<11	332.2
40	—	T Canis min.	27 3	+12 0.6	9.1 — 9.7	<13	335.9
40a	—	Canis min.	34 34	+ 8 40.2	8¼	13.5	405

TABLE I.—VARIABLE STARS.

No.	Class.	Discoverer.	Date.	Observations, 1885.
0*a*	—	Chandler	1881	1 G. 30 Sr.
1	II.	Krüger	1870	6 Ee. 1 En. 3 G. 24,42,27 Sk.
2	II.	Argelander	1858	3 D. 2 Ee. 5 P. 15,19,8 Sk. 27 Sr.
3	II.	Borelly	1872	3 Ee. 7 P. 9,0,0 Sk.
4	I.	Tycho Brahe	1572	—
5	II.	Luther	1855	4 Ee. 1 P.
6	III.	Birt	1831	2 B. 32 Sr.
6*a*	V.	Ceraski	1880	21 Hn. 3 K.
7	II.	Argelander	1861	8 P. 67,39,23 Sk.
8	II.	Hind	1851	10 Ee. 4 P. 1,0,0 Sk.
8*a*	—	Peters	1880	11 P.
8*b*	—	Gould	1874?	1 Hn. 11 Sr.
8*c*	II.	Gould	1872?	—
9	II.	Hind	1850	12 P. 11,2,0 Sk.
10	II.	Peters	1865	10 P.
11	II.	Argelander	1857	6 Hn. 10 P. 7,6,0 Sk.
12	II.	Fabricius	1596	35 B. 37 G. - Hg. 15 K. 18,23,17 Sk.
13	II.	Krüger	1873	- Hg. 21 Hn. 43,39,53 Sk.
14	II.	Argelander	1866	11 P. 9,6,8 Sk.
15	II.	Auwers	1870	12 Hn. 37,30,16 Sk.
16	II.?	Schmidt	1854	36 Sr.
17	V.	Montanari	1669	10 B. 3 Hg. 1 Sr.
18	II.	Schönfeld	1861	6 Ee. 12 H. 6 P. 8,14,9 Sk.
19	V.	Baxendell	1848	9 Zr.
20	—	Hind	1861	3 K. 8 P. 5,5,8 Sk.
21	II.	Hind	1849	6 Ee. 9 P. 19,9,5 Sk.
22	II.	Oudemans	1855	6 Ee. 9 P. 19,9,6 Sk.
22*a*	—	Gould	1874?	—
23	II.	Auwers	1871	12 P. 15,8,7 Sk.
24	II.	Hind	1848	2 Ee. 4,1,0 Sk.
25	III.	Fritsch	1821	34 Sr.
26	II.	Schmidt	1855	17,11,10 Sk. 21 Sr.
27	II.	At Bonn	1862	12 Hn. 3 P. 11,19,0 Sk.
27*a*	II.	Dunér	1881	21 D. 5 P.
28	II.	Webb	1870	13 Ee. 1 K. 10 P. 25,19,23 Sk.
29	III.	J. Herschel	1834	14 Sr.
29*a*	—	Bond	1863	11 Ee. 10 P.
30	III.	J. Herschel	1836	9 Zr.
31	II.?	Schmidt	1866	2 B. 20 Sr.
31*a*	—	Schönfeld	1883	13 Sr.
32	IV.	Gould	1871	71 Sr.
33	II.	Schmidt	1861	—
34	IV.	Winnecke	1867	29 Sr.
35	II.	Krüger	1874	11 P. 5,21,15 Sk.
36	IV.	Schmidt	1844	22 Sr. 10 Zr.
37	II.	Hind	1848	6 K. 11 P. 0,0,7 Sk.
38	II.	At Bonn	1854	8 Hn. 0,14,23 Sk.
38*a*	II.	Gould	1872	—
38*b*	II.	Baxendell	1880	1 Ee. 6 K.
38*c*	II.?	Gould	1873	20 En. 64 Sr.
39	II.	Hind	1856	2 Ee. 7 K. 0,14,25 Sk.
40	II.	Schönfeld	1865	1 Ee.
40*a*	II.	Baxendell	1879	2 Ee. 2 K.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec 1875	Max.	Min.	Per.
			h. m s.	° ′	m.	m.	d.
41	—	S Geminorum	7 35 32	+23 44.6	8.2 — 8.7	<13	294.2
42	—	T Geminorum	41 48	+24 2.7	8.1 — 8.7	<13	288.1
42a	—	S Puppis	43 6	—47 8.8	7½	9	—
43	—	U Geminorum	47 41	+22 19.7	8.9 — 9.7	13.1	Irr.
43a	—	Puppis	55 0	—12 32	8½	<14	310
44	—	R Cancri	8 9 40	+12 0.5	6.2 — 8.3	<11.7	354.4
45	—	V Cancri	14 36	+17 40.9	6.8 — 7.2	<12	272
46	—	U Cancri	28 37	+19 19.5	8.2 — 10.4	<13	305.7
47	—	S Cancri	36 48	+19 29.0	8.2	9.8	9.5
48	—	S Hydræ	47 3	+ 3 32.4	7.5 — 8.5	<12.2	256.4
49	—	T Cancri	49 32	+20 19.7	8.2 — 8.5	9.3 — 10.5	481.2
50	—	T Hydræ	49 35	— 8 39.8	7.0 — 8.1	<12.5	289.4
50a	—	R Carinæ	9 29 6	—62 14.2	4.4	9.3	313
51	—	R Leonis min.	38 4	+35 5.2	6.1 — 7.5	<11.0	374.7
52	1752	R Leonis	40 50	+12 0.5	5.2 — 6.4	9.4 — 10.0	312.6
52a	—	l Carinæ	41 49	—61 55.9	3.7	5.2	31.2
52b	—	Leonis	53 3	+21 51.6	8½	8.6<13	280 ?
52c	—	Antliæ	10 4 22	—37 7.1	6½	<8	—
52d	—	Carinæ	5 23	—60 56.3	6½	9	—
52e	—	U Leonis	17 21	+14 38.1	9½	Inv.	—
52f	1869	Hydræ	31 22	—12 44.1	4½	6	—
53	1880	R Ursæ maj.	35 47	+69 25.9	6.0 — 8.1	12	303.4
54	—	η Argus	40 13	—59 1.6	>1	6.3	Irr.
54a′	—	T Carinæ	50 18	—59 51.2	6.2	6.9	—
55	—	R Crateris	54 25	—17 39.2	>8	<9	—
56	—	S Leonis	11 4 23	+ 6 8.5	9.0 — 9.7	<13	187.6
57	—	T Leonis	32 2	+ 4 3.9	10 ?	<13	—
58	—	X Virginis	55 27	+ 9 46.1	7.8 ?	<10	—
59	—	R Comæ	57 51	+19 28.8	7.4 — 8.0	<13	363
60	—	T Virginis	12 8 12	— 5 20.4	8.0 — 8.8	<13	337
61	—	R Corvi	13 10	—18 34.5	6.8 — 7.3	<11.5	318.6
61a	—	— Virginis	27 26	— 3 43.8	8	14	210±
62	—	T Ursæ maj.	30 42	+60 10.6	7.0 — 8.3	12.2	255.6
63	2147	R Virginis	32 10	+ 7 40.6	6.5 — 7.5	10.0 — 10.9	145.7
63a	—	R Muscæ	34 28	—68 43.3	6.6	7.3	0.9
64	—	S Ursæ maj.	38 28	+61 46.7	7.7 — 8.2	10.2 — 11.1	224.8
65	—	U Virginis	44 46	+ 6 14.0	7.7 — 8.1	12.2 — 12.8	207.4
66	—	W Virginis	13 19 35	— 2 43.4	8.7 — 9.2	9.8 — 10.4	17.3
67	—	V Virginis	21 21	— 2 31.4	8.0 — 9.0	<13	251
68	2275	R Hydræ	22 53	—22 38.0	4.0 — 5.5	10 ?	469.3
69	2289	S Virginis	26 29	— 6 33.0	5.7 — 7.8	12.5	374.0
69a	—	Virginis	14 3 37	—12 42.7	9	14	—
69b	—	R Centauri	7 35	—59 19.8	6	10	—
70	—	T Bootis	8 14	+19 39.1	9.7 ?	<13	—
71	—	S Bootis	18 41	+54 22.7	8.1 — 8.5	13.2	272.4
72	—	R Camelopardi	27 8	+84 23.6	7.9 — 8.6	12 ?	266.2
73	2445	R Bootis	31 41	+27 16.9	5.9 — 7.5	11.3 — 12.2	223.0
73a	2459	Bootis	37 56	+27 3.6	5.2	6.1	370 ?
73b	—	Bootis	48 33	+18 12.1	9.1	12.0 — 13.6	173.9
74	2506	δ Libræ	54 18	— 8 1.2	4.9	6.1	2.3
74a	—	Libræ	15 3 37	—19 33.9	10	<13.5	700±
74b	—	R Triang. Austr.	6 37	—66 2.1	6.6	8.0	3.4
75	—	U Coronæ	13 6	+32 6.4	7.6	8.8	3.5
76	—	S Libræ	14 18	—19 56.1	8.0	12.5 ?	—
77	—	S Serpentis	15 48	+14 46.9	7.6 — 8.6	12.5 ?	361.0

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1885.
41	II.	Hind	1848	–,–,29 Sk.
42	II.	Hind	1848	–,–,31 Sk.
42*a*	—	Gould	1874?	—
43	II.?	Hind	1855	2 Hg. 45 K. 49,54,54 Sk.
43*a*	II.	Pickering	1881	—
44	II.	Schmidt	1829	1 Ee.
45	II.	Auwers	1870	7 P. 8,6,0 Sk.
46	II.	Chacornac	1853	1 Ee. 3 Hn. 6 K. 16,17,8 Sk.
47	V.	Hind	1848	1 Ee. 32 Hn. 2 K.
48	II.	Hind	1848	5,1,22 Sk. 22 Sr.
49	II	Hind	1850	1 En 4 P. 6,5,12 Sk.
50	II.	Hind	1851	4,12,8 Sk.
50*a*	II.	Gould	1871	—
51	II.	Schönfeld	1863	2 P. 24 Sr.
52	II.	Koch	1782	72,34,31 Sk. 38 Sr.
52*a*	—	Gould	1871	—
52*b*	II.	Becker	1882	—
52*c*	—	Gould	1872	—
52*d*	—	Gould	1871	—
52*e*	—	Peters	1878	—
52*f*	—	Gould	1871	13 Sr. 2 Zr.
53	II.	Pogson	1853	3 Hn. 17 K. 2,0,10 Sk. 32 Sr.
54	II.?	Burchell	1827	—
54*a*	—	Thome	1872	—
55	II.	Winnecke	1861	4 Hn. 50,32,26 Sk.
56	II.	Chacornac	1856	—
57	II.	Peters	1865	—
58	II	Peters	1871	4 Ee.
59	II.	Schonfeld	1856	4 K. 9 P.
60	II.	Boguslawski	1840	3,18,5 Sk.
61	II.	Karlinski	1867	25,14,3 Sk.
61*a*	II.	Henry	—	—
62	II.	Hencke	1856	12,10,19 Sk. 34 Sr.
63	II.	Harding	1809	2,0,0 Sk. 31 Sr.
63*a*	IV.	Gould	1871	—
64	II.	Pogson	1853	17 K. 11,14,39 Sk. 32 Sr.
65	II	Harding	1831	12,0,15 Sk. 5 Sr.
66	II.?	Schonfeld	1866	—
67	II.	Goldschmidt	1857	13 P. 4,0,0 Sk.
68	II.	Maraldi	1704	80,0,4 Sk.
69	II.	Hind	1852	7 K.
69*a*	II.	Palisa	1880	10 P.
69*b*	—	Gould	1871	—
70	I.?	Baxendell	1880	—
71	II.	At Bonn	1860	14 Ee. 14 Hn. 6,14,0 Sk.
72	II.	Hencke	1858	10 Ee. 10 P. 51,40,40 Sk.
73	II.	At Bonn	1858	2 G. 0,18,39 Sk. 29 Sr.
73*a*	—	Schmidt	1867	6 Zr.
73*b*	II.	Baxendell	1880	16 Ee.
74	V.	Schmidt	1859	5 Zr.
74*a*	II.	Palisa	1878	—
74*b*	IV.?	Gould	1871	—
75	V.	Winnecke	1869	11 Hn.
76	II.	Borelly	1872	10 P. 19,12,8 Sk.
77	II.	Harding	1828	12 Ee. 9 Hn. 20 P.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
78	2553	S Coronæ	15 16 18	+31 49.1	6.1 — 7.8	11.9 — 12.5	361.0
78a	—	Libræ	34 46	—20 46.5	9	<14	—
79	2639	R Coronæ	43 25	+28 32.5	5.8	13.0	Irr.
80	2647	R Serpentis	44 56	+15 30.8	5.6 — 7.6	<11	357.6
80a	—	V Coronæ	45 4	+39 57.0	7.7	12	360.0
81	—	R Libræ	46 32	—15 51.7	9.2 — 10.0	<13	723
82	2678	T Coronæ	54 16	+26 16.5	2.0	9.5	—
83	—	R Herculis	16 0 37	+18 42.5	8.0 — 9.0	<13	319.0
83a	—	W Scorpii	4 28	—19 48.6	10	<13	224.3
84	—	T Scorpii	9 36	—22 39.9	7	<10	—
85	—	R Scorpii	10 12	—22 38.2	9 ? — 10.5	<12.5	223
86	—	S Scorpii	10 13	—22 35.2	9.1 — 10.5	<12.5	176.9
86a	—	Ophiuchi	14 40	— 7 24.0	9.0	<13.5	326
87	—	U Scorpii	15 16	—17 35.3	9 ?	<12	—
87a	—	Ophiuchi	19 46	—12 8.5	7.5	10.5	365
88	—	U Herculis	20 16	+19 10.8	6.6 — 7.7	11.4 — 11.6	408.3
89	2772	g Herculis	24 32	+42 9.6	5	6.2	Irr.
90	—	T Ophiuchi	26 35	—15 51.8	10	<12.5	—
91	—	S Ophiuchi	27 4	—16 53.7	8.3 — 9.0	<12.5	233.8
91a	—	W Herculis	30 48	+37 35.6	8.0	<14.5	289
91b	—	Urs. Min.	31 40	+72 31.9	8.6	10.5	180 ?
91c	—	R Draconis	32 22	+67 0.7	7.2	13<	245.9
92	2828	S Herculis	46 13	+15 9.2	5.9 — 6.8	11.5 — 12.2	303
93	2839	Ophiuchi	52 30	—12 42.0	5.5	12.5	—
93a	—	V Herculis	53 41	+35 15.5	9.0	11.7	—
94	—	R Ophiuchi	17 0 36	—15 55.5	7.6 — 8.1	<12	302.4
95	2879	α Herculis	8 57	+14 32.1	3.1	3.9	Irr.
95a	2883	U Ophiuchi	10 12	+ 1 21.0	6.1	6.8	0.9
96	2890	u Herculis	12 42	+33 14.1	4.6	5.4	38.5
97	—	Serpentarii	23 9	—21 22.4	>1	?	—
98	2972	X Sagittarii	39 41	—27 46.8	4	6	7.0
99	3035	W Sagittarii	57 2	—29 35.1	5	6.5	7.6
100	—	T Herculis	18 4 22	+31 0.1	7.2 — 8.3	11.4 — 12.1	165.1
101	—	T Serpentis	22 43	+ 6 13.1	9.1 — 10.0	<12.8	342.3
102	—	V Sagittarii	24 4	—18 20.9	7.5 ?	9.5 ?	—
103	—	U Sagittarii	24 32	—19 12.7	7.0	8.3	6.7
104	—	T Aquilæ	39 45	+ 8 36.9	8.8	9.5	Irr.
105	3176	R Scuti	40 49	— 5 50.2	4.7 — 5.7	6.0 — 8.5	71.1
105a	—	κ Pavonis	44 3	—67 23.2	4.0	5.5	9.1
106	3193	β Lyræ	45 28	+33 18.0	3.4	4.5	12.9
107	3224	R Lyræ	51 32	+43 47.1	4.3	4.6	46.0
108	—	S Coron. Austr.	52 43	—37 7.2	9.8	11.5 ?	6.1
109	—	R Coron. Austr.	53 29	—37 7.2	10.5 — 11.5	<12.5	31
110	—	R Aquilæ	19 0 21	+ 8 2.6	6.4 — 7.4	10.9 — 11.2	345.1
111	—	T Sagittarii	9 1	—17 11.2	7.6 — 8.1	<11	381
112	—	R Sagittarii	9 21	—19 31.5	7.0 — 7.2	<12	270.0
113	—	S Sagittarii	12 7	—19 15.1	9.7 — 10.4	<12.7	230
114	3395	R Cygni	33 28	+49 55.1	5.9 — 8.0	13	425.3
115	—	U Vulpeculæ	42 26	+27 0.5	3	?	—
116	—	S Vulpeculæ	43 16	+26 58.7	8.4 — 8.9	9.0 — 9.5	67.5
117	3434	χ Cygni	45 46	+32 36.0	4.0 — 6.0	12.8	406.5
118	3436	η Aquilæ	46 6	+ 0 41.2	3.5	4.7	7.2
119	—	S Cygni	20 2 53	+57 37.6	8.8 — 9.5	<13	322.8
120	—	R Capricorni	4 17	—14 38.2	8.8 — 9.7	<13	347
121	—	S Aquilæ	5 52	+15 14.9	8.9 — 9.9	10.7 — 11.8	147.2

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1885.
78	II.	Hencke	1860	10 Hn. 6 K. 15,50,30 Sk. 35 Sr.
78a	—	Peters	1878	9 P.
79	II.?	Pigott	1795	3 Ee. 5 G. 61,39,26 Sk. 70 Sr.
80	II.	Harding	1826	7 Ee. 1 K. 17 P. 6 Sr.
80a	II.	Dunér	1878	19 D. 3 Ee. 1 En. 27,16,16 Sk.
81	II.	Pogson	1858	—
82	I.	Birmingham	1866	17 B. 13 Hn. 2 K. 3,9,16 Sk.
83	II.	At Bonn	1855	24 Ee. 8 Hn.
83a	II.	J. Palisa	1877	11 P.
84	I.	Auwers	1860	—
85	II.	Chacornac	1853	5 K. 9 P.
86	II.	Chacornac	1854	5 K. 9 P.
86a	II.	Schonfeld	1881	—
[redacted]	I.?	Pogson	1863	—
87a	—	Dunér	1881	6 D.
88	II.	Hencke	1860	18,32,22 Sk. 30 Sr.
89	III.	Baxendell	1857	65 Sr.
90	II.	Pogson	1860	—
91	II.	Pogson	1854	1 Hn.
91a	—	Dunér	1880	13 D. 16 Ee. 7 Hn. 10 P.
91b	II.	Pickering	1881	20,17,70 Sk.
91c	II.	Geelmuyden	1876	89 Sr. 9,13,25 Sk.
92	II.	At Bonn	1856	8 Hn. 35,30,39 Sk.
93	I.	Hind	1848	—
93a	II.	Baxendell	1880	12 Ee.
94	II.	Pogson	1853	—
95	III.	W. Herschel	1795	2 Sr. 18 Zr.
95a	V.	Sawyer	1881	4 Sr.
96	III.	Schmidt	1809?	2 G. 12 Sr. 16 Zr.
97	I.	Fabricius	1604	—
98	IV.	Schmidt	1866	26 Sr.
99	IV.	Schmidt	1866	31 Sr.
100	II.	At Bonn	1857	10 Hn. 7,22,14 Sk. 16 Sr.
101	II.	Baxendell	1860	3 Ee.
102	II.	Quirling	1865	—
103	IV.	Schmidt	1866	—
104	II.	Winnecke	1860	—
105	II.	Pigott	1795	3,0,0 Sk. 65 Sr.
105a	IV.	Thome	1872	—
106	IV.	Goodricke	1784	1 Hg. 19 Zr.
107	II.?	Baxendell	1856	67 Sr.
108	IV.?	Schmidt	1866	—
109	II.?	Schmidt	1860	—
110	II.	At Bonn	1856	7 Ee. 13,3,32 Sk.
111	II.	Pogson	1863	35,17,14 Sk.
112	II.	Pogson	1858	6 P. 24,10,10 Sk
113	II.	Pogson	1860	7 P. –,–,12 Sk.
114	II.	Pogson	1852	9 Hn. 17 P. 10,8,1 Sk.
115	I.	Anthelm	1670	3 K.
116	II.	Hind	1861	31 Hn. 2 K. 7,0,0 Sk.
117	II.	Kirch	1686	2 En. 22 G – Hg. 36 P. 20,27,9 Sk. 28 Sr.
118	IV.	Pigott	1784	—
119	II	At Bonn	1860	9 K. 18 P. 0,2,12 Sk.
120	II.	Hind	1848	—
121	II.	Baxendell	1863	3 Ee 16 K. 30 P.

TABLE I. — *Continued.*

No.	H P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
122	—	R Sagittæ	20 8 22	+10 21.0	8.5 — 8.7	9.8 — 10.4	70.4
123	—	R Delphini	8 58	+ 8 42.7	7.6 — 8.5	12.8	284.0
124	3547	P Cygni	18 11	+37 38.7	3 — 5	<6	—
125	—	U Cygni	15 44	+47 30.1	7.8 ?	9.8 ?	—
126	3557	R Cephei	34 29	+88 45.2	5 ?	10 ?	—
126a	—	— Cygni	37 17	+47 41.8	8	[illegible]	423.
127	—	S Delphini	37 19	+16 38.4	8.4 — 8.6	10.4 — 11.1	275.6
128	—	T Delphini	39 34	+15 56 7	8.2 — 8.9	<13	331.4
129	—	U Capricorni	41 11	—15 14.4	10 2 — 10.8	<13	203.5
130	3654	T Cygni	42 12	+33 55.0	5.5 ?	6 ?	—
131	—	T Aquarii	43 20	— 5 36.5	6.7 — 7 0	12.4 — 12.7	203.2
132	—	R Vulpeculæ	58 49	+23 19.5	7.5 — 8.5	12.5 — 13.0	137.5
132a	—	Capricorni	21 0 19	—24 25.5	9½	14	—
132b	—	T Cephei	7 52	+67 58.9	5.6	9.5	382
133	—	T Capricorni	15 6	—15 41.4	8.9 — 9 7	<13	269.4
134	—	S Cephei	36 45	+78 8 6	7.4 — 8.5	11.5	485
134a	—	Nova Cygni	37 2	+42 18 2			—
135	3845	μ Cephei	39 41	+58 12.4	4 ?	5 ?	Irr.
136	—	T Pegasi	22 2 48	+11 55.7	8.8 — 9.8	<12 5	367.5
137	3981	δ Cephei	24 32	+57 46.6	3 7	4.9	5.4
137a	—	Lacertæ	37 43	+41 43.0	8.6	<13.5	315.
138	—	S Aquarii	50 25	—21 0.6	7.7 — 9.1	<11.5	279.4
139	4078	β Pegasi	57 45	+27 24 2	2 2	2.7	Irr.
140	—	R Pegasi	23 0 22	+ 9 52.1	6.9 — 7.7	12 ?	382.0
141	—	S Pegasi	14 14	+ 8 14.2	7.6	<12.2	—
142	4193	R Aquarii	37 21	—15 58.7	5.8 — 8.5	11 ?	388.0
143	4234	R Cassiopeiæ	52 4	+50 41.5	4 8 — 6.8	<12	425.9

REMARKS.

2. Max. 1885, beginning of February. D.

6a. Min. 1885, March 14, 9h. 32m. G. M. T.; March 19, 9h. 12m. G. M. T. K.

12. Max. 1885, February 4. G. Max. 1885, February 11; magn. 2.9. K.

27a. Max. 1885, February 12. D.

37. Rising to max. 1885, May 11; magn. 7.3. K.

38b. Rising to max. 1885, April 17; magn. 10.1. K.

38c. Max. 1885, January 25±. Min. 1885, January 6.5, February 17. En.

43. Max. 1885, April 8.8; magn. 9.25 (a double max.?); 1885, December 6± ?? K.

46. Max. 1884, December 30; magn. 10.6. K.

47. Minima partially observed, 1885, February 20, March 11. Star disappeared in mist near horizon. K.

53. Max. 1885, June 28; magn. 7.1. Min. 1885, March 10; magn. 13.2. K.

59. Invisible on each occasion. K.

64. Max. 1885, May 5; magn. 7.25. K.

69. Declining. K.

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1885.
122	II.?	Baxendell	1859	25 Ee. 38 Hn. 1 P.
123	II.	Hencke	1859	7 Hn. 18,22,6 Sk.
124	I.	Janson	1600	0,1,16 Sk. 44 Sr. 13 Zr.
125	II.	Knott	1871	8 Ee. 7 En. 12 K. 11 P. 32,43,44 Sk.
126	II.?	Pogson	1856	18 Ee. 7 K. 20,10,12 Sk.
126*a*	II.	Birmingham	1881	3 En. - Hg. 11 P. 60,50,85 Sk.
127	II.	Baxendell	1860	21 Ee. 4 P. –,-,21 Sk.
128	II.	Baxendell	1863	14 Ee. 14 K. 19,19,11 Sk.
129	II.	Pogson	1858	9 P.
130	—	Schmidt	1864	48 Sr.
131	II.	Goldschmidt	1861	11 P.
132	II.	At Bonn	1858	7 Ee. 10 K. 2 P. 8,0,0 Sk.
132*a*	—	Peters	1867	9 P.
132*b*	II.?	Ceraski	1878	25 Ee. 32 K. 25,30,45 Sk.
133	II.	Hind	1854	5 P.
134	II.	Hencke	1858	31 Ee. 56,60,54 Sk.
134*a*	I.	Schmidt	1876	—
135	III.?	Hind	1848	1 B. 35 G. 49,25,34 Sk. 22 Zr.
136	II.	Hind	1863	10 P. 0,2,0 Sk.
137	IV.	Goodricke	1784	22 Zr.
137*a*	—	Deichmüller	1883	4 Ee. - Hg. 11 K. 2 P.
138	II.	Argelander	1853	8 P. 13,1,2 Sk.
139	III.	Schmidt	1847	3 B. 39 Sr.
140	II.	Hind	1848	4 Ee. 4 P.
141	II.	Marth	1864?	0,1,0 Sk.
142	II.	Harding	1811	8 P. 19,8,8 Sk. 18 Sr.
143	II.	Pogson	1853	5 D. 5 Ee. 6 G. 19 P. 41,24,28 Sk.

78. Max. 1885, May 5±? ; magn. 7.6. K.

79. Estimated magn. 5.8 to 6.0. G.

80. 1885, April 20; magn. 11.5. K.

80*a*. Max. 1885, August. Computed max. 1878, October 21.7, $+356^d.52$. E. Computed min. 1879, May 3, $+356^d.52$ E. D.

85. Past max. 1885, May 11. K.

86. Past max. 1885, May 11. K.

87*a*. Max. 1885, about end of March. D.

91*a*. Max. 1885, September. D.

117. Max. 1884, November 23. G.

121. Min. 1885, June 16; magn. 11.0. Star called T *Aquilæ* by Baxendell. K.

125. Max. 1885, May 16; magn. 7.65. K. Max. 1885, middle of July. En.

128. Max. 1885, Aug. 17; magn. 10.3. Star called S *Delphini* by Baxendell. K.

132. Max. 1885, July 20; magn. 8.2. K.

132*b*. Max. 1885, April 2; magn. 6.8. Min. 1885, September 15; magn. 9.6. K.

135. Near max. 1885, May 11. G.

TABLE II. — ADDITIONAL STARS.

No.	R. A. 1875.	Dec. 1875.	Observations, 1885.	Rem.
	h. m.	° ′		
1	0 6.8	+14 30	3 B.	
3	15.4	—20 45	8 M.	
—	35.9	+40 37	13 D. 27 Ee. - Hg. 17 Hn. 47 P.	A.
9	37.8	+ 6 37	6 P.	
21	1 6.2	+30 53	3 M.	
23	7.7	+34 57	3 M.	
25	15.0	+ 9 2	5 P.	
31	19.5	— 4 37	3 M.	
37	25.7	— 7 22	3 M.	
—	28.2	+11 55	5 G.	B.
43	38.5	—29 40	3 M.	
45	39.1	+ 7 56	3 M.	
47	47.7	+ 8 10	4 B. 3 M.	
57	2 0.8	— 9 11	3 M.	
59	10.4	+58 22	3 M.	
61	15.2	+54 47	3 M.	
63	19.0	+ 9 56	3 M.	
—	28.2	—13 38	8 Sr.	C.
—	49.8	+58 14	42,35,45 Sk.	D.
73	3 37.6	+ 9 0	4 P.	
77	41.8	— 0 17	1 M.	
81	46.5	+ 7 24	3 G. 5 M.	E.
85	49.5	+39 39	3 M.	
87	57.8	+23 38	5 P.	
91	4 10.5	—10 24	3 M.	
93	14.6	+19 31	3 K. 5,5,3 Sk.	
117	5 1.4	— 8 40	1 M.	
123	5.8	+ 0 22	1 M.	
129	6.6	— 0 15	1 M.	
145	28.3	+10 10	15 G.	F.
151	29.3	— 3 20	1 M.	
—	48.4	+20 9	3 D. 7 G. 6 P. 5 Sr.	G.
—	7 16.2	+23 47	26,-,0 Sk.	H.
205	8 2.4	+19 48	5 P.	
—	49.1	+12 7	- K.	I.
—	51.5	+11 18	- K.	J.
229	9 20.1	+14 50	3 M.	
243	30.4	+15 48	2 M.	
293	10 45.5	—20 35	4 M.	
303	11 10.0	— 3 22	5 M.	
311	12 7.5	+ 0 17	3 M.	
315	10.7	+30 49	2 M.	
327	24.0	+ 5 6	3 M.	
331	26.9	—19 56	1 M.	
337	32.7	+17 12	3 M.	
339	32.9	+17 11	1 M.	
348	44.6	+32 23	4 M.	
353	13 8.2	— 9 40	3 M.	
361	24.0	— 8 55	3 M.	
375	47.8	+11 41	4 M.	
381	56.4	— 1 47	7 M.	
—	14 15.4	— 1 25	9 B.	K.
—	24.7	+39 36	25 D.	L.
413	45.4	—11 49	6 M.	

TABLE II. — *Continued.*

No.	R. A. 1875.	Dec. 1875.	Observations, 1885.	Rem.
	h. m.	° ′		
429	15 10.7	— 3 43	5 M.	
447	36.5	—10 31	3 M.	
459	16 1.2	—21 11	10 P.	
465	9.1	+11 50	3 M.	
471	22.4	—19 14	3 P.	
479	31.7	+72 32	4 M.	
483	44.7	— 5 58	5 M.	
491	53.2	— 4 2	3 M.	
503	17 37.6	—18 36	3 M.	
509	18 2.7	+28 44	18 B. 37 Hn. 15 Zr.	
—	20.5	+38 40	2,4,3 Sk.	M.
517	28.0	+36 54	5 M. 3,5,5 Sk.	N.
—	38.5	+36 50	3,4,8 Sk.	O.
—	39.1	+39 11	3,6,7 Sk.	P.
521	43.1	— 8 3	3 M. 6,5,12 Sk.	Q.
529	52 8	+14 12	2 M.	
535	57.7	— 5 52	- K. 2 M. 5,5,11 Sk.	R.
537	59.8	+56 63	3 M.	
543	19 19.0	—21 30	3 M.	
545	23.9	+ 2 39	3 M.	
549	27.1	+17 28	6 Ee. 3 M.	
555	35.3	+12 53	10 Ee. 3 G. 1 M.	S.
557	37.9	+35 56	3 M.	
561	39.1	+33 51	3 M.	
—	39.9	+34 7	2 M.	
—	50.3	+16 17	25 G. 16 Sr.	T.
567	20 7.1	—22 21	7 P.	
—	9.8	—21 42	20,12,20 Sk.	U.
573	19.7	+ 9 39	3 M.	
575	23.6	—12 39	3 G. 3 M.	
—	24 3	+39 34	18 En. - K.	V.
585	39.6	— 0 48	3 M.	
—	39.7	+17 38	14 G.	W.
589	42.6	+45 36	5 M. 37,24,14 Sk.	X.
591	44.5	+45 23	3 M. 37,24,14 Sk.	Y.
—	46.1	+27 47	7 G. 37 Sr.	Z.
595	59.6	+66 13	3 M.	
597	21 0.1	+67 41	3 M.	
601	1.4	—21 51	7 P.	
603	7.9	+67 59	2 M.	
607	12.2	+66 6	3 M.	
—	31.3	+44 49	54 G. 55 Sr.	AA.
—	38.2	+37 27	0,0,14 Sk.	
613	45.2	+ 6 4	3 M.	
615	56.5	—17 14	9 P.	
—	22 29.3	— 8 15	- K.	BB.
—	31.6	+57 47	8 En.	CC.
635	23 14.8	+55 26	3 M.	
645	33.8	— 1 26	6 M.	
—	39.9	+ 2 47	1 G.	DD.
653	54.9	+59 40	1 M.	

REMARKS.

A. New star in the nebula of Andromeda, announced by Hartwig.

B. This star is 100 *Piscium*. Variation discovered by Borelly. Near a maximum, 1885, November 30. G.

C. Discovered by Sawyer in 1884. Period not determined.

D. DM. +58° 539.

E. Discovered by Gore in 1885.

F. The observations show a variation of about half a magnitude. The star has been called T *Orionis*. G.

G. Discovered by Gore in 1885. Magn. 6.15, December 18, 1885. G.

H. DM. +23° 1699. Sk.

I. 60 *Cancri*. K.

J. DM. +11° 1954. Variation suspected by Baxendell. K.

K. 103 *Virginis*. B.

L. This star has been called V *Bootis*. Max. 1885, June 1; magn. 7.2. Min. 1885, January 28, October 23; magn. 9.4. Computed max. 1884, September 3, +266.5 E. Computed min. 1885, January 29, +266.5 E. D.

M. Birmingham 442. Sk.

N. Birmingham 448. Sk.

O. Birmingham 458. Sk.

P. Birmingham 459. Sk.

Q. Birmingham 464. Sk.

R. Birmingham 483. Sk.

S. About magn. 8, 1885, August 19, September 9, November 4. G.

T. Discovered by Gore, 1885. Confirmed by Espin and Sawyer. Variation from about magn. 5.6 to 6.4. Period short, about 8½ days (G.), 8 days (Sr.).

U. Birmingham 545. Sk.

V. DM. +30° 4208. Found 1885, July 9, as a splendid red star, magn. 7.9, since varying to 9.2; no period found; diminishing irregularly. En. Below magn. 8, 1885, August 13, September 7, September 9.

W. Birmingham 569. Discovered by Espin and confirmed by Gore. Period perhaps 111± days. G.

X. DM. +45° 3271. Sk.

Y. DM. +45° 3289. Sk.

Z. Discovered by Sawyer, 1885, and confirmed by Gore. Period 4.437 days. Sr.

AA. Birmingham 587. Discovered by Gore, 1885, and confirmed by Sawyer. Near max. 1885, January 1, August 19; near min. 1885, June 9. G. Period 120± days. Sr.

BB. Variation suspected by Hind. K.

CC. DM. +57° 2568. Fine orange red. Observed variation from magn. 7.0 to 8.0. Period probably short. En.

DD. 19 *Piscium*. Magn. 5.5, October 3, 1885. G.

Professor Safarik also reports the following numbers of observations of suspected variable stars, which are given in the same form as in the tabular statements: star in *Auriga*, 0,0,1; in *Canis Major*, 0,0,2; in

Capricornus, 0,0,13; in *Monoceros*, 0,3,3; two in *Ophiuchus*, 0,14,9; in *Perseus*, 0,0,8; in *Virgo*, 0,0,9.

He has made observations of the minor planets as follows: Ceres (1), 6,19,28; Pallas (2), 0,0,24; Juno (3), 4,23,12; Vesta (4), 0,0,4; and observed the periodical comet Pons three times in December, 1883, and eight times in January, 1884.

The observations of asteroids just mentioned deserve attention, and it is to be hoped that other observers will adopt the practice of observing such objects, and of including them in their statements with regard to variable stars. The instance of Iapetus shows that the variations of a body shining by reflected light may become an interesting object of study, and may instruct us with regard to its period of rotation. Again, supposing it to be proved, either that a certain asteroid varies only in accordance with its distance from the Sun and from the observer, or that its other variations have a sufficiently definite character to allow them to be computed, it would form a valuable instrument of comparison between widely separated stars. It is probable that stellar magnitudes, as estimated, or even as measured by many kinds of photometric apparatus, are subject to systematic errors dependent upon the relative frequency of stars in different parts of the sky. These errors might be detected and eliminated by the comparison of asteroids with the stars differing but little from them in brightness, among which they were apparently moving at different times.

REPORTS OF COMMITTEES.

THIRD REPORT OF THE COMMITTEE ON STANDARDS OF STELLAR MAGNITUDES.

THE work described in the previous reports of this Committee (*Proc. Amer. Assoc.* XXX, p. 1, XXXI, p. 1) has been continued during the past year. The scrutiny of the regions from which the standards are selected has been carried on by several members of the committee. The maps of these regions were originally made as stated in the last report with the fifteen inch telescope of the Harvard College Observatory. The most favorable nights were not always employed in this work, and the proximity of the electric lights in Boston doubtless obscured some of the fainter stars. The careful revision with the telescope of the Washburn Observatory, which has also a slightly larger aperture, has doubtless rendered these maps nearly complete, so far as the limit of visibility of telescopes of this size is concerned. A second revision has been made with the Princeton telescope whose aperture is twenty-three inches. Much fainter stars are thus detected. Another revision will probably be undertaken with the Washington telescope of twenty-six inches aperture which should reveal still fainter stars. To secure the faintest stars visible with the largest telescope in the world, hectograph copies have been made of the regions following, γ *Pegasi*, ϵ *Orionis*, η *Virginis*, η *Serpentis*. Copies have been sent to all observatories containing telescopes of the largest size, with the request that all stars visible may be added. If this request is acceded to, we shall have the means also of determining the comparative advantages of telescopes of different forms. Engravings of these charts have been published in the *Astronomical Register*, XXIII, 39, and the *Sidereal Messenger*, IV, 24. Replies have already been received including important additions to the charts from the Melbourne and Strasburg observatories. It is hoped that the results of observations with all the larger telescopes may be received before July, 1886, in order that they may be included in the report for next year.

Meanwhile stars suitable as standards have been selected from each of the twenty-four maps. An attempt was made to choose such as should differ in brightness, on the average, by about half a magnitude, beginning with the brightest stars in the region which are generally not far from the tenth magnitude, and extending to the faintest objects seen. The magnitude of most of these was estimated twice when the charts were made and once when the standards were selected. Measurements were also made with the photometer, designated as I in the *Harv. Observ. Annals*, XI, p. 7, figs. 5 and 6. Each star was observed on three evenings, three settings being made on each evening. The construction of this photometer does not permit the measurement of bright stars. The two brightest stars of each chart, if not too faint, are now being measured with the large meridian photometer at Cambridge. Three sets, each consisting of four settings, are taken of each star. This portion of the work is nearly completed. All stars that were not too faint were also measured on three evenings with the form of wedge photometer described *Proc. Amer. Acad.* XVII, 231. Three settings were made each evening. With this instrument the observations are differential, and it is necessary to determine the constants independently. The constant expressing the opacity of the wedge was found by means of a special photometric investigation. In each series of measures the light of the bright star preceding the group was reduced by a piece of shade glass and then measured by the wedge. Assuming the opacity of the shade glass, a means is thus afforded of reducing all the measures to the scale of the meridian photometer since the preceding bright stars, or leading stars, have all been measured with that instrument. Unfortunately, the shade glass was of a bluish tint and appears to be more opaque to the red stars like *a Tauri*, than to the blue stars. The observations were accordingly reduced by means of the observations of the same stars with the other photometers.

The various determinations of the brightness of the stars selected as standards are compared in Table I. The twenty-four divisions correspond to the various groups of standards and each is preceded by the name, Harvard Photometry number and magnitude of the leading star. The successive columns give for each star selected as a standard, the letter employed to designate it, the amount it follows the leading star in right ascension, and its declination. The latter is reckoned from the southern edge of the

zone 10′ wide from which the standards are selected. The declinations of the leading stars would therefore be 5′.0. The fourth column gives the magnitudes assumed provisionally for the standards. It is derived from the mean of the measures described above. Magnitudes have been assumed for stars too faint to be measured with any of these photometers. The limiting magnitudes of the stars observed at Cambridge, may be taken at 15.0, although stars have been measured as faint as 15.5. Standards contained on the original charts and not measured will therefore be called 15.0. Those added by the Washburn telescope will be somewhat fainter, and will be called 15.1. The aperture of the Princeton telescope should render visible stars nine-tenths of a magnitude fainter, if no other cause entered. These stars should therefore vary in magnitude from 15.2 to 16.0, or have a mean magnitude of 15.6. For stars south of the equator, one-tenth of a magnitude is subtracted from the magnitudes thus indicated on account of the atmospheric absorption. The fifth column gives the residuals expressed in tenths of a magnitude, found by subtracting the provisional magnitude from that found by the three photometers respectively. Italics indicate that the residuals are negative. When the magnitude depends upon a single instrument an A is inserted. A discordance of a magnitude or more is indicated by a B.

TABLE I.

Des.	Δ α m. s.	Δ δ ′	Prov. Mag.	P W I	Des.	Δ α m. s.	Δ δ ′	Prov. Mag.	P W I
1. γ PEGASI. H. P. 23.				3.0	2. θ CETI. H. P. 220.				3.8
a	2 25	6.0	11.9	. *1* 1	a	4 3	0.7	11.2	. A .
b	2 59	4.5	12.5	. *1* 1	b	4 48	11.0	12.0	. 0 *1*
c	2 37	0.4	13.4	. 0 1	c	3 28	2.9	13.0	. 2 *3*
d	2 49	4.5	13.6	. 0 *1*	d	3 20	0.2	13.0	. *2* 3
e	3 47	2.0	13.9	. 2 *2*	e	3 37	0.7	13.6	. . A
f	3 27	7.5	14.0	. . A	f	2 49	8.1	13.8	. . .
g	2 44	7.0	14.1	. . A	g	3 33	8.5	14.4	. . .
h	3 8	7.5	14.6	. . A	h	3 25	0.1	14.9	. . .
i	3 21	8.0	15.0	. . .	i	3 46	2.7	15.5	. . .
j	3 32	5.7	15.6	. . .	j	3 49	5.0	15.5	. . .
k	3 16	1.3	15.6	. . .					

TABLE I.—*Continued.*

3. α PISCIUM. H. P. 320. 4.0				
a	3 48	7.9	12.0	. 2 2
b	5 37	1.2	12.0	. 0 0
c	5 33	3.8	13.0	. 1 1
d	4 9	5.6	13.4	. 0 0
e	2 19	4.1	13.9	. 0 0
f	2 27	8.0	14.3	. . A
g	2 39	2.0	14.2	. . A
h	5 4	4.1	15.0	. . .
i	5 10	5.2	15.6	. . .
j	3 1	2.2	15.6	. . .

4. α CETI. H. P. 482. 3.7				
a	2 19	7.2	10.9	. 1 1
b	3 37	3.7	10.8	. 2 2
c	3 27	6.8	11.8	. 0 1
d	2 45	7.8	12.9	. 1 1
e	3 23	1.9	13.2	. 2 3
f	5 2	9.6	13.6	. . A
g	5 3	4.8	13.6	. . A
h	4 1	7.0	14.4	. . A
i	3 40	5.5	15.0	. . .
j	3 33	6.1	15.6	. . .
k	2 25	4.6	15.6	. . .

5. γ ERIDANI. H. P. 653. 3.0				
a	5 38	6.8	9.4	0 1 .
b	2 40	8.3	10.7	. 1 1
c	5 48	1.7	11.9	. 2 2
d	5 21	6.2	12.2	. 0 1
e	3 12	0.0	12.4	. 1 1
f	3 44	3.9	13.2	. 1 0
g	3 45	2.0	13.4	. . A
h	3 53	0.7	13.8	. . A
i	3 29	5.0	14.9	. . .
j	3 10	5.9	15.0	. . .
k	3 30	6.0	15.0	. . .

6. α TAURI. H. P. 797. 1.0				
a	3 59	5.8	9.0	4 1 5
b	4 30	6.0	10.0	6 1 6
c	3 5	9.3	11.2	B 9 0
d	3 13	10.8	12.0	. . .
e	3 23	7.0	12.6	. 2 1
f	2 41	1.0	13.1	. 1 1
g	2 44	7.8	13.6	. 1 2
h	3 6	2.5	14.0	. . .
i	3 5	4.0	14.5	. . .
j	2 51	6.8	15.0	. . .

7. ε ORIONIS. H. P. 1029. 1.8				
a	2 1	7.6	6.8	1 1 .
b	2 42	7.7	7.8	1 0 .
c	2 52	4.2	8.7	. A .
d	3 7	1.8	10.2	. 2 2
e	3 16	7.8	11.4	. 0 1
f	2 26	2.2	12.3	. 2 2
g	2 14	4.0	13.3	. 0 0
h	3 31	6.3	13.3	. . A
i	3 49	2.2	14.1	. . A
j	3 40	3.8	14.6	. . A
k	3 49	6.0	15.0	. . .
l	3 21	8.8	15.5	. . .

8. γ GEMINORUM. H. P. 1249. 2.0				
a	3 40	5.0	6.4	2 3 .
b	2 18	3.1	9.5	7 0 7
c	2 24	9.2	11.0	. 2 1
d	2 30	1.2	11.4	. 0 1
e	3 43	3.0	12.0	. 2 3
f	3 50	9.0	12.2	. 1 0
g	3 1	6.3	12.8	. 3 3
h	2 35	3.6	13.7	. 0 0
i	2 47	4.9	13.5	. . A
j	2 32	5.8	13.8	. . A
k	2 34	6.8	14.5	. . .
l	2 41	4.9	15.0	. . .
m	2 49	7.1	15.1	. . .

TABLE I.—*Continued.*

9. α Can. Min. H. P. 1442. 0.5				
a	4 33	0.7	10.3	*6* *1* 7
b	5 56	1.2	10.8	. *3* 2
c	2 38	0.2	12.0	. *3* 2
d	2 13	3.0	12.0	. *2* 2
e	2 55	1.3	12.7	. *1* 1
f	2 41	7.8	12.8	. *1* 2
g	2 12	6.5	13.8	. . A
h	2 21	4.2	13.9	. . A
i	2 34	2.0	13.8	. . A
j	2 17	1.4	14.4	. . A
k	2 24	2.9	15.0	. . .
l	2 27	5.4	15.6	. . .
m	2 11	7.9	15.6	. . .

10. ε Hydræ. H. P. 1608. 3.6				
a	5 23	0.2	9.6	0 *1* .
b	5 4	5.0	10.1	*2* 2 .
c	4 0	1.0	11.3	. *3* 3
d	2 49	6.5	11.8	. *1* 2
e	2 17	9.0	13.0	. *1* 2
f	2 50	2.8	13.0	. *2* 1
g	3 25	7.3	13.9	. . A
h	2 41	4.0	14.2	. . A
i	2 29	5.7	14.4	. . A
j	2 48	5.5	15.0	. . .
k	3 18	5.5	15.1	. . .

11. α Leonis. H. P. 1797. 1.4				
a	3 11	9.0	8.7	. *2* 2
b	4 14	10.8	12.0	. 1 *2*
c	2 16	8.8	12.7	. 0 0
d	2 26	1.8	13.2	. *2* 1
e	2 46	4.9	13.2	. 0 1
f	3 10	4.5	13.6	. 1 0
g	2 18	4.5	14.6	. . .
h	2 12	2.5	14.8	. . A
i	2 52	4.5	15.0	. . .
j	2 57	7.2	15.6	. . .
k	2 59	3.0	15.6	. . .

12. θ Leonis. H. P. 1951. 3.5				
a	2 38	5.7	9.7	*1* 1 .
b	3 15	6.2	10.8	*6* *2* 9
c	4 1	4.7	11.6	. *2* 2
d	2 44	5.8	12.8	. *1* 2
e	3 52	5.8	12.8	. *3* 3
f	3 44	4.3	13.6	. *2* 3
g	2 36	2.1	13.8	. *3* 2
h	2 25	6.2	14.2	. . A
i	3 8	2.9	15.0	. . .
j	2 58	3.6	15.6	. . .
k	3 40	5.1	15.6	. . .

13. η Virginis. H. P. 2088. 4.0				
a	2 3	9.3	11.3	. A .
b	5 10	1.8	11.5	. A .
c	4 27	3.5	11.8	. *1* 0
d	5 1	7.8	12.6	. *1* 0
e	4 25	5.1	12.8	. 1 0
f	5 26	3.2	13.0	. 0 1
g	5 59	3.2	13.9	. 2 *2*
h	4 38	3.6	14.2	. . A
i	5 54	6.0	14.5	. . .
j	5 33	3.0	14.9	. . .
k	4 49	4.6	15.5	. . .
	5 13	4.2	15.5	. . .

14. α Virginis. H. P. 2263. 1.2				
a	3 24	7.7	10.2	. *1* 0
b	2 22	6.8	11.4	. 0 0
c	5 28	9.8	11.4	. *1* 1
d	2 51	5.2	12.0	. *2* 1
e	3 4	6.0	12.6	. 0 *1*
f	3 49	7.7	13.1	. 0 0
g	2 12	5.2	13.6	. 1 *1*
h	2 33	5.4	14.1	. . A
i	2 31	1.9	14.6	. . A
j	2 4	4.0	14.9	. . .
k	2 47	6.4	15.5	. . .
l	2 26	4.2	15.5	. . .

TABLE I.—*Continued.*

15. α Bootis. H. P. 2400. 0.0

a	5 21	0.9	7.2	2 2 .
b	4 36	5.2	9.2	2 2 5
c	6 6	1.3	10.4	. 2 2
d	5 31	6.3	11.8	. 3 2
e	3 59	3.2	12.2	. 1 2
f	3 36	1.5	12.8	. 0 1
g	3 24	0.5	13.9	. . A
h	3 0	8.3	14.7	. . .
i	3 2	6.4	15.0	. . .
j	4 27	5.5	15.6	. . .
k	5 43	5.0	15.6	. . .

16. β Libræ. H. P. 2539. 2.7

a	2 7	4.8	9.4	2 1 .
b	3 36	6.0	10.1	2 2 .
c	2 41	8.8	11.0	. 8 8
d	3 47	6.1	12.0	. 1 1
e	3 23	2.8	12.8	. 1 1
f	2 25	1.0	13.3	. 2 2
g	2 19	3.9	14.9	. . A
h	2 22	6.4	14.9	. . .
i	2 18	7.4	15.0	. . .
j	2 25	4.4	15.5	. . .

17. δ Ophiuchi. H. P. 2726. 2.8

a	2 57	0.0	10.6	0 0 1
b	5 40	9.6	10.7	. . A
c	2 51	6.6	12.0	. 0 1
d	3 0	4.3	12.7	. 2 2
e	2 54	3.5	13.4	. 0 1
f	2 40	3.5	14.1	. . A
g	2 40	4.5	14.9	. . A
h	2 32	5.7	14.9	. . .
i	2 21	6.2	15.5	. . .
j	3 10	6.9	15.5	. . .

18. η Ophiuchi. H. P. 2868. 2.6

a	3 8	10.5	9.3	B 4 7
b	2 52	1.0	11.8	. 4 5
c	2 14	4.3	12.3	. 4 4
d	3 7	8.1	13.1	. 2 2
e	3 45	8.0	13.6	. 2 1
f	2 0	1.5	13.8	. 0 1
g	2 9	7.3	14.2	. . A
h	2 42	2.0	14.3	. . A
i	2 45	7.2	14.5	. . A
j	2 38	6.0	15.0	. . A
k	2 33	4.1	15.0	. . .
l	2 28	5.3	15.5	. . .

19. η Serpentis. H. P. 3090. 3.4

a	5 0	3.6	9.7	1 1 .
b	3 42	8.8	11.8	. 0 1
c	4 36	5.0	12.4	. 0 0
d	2 33	6.5	13.0	. 2 2
e	3 16	0.5	13.0	. 1 2
f	5 18	5.2	14.2	. . A
g	4 40	5.0	15.6	. . A
h	4 24	. 8.4	15.0	. . .
i	5 4	8.5	15.5	. . .

20. δ Aquilæ. H. P. 3343. 3.5

a	4 2	3.0	8.2	2 1 .
b	3 23	5.8	9.0	1 0 .
c	5 20	6.0	11.2	. 5 4
d	4 6	8.5	11.1	. 1 1
e	2 46	4.0	11.5	. 1 1
f	2 35	2.8	12.4	. 1 0
g	2 17	4.8	12.6	. . A
h	2 23	3.0	13.3	. . A
i	2 56	2.6	13.3	. . A
j	2 32	4.5	14.6	. . A
k	2 27	3.5	14.9	. . A
l	2 47	2.5	15.1	. . .
m	3 23	6.8	15.1	. . .

TABLE I.—*Continued.*

21. θ AQUILÆ.		H.P. 3514.	3.4	
a	2 42	1.8	8.1	*2 2* .
b	3 10	6.0	10.3	*5 1 5*
c	3 22	1.8	11.4	. *3* 3
d	4 3	2.2	12.0	. *4* 3
e	2 9	0.8	13.2	. *4* 3
f	2 14	7.5	13.4	. *4* 3
g	2 55	3.8	13.7	. . A
h	2 21	6.5	14.0	. . A
i	2 8	4.0	14.4	. . A
j	2 15	6.0	14.9	. . A
k	2 23	4.5	15.2	. . A
l	2 26	7.5	15.5	. . .
m	2 29	2.7	15.5	. . .

22. β AQUARII.		H.P. 3795.	3.1	
a	2 21	10.1	9.5	. A .
b	4 0	2.0	11.9	. *1* 1
c	2 52	8.4	12.4	. 0 1
d	4 10	1.0	12.9	. 0 0
e	4 9	8.1	13.2	. 1 *2*
f	3 6	6.9	13.4	. . A
g	3 32	7.7	13.8	. . A
h	3 54	6.0	14.0	. . A
i	2 59	2.5	14.9	. . A
j	3 12	3.2	14.9	. . .
k	3 40	5.5	15.1	. . .
l	2 59	7.7	15.5	. . .
m	2 34	7.5	15.5	. . .

23. α AQUARII.		H. P. 3890.	3.2	
a	6 10	2.9	9.3	. A .
b	3 21	2.2	12.5	. 0 0
c	2 49	5 9	12.9	. 0 0
d	3 45	1.4	13.0	. *1* 0
e	2 20	2.8	13.7	. . A
f	3 6	5.4	14.2	. . A
g	2 27	4.1	14.7	. . A
h	2 9	3.4	15.1	. . A
i	2 14	4.9	15.0	. . .
j	2 40	3.2	15.0	. . .
k	3 22	6.2	15.5	. . .

24. α PEGASI.		H. P. 4080.	2.6	
a	4 56	5.0	9.9	. A .
b	2 32	8.7	10.5	. A .
c	3 23	9.3	11.6	. *1* 0
d	1 54	8.3	12.1	. A .
e	3 38	9.3	13.3	. 1 *1*
f	3 20	6.0	14.3	. . A
g	2 31	3.8	14.6	. . A
h	2 18	1.0	15.2	. . A
i	2 34	2.0	15.0	. . .
j	3 30	5.3	15.1	. . .
k	3 1	5.7	15.6	. . .

An additional series of standards is contained in Table II. This consists of stars in the immediate vicinity of the North pole. The selection is that recommended in a circular issued by the Harvard Observatory in 1879. See also *Astron. Nach.*, XCV, 29, *Nature*, XX, 14, *Astron. Reg.*, XVII, 175. These stars afford a convenient means of insuring uniformity of scale throughout the

other series. They are also always above the horizon and at nearly the same altitudes. A similar series of stars in the immediate vicinity of the South pole is much to be desired to complete the system. The first column of Table II gives a designation of the stars. The first eleven are denoted by the DM. numbers, the remainder by the letters employed in the circular referred to above. The approximate right ascensions and declinations for 1880 are given in the second and third columns. The fourth column gives the approximate magnitude and the last column the residuals from six series of measures which have been made of them. In this column P denotes the results obtained with the small meridian photometer (H. C. Annals, XIV, p. 397); i, photometer I attached to the large telescope with the aperture reduced

TABLE II.

Desig.	α 1880.	δ 1880.	Magn.	P	i	I	J	I′
	h. m.							
88° 8	1 14	88° 40′	2.2	A	.	.	.	.
86° 269	18 11	86 37	4.3	A	.	.	.	.
87° 51	6 44	87 14	5.3	A	.	.	.	.
88° 112	19 44	88 57	6.5	A	.	.	.	.
88° 4	0 51	88 23	7.0	*5*	5	.	.	.
88° 9	2 3	88 36	8.6	*6*	6	.	.	.
89° 3	2 28	89 36	9.2	*4*	5	.	.	.
89° 35	17 50	89 48	9.8	.	A	.	.	.
89° 37	19 28	89 54	10.5	.	*3*	6	.	.
89° 1	0 19	89 45	10.5	.	A	.	.	.
89° 26	13 23	89 49	10.6	.	A	.	.	.
a	19 30	89 54	12.2	.	.	.	.	A
b	19 30	89 55	12.4	.	*6*	0	2	5
h	22 0	89 50	12.8	.	.	2	.	*1*
k	23 10	89 53	13.2	.	.	*2*	1	2
d	14 0	89 57	13.3	.	.	1	*1*	0
l	0 0	89 54	14.0	.	.	*2*	0	1
c	18 30	89 58	14.0	.	.	*1*	*1*	1
e	9 10	89 58	14.8	.	.	0	*2*	3
f	3 4	89 58	14.8	.	.	1	*1*	*1*
g	0 10	89 57	15.7	.	.	*1*	*3*	4

to 12.7 cm.; I, the same with the full aperture of 38 cm.; J, similar observations with photometer J (H. C. Annals, XI, p. 8); I′, observations I repeated in August, 1885, three observers each making three settings. The observations i, I and J were made in 1878 and 1879, and in general depend on five settings on each of ten nights.

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APPENDIX C.

Possibility of Errors in Scientific Researches, due to Thought-Transference.

If the theory of Richet is true, an important error may enter many scientific researches in which an assistant is aware of facts a knowledge of which the observer intentionally avoids. An excellent example occurs in the revision of the northern stars, contained in the Durchmusterung of Argelander, which has been undertaken by the Astronomische Gesellschaft. It was provided that the observers, after familiarizing themselves with the scale of magnitudes of the Durchmusterung, should estimate the brightness of each star observed. The Durchmusterung magnitude was then read aloud by the recorder, to enable the observer to continually correct his estimates of the scale of magnitudes. Let M represent the number of cases in which the difference between the estimated and catalogue magnitudes was D. If the number of observations is large, we should in general expect that the relation between N and D would be represented by a smooth curve. If no errors entered but those due to accident, this would become the probability curve. On the other hand, if any thought-transference occurs between the recorder and observer, we should expect an increase in the value of N when D is zero; that is, of cases in which the magnitude was estimated correctly. It is accordingly only necessary to count the values of N corresponding to various values of D. These results may then be compared with that given by the law of frequency of error; or a curve may be constructed with the various values of N and D, not including those in which D equals zero. The value of N, when D is zero, is now derived from the curve passing through the other points. If the actual value of N, when D is zero, in general exceeds that given from the curve, we may infer that thought-transference occurs, unless some other explanation can be found. The amount of material available for this discussion is very large. The number of stars to be observed exceeds a hundred thousand, each of which is measured on at least two nights. More than a dozen observatories participated in the work; so that the test may be applied to many different persons. The stars between +50° and +55° were observed at the Harvard College

Observatory. A count has been made of the residuals in 0, 6, 12, and 18 hours of right ascension. This furnishes sufficient material for the present investigation, although only about one-sixth of the entire work. Similar estimates of magnitude were also made in connection with observations with the meridian photometer, and thus the results of a number of observers and recorders could be tested. The various series employed are compared in the successive lines of Table I., where a comparison is also made with the result derived from the theory of probabilities, assuming that no error enters but that due to accident. The successive columns give a number for reference, the initial of the observer who becomes the percipient if any thought-transference occurs, and the recorder or agent. The letters C., E., M., P., R., and W. indicate Messrs. Cutler, Eaton, McCormack, Pickering, Rogers, and Wendell respectively. When the results of various persons are combined, they are indicated by V. The fourth column gives the number of observations contained in the series; the fifth, the average value of the residual, or arithmetical sum of all the residuals divided by their number. The sixth column gives the number of cases in which the residual is zero; and the seventh, the ratio of these numbers to the numbers in the fourth column. This quantity is, therefore, the observed proportion of zeros. From the average deviation we may compute what proportion of residuals should be zero according to the theory of probabilities. The average deviation of each series was next multiplied by .845, which gives the probable error according to the formula of Peters, and .05 was divided by this quantity. The quotient gives the fraction of the probable error which an error must not exceed to give a residual zero. A table of the frequency of error then gives the proportion of the observations whose error should fall within this limit, or which should give residuals zero. These computed proportions are given in the last column but one of Table I. The last column is found by subtracting the computed from the observed proportion of cases in which the residual is zero. About four-fifths of the stars are estimated in the Durchmusterung as fainter than the magnitude 7.9; and these only are employed, since the brighter stars are much more difficult to estimate. In line 7 all the stars are included, and all are brighter than this limit. This is probably the cause of the larger average deviation.

The first four lines of the table give the results of the observations of Professor Rogers, in 0, 6, 12, and 18 hours of right ascension, respectively, with the meridian circle. It is impossible to determine whether the conditions in this case were favorable to thought-transference, as Mr. McCormack is not now living. He was instructed to

TABLE I.

No.	Percipient.	Agent.	No. of Observations.	A. D.	No. of Zeros.	Observed Proportions.	Computed Proportions.	O-C
1	R.	M.	981	.223	101	.195	.143	+.052
2	R.	M.	759	.244	129	.170	.129	+.041
3	R.	M.	458	.231	92	.201	.135	+.066
4	R.	M.	930	.233	152	.163	.134	+.029
5	P.	E.	514	.240	74	.144	.131	+.013
6	P.	E.	540	.200	88	.163	.158	+.005
7	P.	E.	492	.332	51	.104	.096	+.008
8	P.	C.	513	.226	75	.153	.139	+.014
9	P.	W.	580	.213	108	.186	.147	+.039
10	P.	W.	609	.190	97	.159	.167	−.008
11	W.	E.	163	.198	30	.184	.160	+.024
12	W.	C.	141	.160	30	.233	.192	+.041
13	W.	P.	402	.199	82	.204	.159	+.045
14	W.	P.	486	.180	100	.206	.177	+.029
15	R.	V.	3,128	.232	564	.180	.135	+.045
16	P.	V.	3,248	.231	493	.152	.136	+.016
17	W.	V.	1,192	.187	242	.203	.170	+.033
18	V.	V.	7,568	.226	1,299	.172	.139	+.033

record the estimated magnitude before calling out the catalogue magnitude; and, if he did not look at the catalogue magnitude until then, no thought-transference would be indicated. Line 5 gives the observations made in series 1 to 100 with the meridian photometer, or between Feb. 28, 1882, and Jan. 23, 1883. The observer had probably not as yet acquired a fixed habit of estimating the magnitudes. Line 6 relates to series 101 to 400 between the dates Jan. 23, 1883, and Feb. 7, 1885. Line 7 relates to similar estimates of the magnitudes of the standard stars of the Uranometria Argentina, and are the only estimates not relating to the Durchmusterung magnitudes. Line 9 contains the observations contained in series 301 to 400, between July 25, 1884, and Feb. 7, 1885; and line 10, those from series 401 to 450, between Feb. 10, 1885, and April 25, 1885. The same distinction applies to lines 12 and 13. A portion of the last five series were recorded by Professor Searle, but not enough to render a subdivision desirable. Lines 15, 16, and 17 give the results of all of the observations by the three percipients respectively; and line 18 gives the results of all combined.

Table II. gives the details of the count of the number of residuals

of various magnitudes. These magnitudes are given in the first column, and the successive columns give the number of residuals in the first fourteen lines of Table I. When the residual is larger than one magnitude, it is indicated by an L in the first column. The numbers at the top of the columns of Table II. have the same meaning as those in the first column of Table I.

TABLE II.

Residual.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
+L.	1	4	–	2	–	–	14	–	–	–	–	–	–	1
+1.0	2	–	–	–	–	–	5	–	–	–	–	–	–	–
+0.9	2	1	–	2	1	–	4	–	–	–	–	–	1	–
+0.8	1	11	–	2	2	2	10	–	1	–	1	–	–	1
+0.7	6	5	4	8	–	1	13	1	3	–	–	1	–	–
+0.6	6	13	8	10	4	3	9	5	2	1	–	1	4	2
+0.5	27	30	8	31	5	8	21	8	8	3	4	1	3	5
+0.4	37	27	25	49	12	17	23	16	16	11	7	2	8	9
+0.3	85	65	40	77	28	20	33	31	22	80	12	6	21	13
+0.2	111	85	62	131	55	44	40	59	37	37	12	7	34	43
+0.1	132	95	65	111	73	63	43	46	78	70	29	16	44	66
0.0	191	127	62	152	74	88	51	75	108	97	30	33	82	100
−0.1	114	95	41	104	54	84	51	68	85	113	26	34	52	99
−0.2	80	72	43	91	69	77	58	63	80	101	13	20	53	70
−0.3	68	49	28	67	51	52	34	53	50	72	14	10	46	45
−0.4	37	28	12	36	22	35	31	52	44	42	4	5	30	14
−0.5	19	16	12	26	24	15	20	20	20	17	6	3	17	12
−0.6	14	6	5	7	16	14	10	12	10	7	–	1	2	10
−0.7	9	5	4	6	10	6	8	2	7	4	3	–	2	3
−0.8	6	4	2	2	8	4	5	2	4	2	2	–	1	1
−0.9	2	2	–	2	3	–	1	–	1	–	–	–	–	–
−1.0	4	3	1	2	2	1	2	–	2	2	–	–	–	–
−L.	18	8	6	6	1	–	–	–	2	–	–	–	2	2

Every residual in the last column of Table I., with one exception, is positive. The actual number of residuals equal to zero is, therefore, in excess of that given by theory; and this effect is most marked in the cases of Professor Rogers and Mr. Wendell. It would not be safe, however, to infer from this the existence of any thought-transference until all other explanations of this deviation have been carefully considered. If the probable error is diminished in any series of observations, the theoretical number of zero-residuals would be increased. But, in almost every series of observations, the num-

ber of large residuals is greater than that given by theory, on account of various sources of large errors. Such, in the present case, are variability of the stars, clouds, error in identification or of record. According to theory, the entire number of residuals exceeding a magnitude should not exceed half a dozen, or one tenth part of its actual amount. On the other hand, the estimated magnitudes differ systematically from those of the catalogue, as is shown in several series by the difference in the number of positive and negative residuals. The effect of this would be to diminish the theoretical proportion of zero residuals. Moreover, if thought-transference really exists, the excess of zero-residuals should not be included in deducing the probable error. The latter would then become larger, and the computed proportion of zero-residuals would be diminished. If the recorder should enter the catalogue magnitude by mistake for that estimated, the number of zero-residuals would be increased. But, with the careful recorders employed, it can hardly be supposed that this effect could be sensible.

A comparison must next be made, of the number of zero-residuals with those of other magnitudes. In Table III., the first column gives the magnitudes of the residuals, as in Table II. The next three columns give for the three observers the proportion of residuals of each magnitude which constitute series 15, 16, and 17. The next three columns give the residuals found by subtracting from these quantities the theoretical proportions, according to the law of the frequency of error. A correction is first applied for the constant differences in the estimated scales from that of the Durchmusterung. In the case of Professor Rogers, his estimates, on the average, were too faint by .02. Mr. Wendell's estimates, and my own, were too bright by .05 and .07, respectively.

The last three columns show that the agreement with the probability-curve is all that can be desired, except for the residual's zero. The graphical comparison by drawing a smooth curve through the given point is not needed. The zero-residuals, however, show a marked result of observation over theory, which is much too great to be ascribed to accident, at least in the case of Messrs. Rogers and Wendell.

One other source of error remains to be considered. The stars in the Durchmusterung are not distributed regularly, according to magnitude. There is an excess of those in which the tenth of a magnitude is either 0 or 5, and a deficit for 1, 4, 6, and 9 tenths. If a similar irregularity occurred in the scale of an observer, we should expect an excess of residuals 0 and 5, as compared with the other

TABLE III.

Residual.	15.	16.	17.	15.	16.	17.
+L.	0.002	0.004	0.001	+0.002	+0.004	+0.001
+1.0	0.001	0.002	0.000	+0.001	+0.002	0.000
+0.9	0.002	0.002	0.001	+0.001	+0.001	+0.001
+0.8	0.004	0.005	0.002	+0.001	+0.002	+0.002
+0.7	0.007	0.006	0.001	−0.001	+0.001	−0.001
+0.6	0.014	0.007	0.006	−0.004	−0.003	+0.002
+0.5	0.033	0.016	0.011	−0.003	−0.004	0.000
+0.4	0.044	0.029	0.022	−0.014	−0.009	−0.005
+0.3	0.085	0.052	0.044	−0.003	−0.009	−0.013
+0.2	0.124	0.084	0.080	+0.011	−0.006	−0.016
+0.1	0.130	0.114	0.130	0.000	−0.002	−0.008
0.0	0.181	0.151	0.205	+0.045	+0.019	+0.040
−0.1	0.113	0.140	0.168	−0.012	+0.005	+0.003
−0.2	0.094	0.138	0.131	−0.010	+0.015	−0.007
−0.3	0.068	0.096	0.096	−0.008	−0.004	−0.002
−0.4	0.036	0.070	0.044	−0.012	−0.001	−0.014
−0.5	0.023	0.036	0.032	−0.005	−0.010	+0.004
−0.6	0.010	0.023	0.011	−0.004	−0.004	0.000
−0.7	0.008	0.011	0.007	0.000	−0.002	+0.004
−0.8	0.004	0.008	0.004	+0.001	0.000	+0.003
−0.9	0.002	0.002	0.000	+0.002	+0.002	0.000
−1.0	0.003	0.003	0.000	+0.003	+0.003	0.000
−L.	0.012	0.001	0.004	+0.012	+0.001	+0.004

residuals. Unfortunately, the only means of determining the irregularity in an observer's scale is by counting the number of times he has employed each tenth of a magnitude. It then becomes difficult to decide how far this irregularity is caused by that of the Durchmusterung. A discussion of the magnitudes 7.5 to 9.2 shows, that, in the Durchmusterung, the proportion of stars having the tenth of a magnitude 0 or 5 is .22, instead of .10. About .05 are in each class, differing one-tenth from these, or having the tenths, 1, 4, 6, and 9. About .09 differ two-tenths, or equal 2, 3, 7, and 8, each. These proportions, for Professor Rogers, become .16, .07, and .10. My early estimates were mainly made in half magnitudes; and in line 5 of Table I. the proportions are, accordingly, .22, .04, and .10. Later, my scale became the same as Mr. Wendell's, and gave the proportions, .14, .08, and .10.

This source of error will be eliminated if the scale, either of the catalogue or of the observer, is rendered uniform, however great

the irregularity is in the other. Accordingly, a re-count of the residuals was made, selecting the first twenty-five in each series for which the Durchmusterung magnitude was 8.3; and an equal number for each of the magnitudes, 8.4, 8.5, to 9.2. This count was made for each of the series given in lines 1, 2, 3, 4, 13, and 14. Lines 11 and 12 were also included with 13. It did not seem necessary to recount my own estimates, since the evidence of thought-transference is here very slight. The results of this count are given in Table IV., which has a form similar to Table III. The four series of Professor Rogers are combined, as in line 15 of Table I. Occasionally there were not a sufficient number of estimates of a given magnitude, and in these cases the proportion of each was assumed from what observations were actually made.

TABLE IV.

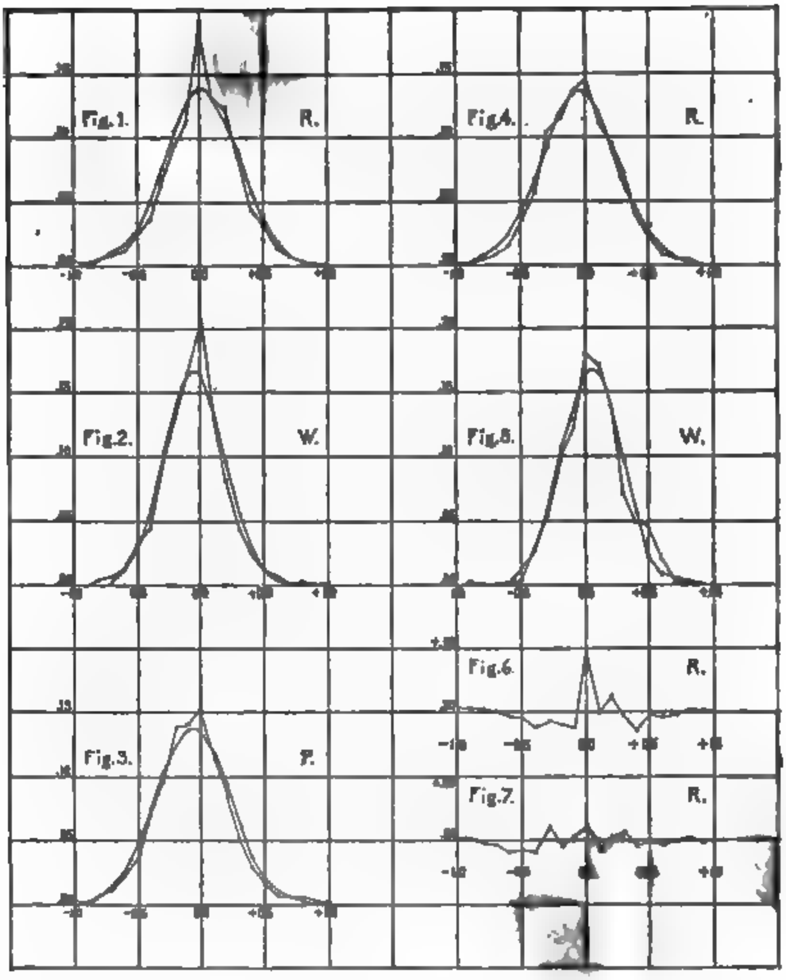
Residual.	13.	14.	15.	13.	14.	15.
+L.	0.000	0.000	0.015	0.000	0.000	+0.015
+1 0	0.000	0.000	0.000	0.000	0.000	0.000
+0.9	0.000	0.000	0.000	0.000	0.000	—0.001
+0.8	0.004	0.000	0.004	+0.002	—0.002	+0.001
+0.7	0.004	0.000	0.005	0.000	—0.004	0.000
+0.6	0.004	0.034	0.000	—0.007	+0.023	—0.003
+0.5	0.062	0.032	0.022	+0.037	+0.005	—0.002
+0.4	0.087	0.012	0.039	+0.030	—0.045	—0.004
+0.3	0.081	0.064	0.072	—0.015	—0.032	+0.006
+0.2	0.133	0.143	0.095	—0.005	+0.005	+0.003
+0.1	0.130	0.212	0.110	—0.035	+0.047	—0.008
0.0	0.176	0.183	0.145	+0.011	+0.018	+0.012
—0.1	0.104	0.140	0.137	—0.034	+0.002	+0.003
—0.2	0.092	0.113	0.115	—0.006	+0.015	—0.003
—0.3	0.081	0.027	0.105	+0.023	—0.031	+0.010
—0.4	0.028	0.032	0.058	0.000	+0.004	—0.009
—0.5	0.000	0.004	0.036	—0.011	—0.007	—0.007
—0.6	0 000	0.004	0.015	—0.003	+0.001	—0.009
—0.7	0.000	0.000	0.009	—0.001	—0.001	—0.004
—0.8	0.000	0.000	0.005	0.000	0.000	—0.002
—0.9	0.004	0.000	0.002	+0.004	0.000	+0.001
—1.0	0.000	0.000	0.001	0.000	0.000	+0.001
—L.	0.000	0.000	0.001	0.000	0.000	+0.001

In each of the three last columns of Table IV. the differences corresponding to the zero-residuals are greatly diminished. They are

still positive; but this may be due to the fact that the reduction is only approximate, and the number of observations insufficient to render the accidental errors very small. At least, the differences, which in Table III. were so large that it was impossible to assign them to chance, are now not much greater than the other quantities in the same column, and do not require any special cause to account for them. The reality of this small positive excess is, however, confirmed by some other facts. It is perceptible in line 7 of Table I., although in the Uranometria Argentina there is no perceptible excess of the magnitudes 0 and 5 tenths. If due to the irregularity of the scale, it should be more marked in line 5 than in the following line. Finally, it is difficult to understand why the effect appears so much more marked in Mr. Wendell's observations than in mine, when we were both employing the same scale. All these deviations are, however, so small that much weight should not be assigned to them.

The results of Tables III. and IV. are represented on the opposite page. Horizontal distances indicate the magnitude of the residuals, and vertical distances the corresponding number of residuals expressed as a fraction of the whole number of residuals. The smooth curves indicate theoretical values, the broken lines the results of observation. Figs. 1, 2, and 3 show the proportion of residuals of various magnitudes corresponding to the observations of Professor Rogers, Mr. Wendell, and myself. They, therefore, show the results of the second, fourth, and third columns of Table III. The preponderance of zero-residuals is well shown, in Figs. 1 and 2, by the projection of the broken lines above the curves. Fig. 4 represents the corresponding values from Professor Rogers's observations, after correction for the inequality in the scale of the Durchmusterung. These quantities are also given in the fourth column of Table IV. The mean of the second and third columns of Table IV. are shown in Fig. 5. It gives the result of Mr. Wendell's observations after correction for inequality of scale. Figs. 4 and 5 show how greatly the excess of zero-residuals is reduced by the application of these corrections. Figs. 6 and 7 show the differences between the observed and computed proportion of residuals in Professor Rogers's observations before and after the correction for inequality of scale. They represent the fifth column of Table III., and the last column of Table IV.

It is extremely desirable that a discussion similar to this may be made at the other observatories taking part in the revision of the Durchmusterung. The apparent absence of thought-transference in the observations at Cambridge by no means proves that it may not



GRAPHICAL REPRESENTATION OF THE RESULTS OF TABLES III. AND IV.

exist elsewhere. The time required to apply this test is so small that it is to be hoped that the opportunity will not be neglected, to search for a phenomenon, which, if real, would exert so wide an influence on human affairs.

E. C. PICKERING.

FORTIETH

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE DECEMBER 3, 1885, AND LAID BEFORE THE BOARD OF OVERSEERS, JANUARY 13, 1886.

CAMBRIDGE, MASS.
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REPORT.

To the President of the University:

Sir, — By the death of Mr. Robert Treat Paine during the past year the Observatory of Harvard College has lost one of the oldest of its friends, well acquainted with the theory and practice of several departments of astronomy, and always deeply interested in the work of this institution. He was an assiduous computer and observer of astronomical and meteorological phenomena, and a member of the Visiting Committee of the Observatory from the time of its first organization, forty years ago. He has perpetuated his services to his favorite science by the bequest to this Observatory of his entire fortune, amounting to more than a quarter of a million of dollars. Upon the settlement of the estate, one half of this sum will become immediately available.

The resignation of another of the original members of the Visiting Committee, Mr. J. Ingersoll Bowditch, is a matter of extreme regret. His services to the Observatory have been continuous, and without his active aid and advice no important steps have been taken in the extension of its work, which has frequently been rendered possible only by his exertions.

During the year the work of which is to be described in the following paragraphs, the resources of the Observatory have been materially diminished, owing to circumstances stated in the last report. It was there mentioned that no means were available for retaining the services of five of the assistants previously employed. But the salary of one of these assistants for another year was soon after provided by means of a subscription undertaken by some of his colleagues, who furnished a part of the required sum from their own scanty means. Without this relief, it would have become necessary to interrupt the regular use of the large telescope in order to continue work with the meridian instruments. As it is, the reduction in the amount of work carried on is much less than would have been expected; but this result is due to extraordinary efforts on the part of the observers, who have performed without assistance the work in which they were previously aided by recorders. This has required an increase in the time spent in observation, and has rendered the work much more

laborious. While this evidence of enthusiasm and devotion to science is most gratifying, it is obvious that it cannot long be continued without injury to health. Indeed, the effects of over-fatigue and exposure during the long, cold nights of last winter were manifest in more than one instance.

EAST EQUATORIAL.

Eclipses of Jupiter's Satellites. — Photometric observations of these eclipses have been continued upon the system adopted in 1878. In all, three hundred and nineteen eclipses have now been observed, thirty-five since the end of October, 1884. The original plan contemplated the prosecution of the work through an entire revolution of Jupiter around the Sun, more than half of which has already been completed. The importance and extent of this work has made it advisable to delay the adoption of a definite plan of reduction until a sufficient amount of experience had been gained; but there is now no reason to defer the consideration of this subject.

Revision of Zone Observations. — The reduction of the observations of the stars between the declinations $+0° 50'$ and $+1° 0'$ has been carried on. To determine the scale of the wedge photometer employed in the work, the constants of the instrument have been photometrically examined, and the initial point of the scale for each evening of observation has been determined by means of the stars in each series which also occur in the working list of the meridian photometer. In the entire zone there are about three hundred such stars, all of which have now been measured with the meridian photometer. The magnitudes of the fainter stars observed in the zone with the wedge photometer have been recomputed by the aid of these data. The work of the wedge has thus been made homogeneous with that of the meridian photometer.

Revision of DM. Magnitudes. — The observations of DM. stars between the declinations $+49° 50'$ and $+50° 0'$, which were begun with the wedge photometer during the previous year, have been completed, as well as a similar series between $+54° 50'$ and $55° 0'$. The resulting magnitudes have been computed, as in the case of the zone between $+0° 50'$ and $+1° 0'$, by means of the stars occurring in these zones which are also under observation with the meridian photometer. It is desirable that similar observations should be made in other zones at intervals of 10°. This would extend the scale of the meridian photometer about one magnitude, and would furnish means for determining the scale of the Durchmusterung between the magnitudes 9.0 and 9.5.

The extensive use thus made of the wedge photometer seems to show that the instrument used here is not capable of the great degree

of precision which is claimed for that employed by Professor Pritchard. To determine whether this difference is due to the form of the instrument, Professor Pritchard was requested and has kindly consented to superintend the construction for this Observatory of a wedge photometer made upon his plan.

Standards of Stellar Magnitudes. — The photometric observations of faint stars selected as standards of magnitude in twenty-four regions following bright stars, and mentioned in previous reports, have been provisionally completed. A few of these stars were bright enough to be observed with the meridian photometer; but most of them were observed with the East Equatorial, with the aid of the wedge photometer or of Photometer I. The results have been published in the report to the American Association for the Advancement of Science by the committee in charge of the subject of stellar magnitudes.

Comparison Stars for Variables. — Another extensive investigation begun with the wedge photometer is the determination of the magnitudes of stars which have been used as standards of comparison for variable stars. Besides the stars of this class named in published works, many others, employed in observations not yet published, have been added to the list. Observers of variable stars are requested to communicate to this Observatory the comparison stars employed by them, and not identical with those contained in published lists. When these comparison stars are sufficiently bright, they are measured by the meridian photometer. Their magnitudes, thus determined, serve as a basis for the measurement of the fainter stars by the wedge. Two observations of each star not determined by the meridian photometer, and of a sufficient number of the others, are made with the wedge on each evening when the region occupied by the comparison stars of any particular variable is observed. Two complete observations of this kind, made on different evenings, are regarded as sufficient; but when moonlight or other causes render the work of either evening imperfect, it is repeated on a third evening. Some incidental attention is also paid, during this work, to enlarging the lists of comparison stars when they can thus be better adapted for future use.

Temporary Star in the Nebula of Andromeda. — This remarkable object, as well as a series of comparison stars selected for it, was repeatedly observed with the wedge photometer immediately after the announcement of its discovery. It is still occasionally observed. It is proposed to determine photometrically the brightness of the stars with which it has been compared by observers at other stations, so far as these are made known.

Comets. — These objects have occasionally been observed as in former years, chiefly by Mr. Wendell. The observations have been confined to special times when they would be of most immediate value. Comet 1884 III. (Wolf) was observed on one night; two others, discovered by Barnard and by Brooks, were respectively observed on seven and on four nights.

Spectra and Color of Stars. — The observations upon the spectra and color of stars have been completed upon the restricted plan indicated in the last report, but are not as yet reduced. It is probable that photography will soon afford a more satisfactory solution of this problem. The work of the East Equatorial in this respect, and also in some others, has been considerably diminished during the year by the frequent employment of the observers in recording the observations made with the meridian photometer, since the diminished resources of the Observatory did not permit the engagement of special assistance in the work of recording.

Meridian Circle.

The work done with this instrument during the year has been more varied in character than usual, while its amount has been somewhat diminished by the financial necessity of dispensing with an assistant to the principal observer. A considerable portion of the year has been occupied with the investigation of the errors of the East Circle of the instrument. This work is now completed. The spaces of 30°, 15°, 5°, 1°, have been independently investigated; the labor thus performed is equivalent to that of reading a single microscope about forty thousand times.

In order to perfect the revision of the zone between +50° and +55°, some additional observations were found to be desirable, when the reduction of the preliminary revision, finished in August, 1884, had been completed. The doubts remaining to be settled chiefly related to possible errors of an entire minute in the circle readings, and to unusually large discrepancies between the positions here obtained and those given in the Durchmusterung. The new revision was begun January 8, 1885, and will be completed with six more nights of observation. Each star concerning which any doubt existed has ordinarily been twice observed. The whole number of observations is 937.

The observation of a polar catalogue was begun April 30, 1885. The working list includes 30 fundamental stars and 153 others. For the present year, the plan of the work requires two observations of each star at each culmination. The number of observations hitherto made is 384.

A series of 275 observations was carried on simultaneously with corresponding observations made with the almucantar, in order to compare the results given by the two instruments, and to test the precision of the Clark level attached to the meridian circle.

Observations of particular stars have also been made at the request of Dr. Auwers, Dr. Elkin, and Professor Holden. These observations are respectively 173, 24, and 12 in number. The first two series will furnish material for the determination of heliometer constants, and the last for the reduction of latitude observations.

The total number of observations with the meridian circle is accordingly 2808, including 1003 observations of fundamental stars employed in the revision of the zone and in the polar catalogue.

The reduction of the first revision of the zone observations is complete. The observations of fundamental stars made up to last October, during the second revision, are also reduced. The reduction of the miscellaneous observations is finished, with the exception of a few of those made at the request of Dr. Auwers.

The report of the longitude operations between Cambridge and Montreal in the summer of 1883 has been completed. It will appear, in connection with the report of Professor McLeod, in the forthcoming volume of the Proceedings of the Royal Society of Canada.

Meridian Photometer.

The number of series of observations made during the year by Mr. Wendell and myself is 202, an increase of 61 over those of the previous year; and the number of separate settings somewhat exceeds 50,000, against 27,500 for the previous year. The accordance of the results continues satisfactory. The average deviation of the separate measures of the standard circumpolar stars, expressed in magnitude, is 0.12. Although this exceeds the corresponding result for the previous year by only a single hundredth of a magnitude, the difference may not be purely accidental, but have its cause in the variability of one or more of the standard stars.

The list of objects to be observed with the meridian photometer has been somewhat extended. The stars first selected from the Durchmusterung for observation were those the magnitude of which had been estimated at each of two observatories during the zone observations carried on during recent years under the general direction of the Astronomische Gesellschaft. To increase the regularity of distribution of the stars observed, other zones of 20′ in width were added, so that the entire series includes zones at intervals of five degrees from the equator to the pole. The middle of each zone is defined by the parallel of declination at 0°, +5°, +10°, etc.

This system ensured a regular distribution of the stars to be observed, and supplied a sufficient number of stars of the eighth and ninth magnitudes to be employed in the determination of the various scales of magnitude in use. The number of stars between the sixth and eighth magnitudes, however, hardly seemed large enough for this purpose. To supply the deficiency, the zones were extended to a width of 60′ for for stars of the magnitudes 7.0 to 7.9 inclusive, and to a width of 120′ for those of the magnitudes 6.1 to 6.9 inclusive. The magnitudes of the stars brighter than these have already been published in Volume XIV. of the Observatory Annals.

The miscellaneous work with the meridian photometer, mentioned in the last report, has been continued. Two special features of this work have already been mentioned in connection with the observations made with the wedge photometer.

Almucantar.

Mr. Chandler has continued his observations with this instrument, which in his hands has exhibited a surprising efficiency and accuracy, fully confirming the anticipations expressed in the last report. The work of observing was suspended after July 1, to allow time for the reduction of the previous observations, the number of which between November 1, 1884, and July 1, 1885, was approximately one thousand. The following subjects have been investigated: right ascensions of circumpolar stars; declinations of stars near the equator; the latitude of this Observatory; effects of personal equation at different declinations, obtained by observers with the meridian circle and with the almucantar. The positions obtained by means of the almucantar have such a precision as to have already led to the detection of errors in the accepted places of several important stars occurring in fundamental catalogues, and generally regarded as accurately determined.

Miscellaneous.

Variable Stars. — Messrs. Parkhurst, Eadie, Hagen, and Zaiser, have continued their valuable coöperation with this Observatory throughout the year. The large and rapidly increasing number of observations of variable stars secured by these gentlemen will form a highly important source of information with regard to such objects. Steps have been taken to provide for the reduction of these observations, which will require however the photometric determination of the brightness of the comparison stars before much progress in it can be made.

The request to astronomers made last year in the pamphlet "Recent Observations of Variable Stars" was kindly complied with by the

principal observers of variable stars in Europe and America. Their names, together with a summary of their work, are given in the pamphlet "Observations of Variable Stars in 1884," which was published by this Observatory early in 1885.

Stellar Photography. — By the aid of the Bache Fund an important investigation has been undertaken in stellar photography. A Voigtlander portrait lens of 8 inches aperture and 44 inches focus has been reground and mounted equatorially by Messrs. Alvan Clark & Sons. It is driven by clockwork, having a Bond spring governor controlled electrically by a sidereal clock substantially mounted in the clock-room of the Observatory. A wide field of work is opened with this instrument. Many photographs have been taken of the trails left by a star when the telescope is not driven by clockwork. An equatorial star will leave its mark in this way when no brighter than the sixth magnitude. Much fainter polar stars will leave an impression, since their motion is slow. Stars as faint as the fourteenth magnitude have thus been photographed without clockwork. The positions of faint polar stars may be determined with great accuracy from their trails, which are very well defined and minute. They also appear to afford an excellent measure of stellar brightness. Attempts have also been made to prepare star-charts by photography. Regions 5° square will bear an enlargement of three times, and by photolithography will furnish charts of the dimensions and scale of those of Peters and Chacornac. The most striking results have been obtained with stellar spectra. Replacing the slit spectroscope by a large prism placed in front of the lens, photographs have been obtained of stars as faint as the eighth magnitude in which lines are shown with sufficient distinctness to be clearly seen in a paper positive. As all the stars in a large region are thus photographed, more than a hundred spectra have been obtained on a single plate. Mr. William H. Pickering has rendered important aid in this investigation, especially in the photographic process. A portion of Mr. Pickering's investigation on the possibility of photographing the solar corona without the aid of an eclipse was also conducted here. The conclusion thus reached was that, unless the effect of the air could be reduced three hundred times, no satisfactory results could be obtained with our present means. The details of this investigation will be found in *Science* VI., 362, 387.

Time Signals. — The time service of the Observatory has not fully succeeded at all times during the year in maintaining the high standard of former years. Inevitable interruptions and delays have occurred from various causes, among which may be specially mentioned the want of a duplicate mean time clock and the increasing number

of telephone and telegraph wires used in Boston, with its natural consequence of frequent interference with wires previously in use. By persevering exertion, however, a satisfactory system for transmitting the signals has now been established. Much delay also occurred in making the arrangements necessary for the new time-ball, which was not brought into operation until October 1, since which time it has been regularly dropped. It is made upon the plan adopted by the United States Government offices, and its cost has been largely provided for by an appropriation made by the city government of Boston.

The list of subscribers to the time signals remains nearly the same as that given in the last report, the most important accession being that of the Boston and Lowell Railroad.

Telegraphic Announcements. — The telegraphic distribution of important astronomical discoveries or data has been continued during the year under the management of Mr. Ritchie, as before. Announcements have been made of the discovery of nine asteroids, five comets or suspected comets, and one new star. There have been sent to Europe or received therefrom twenty cable messages, and the number of telegrams sent in this country was two hundred and seventy. The Associated Press has been regularly notified of the facts thus distributed, and the information has been widely spread by means of the newspapers.

Height and Velocity of Clouds. — This subject has been studied during the year by Professor W. M. Davis, assisted by Mr. A. McAdie, who stationed themselves respectively on the east balcony of the Observatory and at the Jefferson Physical Laboratory. These stations were connected by telephone, and the observers were thus enabled to decide upon suitable portions of clouds for simultaneous observation with wooden altazimuth instruments. About three hundred pairs of measures were made in the spring of 1885 with generally satisfactory results. The altitudes determined varied from 2000 to 25,000 feet; for altitudes less than 8000 feet the variation between the measures was generally within five per cent of the height. In one instance, consecutive observations of a single cumulus cloud showed its base to be 4500 feet high; its summit rose from the height of 6750 to that of 7300 feet at the rate of 200 feet a minute, while the cloud drifted to S. 43° E. at the rate of 27½ miles an hour.

Additional Researches. — Several investigations have been carried on by the aid of the Rumford Fund. They include the determination of the amount of light reflected specularly and diffusely by various substances at different angles; the absorption of the wedge of shade glass used as a photometer for the large equatorial; the opacity of a

series of photographic images employed as standards by Mr. W. H. Pickering in his photographic researches. The atmospheric refraction at large zenith distances was determined by several series of observations of the Sun and of *a Bootis* when setting. Various measures were also made of the amount of the atmospheric polarization.

During the summer over a thousand vertical angles were measured from the summits of Mt. Moosilauk and of Mt. Mansfield to determine the heights of the mountains of New Hampshire and Vermont by the method of geodetic levelling.

For three quarters of an hour after the explosion of Flood Rock was expected to take place, Professor Rogers watched a mercury surface with a microscope to detect the effect of the earthquake wave. Disturbances occurred beginning 3^{m} 14^{s} after the explosion and lasting nearly three minutes, which were apparently due to this cause. Various experiments were afterwards made to show that no local disturbance could have produced the same effect. This appears to be far the greatest distance to which the effects of an artificial explosion have been traced.

Exhibition of Instruments. — To permit the examination by the public of apparatus which would not generally be readily accessible, several instruments not required for immediate use were sent to the exhibition of the Massachusetts Charitable Mechanics' Association. Gold medals were awarded to two of these instruments, the meridian photometer and the almucantar.

Publications.

The principal publications of the year were Volume XIV., Part II. of the Observatory Annals, and the Catalogue of 1213 Stars which will form a part of Volume XV. The second part of Volume XIV. contains a discussion of observations by ancient and modern observers to determine stellar magnitudes, with an explanation of the methods by which the final results of these observations as given in Part I. of the same volume were ascertained. It also investigates various sources of error likely to affect the determination of the brightness of stars, and compares the general result reached with the meridian photometer and those obtained by other means.

The Catalogue of 1213 Stars, prepared by Professor Rogers, contains the results of observations which he carried on with the meridian circle from 1870 to 1879. The catalogue contains accurate places of many stars which, although moderately bright, had previously been only imperfectly observed; and also furnishes means for the more precise determination of the places of numerous fundamental stars.

The publications named below have appeared either as official communications from the Observatory or as papers prepared by its officers individually: —

Thirty-ninth Annual Report of the Astronomical Observatory of Harvard College.

Observations of Variable Stars in 1884. By Edward C. Pickering. Proc. Am. Acad. of Arts and Sciences, xx. 393.

A Photographic Study of the Nebula of Orion. By Edward C. Pickering. Id. xx. 407.

Third Report of the Committee on Standards of Stellar Magnitude. Edward C. Pickering, Chairman. Proc. Am. Assoc. for the Advancement of Science, 1885, p. 1.

Photometric Observations of Neptune at the Harvard College Observatory. By Edward C. Pickering. The Observatory, viii. 111.

Photometric Observations of Ceres (1), Pallas (2), and Vesta (4) at the Harvard College Observatory. By Edward C. Pickering. Id. viii. 238.

On a hitherto unexplained observation by Capt. Gilliss. By S. C. Chandler, Jr. Sidereal Messenger, iii. 315.

Photometric Measurements at the Harvard College Observatory of the Faintest Stars in the Charts constructed by Palisa. By Edward C. Pickering. Id. iv. 133.

Possibility of Errors in Scientific Research due to Thought Transference. By Edward C. Pickering. Amer. Soc. Psychical Research, i. 235.

Elements of Comet 1884 (Wolf). By S. C. Chandler, Jr. Astronomische Nachrichten, cx. 143.

Orbits of Meteors. By O. C. Wendell. Id. cxi. 189.

Dr. Gould's Star in Sculptor. By S. C. Chandler, Jr. Id. cxi. 333.

On the Latitude of Harvard College Observatory. By S. C. Chandler, Jr. Id. cxii. 113.

Comet-Meteor Radiants. By O. C. Wendell. Id. cxii. 321.

On the Right Ascensions of certain Fundamental Stars. By S. C. Chandler, Jr. Id. cxii. 381, cxiii. 17.

On the determination of the absolute length of eight Rowland Gratings. By W. A. Rogers. Proc. Am. Micros. Soc. for 1885, p. 151.

The Microscope in the Workshop. By W. A. Rogers. Proc. Soc. of Mechanical Engineers, Boston meeting, 1885, p. 1.

An Examination of the Standards of Length constructed by the Société Génevoise. By W. A. Rogers. Proc. Am. Acad., xx. 379.

A Preliminary Examination of Metal Thermometers. By W. A. Rogers. Am. Meteor. Journal, October, 1885, p. 252.

Description of a series of fifty Slides, illustrating the action of a diamond in ruling upon glass, presented to the Royal Microscopical Society of London. By W. A. Rogers. Journal R. Micros. Soc., October, 1885.

On a new form of Section-Cutter. By W. A. Rogers. Proc. Am. Soc. Microscopists, 1885, p. 191.

Additional Observations confirming the Relation: Metre des Archives = Imperial yard + 3.37027 inches. By W. A. Rogers. Proc. Assoc. for the Advancement of Science, 1884, p. 117.

To avoid the diminution of astronomical work, many desirable improvements in the general condition of the Observatory have been postponed. They will be undertaken whenever the financial condition of the institution will permit this to be done. Among them may be mentioned new facilities for the use of the large telescope, including probably the entire remounting of the instrument; the arrangement of books in the library, the binding of many valuable works, the provision of new shelving, and the completion of the card catalogue; the improvement of the drainage of the building and grounds and the protection of the woodwork by painting. A still more pressing need than any of these is the publication of material nearly ready for the press, which would occupy several volumes and is now exposed to the risk of destruction by accident. It is to be hoped that this want, at least, may be soon supplied.

EDWARD C. PICKERING, *Director.*

In Catalogue

A PLAN

FOR

THE EXTENSION

OF

ASTRONOMICAL RESEARCH.

BY

EDWARD C. PICKERING,
DIRECTOR OF HARVARD COLLEGE OBSERVATORY.

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THE

EXTENSION OF ASTRONOMICAL RESEARCH.

No science has been more liberally supported than Astronomy during the last two or three centuries. Many millions of dollars have been expended upon it; but, on the other hand, results have been obtained whose value it is impossible to estimate. Apart from the knowledge it has given us of other worlds and of the laws governing the universe, it has furnished us with information regarding this world which has been of enormous practical importance. It has secured safe and certain communication between distant countries, accurate maps, and the precise determination of time. The pecuniary value of these results would many times repay the total expenditure made for astronomical purposes. Nevertheless, in certain cases large sums of money have been expended with little or no useful return. Striking instances may be mentioned of observatories without proper instruments, large telescopes idle for want of observers, and able astronomers unprovided with means of doing useful work. It is the object of the present pamphlet to show how this waste of resources may be in part remedied, and how money may be most advantageously employed for the extension of astronomical knowledge.

The establishment at a university or college of a new observatory, or the gift of a large telescope, is not in general an aid to astronomical progress. To do useful work such a gift must be accompanied by a far larger donation to provide for the necessary current expenses. But few of the existing observatories are really active, and at most of the active observatories the value of the buildings and instruments represents but a small portion of the total sum devoted to astronomical purposes. The current expenses during a single year at some of the most successful

observatories equal the entire value of the instruments employed. Moreover, the usefulness of existing instruments may be indefinitely increased by the provision of funds for the construction of the subsidiary apparatus, which is frequently demanded by the rapid development of new methods in astronomy. For purposes of instruction it is highly desirable that colleges should be provided with suitable observatories, equipped with such instruments as will illustrate the fundamental operations of astronomical research; but the permanent expenditure required will, in this case, be small; since large instruments will not be essential. The number of large telescopes now lying idle is so great, that it is much more important that they should be employed, than that others should be added to their number.

If the money already devoted to observatories could be re-expended, it is evident that far better results could be attained, — first, by establishing at every prominent college a small observatory intended primarily for purposes of instruction; secondly, by founding a few large observatories for research, each of which could then be equipped with the best instruments, and provided with a sufficient endowment to keep the instruments actively employed. A much greater amount of work would thus be accomplished with the same outlay.

Although so sweeping a remedy is impracticable, some improvement in the present condition of things may be attempted. The following examples show how great results might be attained by the expenditure of comparatively small additional sums: —

1. The largest piece of astronomical work ever undertaken is the recent determination of the precise places of about one hundred thousand of the northern stars. This work was begun nearly twenty years ago, and has been progressing ever since, fifteen observatories belonging to various nations taking part. One of the two American observatories participating in this work — that at Albany — has completed the observation and reduction of the stars assigned to it. The manuscript is prepared, but unfortunately no funds exist for its publication. For the want of from three to five thousand dollars this large amount of valuable material, on which many thousand dollars have been already expended, is now lying idle. It is liable to destruction by fire at any time, and is of course useless until published. International obligations seem to require early action in this case.

2. For several years the eighteen-inch telescope at Chicago was the finest instrument of the kind in the world. No means were provided for an observer, and for part of this time the instrument was idle and unused. Later, an enthusiastic and skilful astronomer gratuitously devoted his evenings to it, although he was obliged to occupy his days with engrossing legal business. His observations for years remained unpublished.

3. For twenty-five years the principal work of the Litchfield Observatory consisted in collecting the material for an extensive series of star charts. When the results were ready for the press no means were provided for their publication, and the Director was finally obliged to publish them at his own expense from the savings of a scanty salary.

Numerous similar examples could be added; but it is evident that cases must constantly occur in which work unfinished, and therefore useless, could be completed by the judicious expenditure of small sums of money. Means might be provided for publishing memoirs too long to be included in astronomical periodicals, for furnishing instruments to astronomers, and for paying the salaries of observers for large telescopes which otherwise would be idle. Great aid would thus be rendered to existing observatories in increasing their activity. The greatest extension of astronomical work in the future will probably be effected by the co-operation of different astronomers and observatories. Such co-operation would be greatly facilitated by the aid of considerable funds available for its encouragement. A proper disbursement in this direction could not fail to secure more important results than could possibly be attained at any given observatory. The difficulties in the way of conducting such work permanently are however considerable. It is the object of the present pamphlet to show that the desired results might be attained by intrusting such a fund to the Harvard College Observatory. It will be necessary to show that the fund would be secure, that it would not be likely to be diverted from its legitimate objects, and that its income would be expended efficiently and, so far as possible, impartially.

A large, long established, and growing university possesses some important advantages over a scientific society, or a board of trustees, in the administration of such a fund. The total property invested is likely to be much greater, and under proper

management the advantages ensuing are shared by each of the funds held in trust. The administration of each fund will be carefully watched. Any neglect in this respect would be disastrous, from the loss of confidence on the part of the community, and the consequent falling off in the gifts on which the growth of such an institution in large measure depends.

No safer depositary can be found than the President and Fellows of Harvard College. One evidence of this is the age of the College, — two centuries and a half, making it the oldest in the country. During all this time it has constantly maintained the respect and confidence of the community. The great security for the future lies in the popular sentiment of Boston and the vicinity, where interest in literary and scientific matters is widespread and permanent. This interest is evinced by the large gifts made to such objects, amounting to several hundred thousand dollars every year. Besides large sums given to the Massachusetts Institute of Technology, the Boston Society of Natural History, the Museum of Fine Arts, the Boston Athenæum, and other similar institutions, a large amount is received every year by Harvard University. The sum permanently invested for the various departments of the University amounted in 1840 to \$646,235.17; in 1850, to \$872,440.52; in 1860, to \$1,145,647.20; in 1870, to \$2,387,232.77; in 1875, to \$3,139,217.99; in 1880, to \$3,959,556.08; and in 1885, to \$4,922,392.69. It therefore appears that the rate of increase is continually becoming larger, and now amounts to about a million dollars every five years. The actual increase of University property amounts to a much larger sum; because the buildings which have been presented to the University, and the numerous gifts for immediate use, are not included in the above statement.

Libraries afford another test of this popular sentiment. The number of books in the various public libraries in Boston and vicinity far exceeds that in any other city in the country; and the annual circulation of the Boston Public Library alone has for several years exceeded a million of volumes, and is the largest in the world.

The financial security of property in this part of the country is indicated by the low rate of interest yielded by State and City bonds. This rate when reduced to par is very nearly three per cent for the bonds issued by the State of Massachusetts and the

City of Boston. It differs but little from that derived from British consols, and is only slightly larger than that yielded by United States bonds. Few securities in the world are safer as judged by this test. For several years during the uncertainties caused by the Civil War the Massachusetts bonds actually commanded a higher price than those of the United States. The hold of the University on the community is indicated by its passage uninjured through several important crises. It survived the Revolutionary War and the Civil War without serious loss. The great fire of Boston in 1872 had a more direct effect upon it. No similar event in modern times has caused so great a pecuniary loss. The value of the property destroyed was estimated at about seventy millions of dollars. The direct loss to the University was very large. Notwithstanding the magnitude of the general loss to the community, the continued prosperity of the University was so generally regarded as important, that a large part of its loss was made up by means of a general subscription amounting to about $186,000.

The financial management of the funds intrusted to the College has been excellent. All are invested together, so that each, however small, receives its share of the benefit of the large total investment, which now exceeds five millions of dollars. During the ten years ending September 1, 1876, the average annual rate of interest amounted to 7.21 per cent. Since then it has, of course, diminished; but the rate during the past year amounted to 5.43 per cent. A large part of the College property is free from taxation, and is likely to continue so, as long as literary and charitable institutions enjoy this privilege anywhere. This exemption alone increases the efficiency of a fund intrusted to such an institution by nearly one third. In other words, a hundred thousand dollars held in trust by an incorporated institution, and free from taxes, yields about the same income as one hundred and thirty thousand dollars held by an individual and used for the same purposes.

The funds permanently invested for the benefit of the Harvard College Observatory are already large. On September 1, 1885, they amounted to $226,988.34. Since then a bequest of $320,000 has been received from the late Robert Treat Paine, of Brookline, one half of which, however, is not at present available for

the uses of the Observatory. The question naturally arises why this Observatory should not from its own means furnish the money desired for carrying on work at other institutions, if greater results can thus be attained than in any other way. The answer is that it should; and I hope that it will do so, as far as its means will allow. But the limit of judicious expenditure upon work actually carried on at Cambridge is not yet reached, and the use of instruments which might advantageously be employed whenever the sky is clear is still restricted to a few hours a day, for want of observers. Moreover, some of the funds are restricted, so that they cannot be used outside of the Observatory, and large expenditures are required for permanent improvements which have been postponed on account of the urgency of other work. These improvements include extensive changes and repairs in the building, a new mounting for the large telescope, by which its efficiency could easily be doubled, and the reorganization of the library. A considerable sum is also needed for the publication of observations already made. Nevertheless, the Observatory is intended to promote astronomical science in general, independently of persons or places. It therefore seems to me that if the required fund cannot otherwise be obtained, a portion of it should be subscribed from the unrestricted funds of this Observatory; also, that the money should be expended free from all conditions except those that would lead to the greatest extension of human knowledge. This rule should be applied to the fund, however obtained. Restrictions often limit the usefulness of an appropriation. Of course, common civility and scientific usage require a perpetual acknowledgment of the service rendered by the persons by whom the fund is established. But whatever action is taken by this Observatory, it should impose no conditions on the expenditure of the proposed fund which would limit its efficiency. In the case of a publication it would generally be desirable to adopt the form of the Harvard Annals, or to make the new work one of the volumes of that series. A detached work thus attains a permanent location where it is more easily found and reference to it is more readily made. But should this condition prove distasteful in any case, it should not be insisted upon.

The rule that the fund must be expended so as to attain the best results, independently of all local and personal conditions,

is simple and effective. No restrictions interfering with this principle should be tolerated. Certain forms and rules may be recommended, but they should not be enforced if they would interfere with the objects desired. Thus administered, the fund would not be a source of direct advantage to Harvard University. The direct and indirect expenses would be considerable, and from a selfish standpoint it is not clear that the plan would benefit this Observatory. On the other hand, Science is an ennobling pursuit only when it is wholly unselfish. The attempt to aid all astronomers and all observatories is a far broader and higher aim than local success. The benefit to science would abundantly compensate for the temporary inconvenience and trouble involved in the proper execution of this plan. It is believed that persons could be found who would provide the necessary means, if they could be assured that the expected results would certainly be attained. At least five thousand dollars a year could be advantageously expended in the way here proposed. This would require a permanent fund of about one hundred thousand dollars. But it is not necessary that a large sum should be contributed at once. There would be some advantages in beginning with the sum required for annual expenditure ; in other words, in securing an annual amount equal to the income of the desired fund. The donors would thus be able to judge whether the money was likely to be expended to their satisfaction, before putting a large sum beyond their control. The apprehension that the contribution might not be continued would evidently cause the greatest effort to be made to secure the best possible results. If the expenditures were made fruitfully, the strongest argument would be supplied for making the yearly contribution permanent, or for securing the entire sum needed. If a yearly subscription of five thousand dollars could be secured for five years, the plan could be effectively tried. The same or other donors could then be asked to establish a permanent fund. A similar plan was tried by this Observatory in 1878 with great success. It was shown that an additional annual income of five thousand dollars would produce an immediate increase in the work done much greater than would be proportional to the expenditure. This increase was guaranteed for five years. At the end of that time a statement of the results attained led to the establishment of a permanent fund of fifty thousand dollars, which has permitted

a great extension of the work in progress here. The indirect results which may be ascribed to this subscription were still more important. Mr. Paine was a member of the Visiting Committee, and was well aware of the wants of the Observatory, and of the efforts made to supply them. In October, 1879, when the results of the first subscription began to show themselves, he made a will which has given to the Observatory his entire fortune, exceeding three hundred thousand dollars. Mr. Boyden was asked to take part in the subscription for the Observatory, and declined. He, however, soon after made his will, bequeathing his property, now exceeding two hundred thousand dollars, to astronomical objects. We thus see that the original subscription of 1878 which amounted to twenty-seven thousand dollars, probably contributed to secure for astronomical purposes a sum more than twenty times as great. The large pecuniary interests involved in the work of this Observatory will require executive as well as astronomical capacities in its successive Directors. The income will be more likely to be expended, and expended judiciously, when the responsibility is placed upon a single person than when it is divided among the members of a committee. The assured position of the Observatory will also permit the needs of other similar institutions to be judged more impartially.

The friends of the Observatory have responded so liberally to supply its urgent needs in the past, that it does not seem proper to attempt to raise money for the present purpose by a subscription of the usual form. It is hoped that the required sum may be offered by those who are interested in science for its own sake; also that the breadth of the plan may secure the assistance of some persons who have not hitherto taken an active part in building up Harvard University.

An advantage of the plan here proposed is the broad field of usefulness it would open for this Observatory. It is fairly certain that a vast amount of money will be applied to astronomical purposes in the future. Nearly every donor would prefer that his gifts should aid all astronomers, or astronomy in general, rather than confer a benefit upon a single institution. If a good beginning could be made, great results might be expected. Each appropriation, judiciously expended, would form an argument for new donations. A proper administration of the fund might secure continual additions to it in the future. There appears to

be no limit to the amount of work which could thus be carried on. It might be many times greater than that now conducted at any single observatory. On the other hand, if money should be furnished for immediate expenses, no loss could be incurred, since plans for its application could be proposed before the money should be contributed.

While the best result would be attained by making the fund unrestricted, the wishes of each donor could be respected. One might desire that astronomical research should be fostered in America; another might recognize the great pecuniary restrictions under which many leading European astronomers are working, and might wish to cultivate an international spirit; a third might desire to aid students; and a fourth to secure publication for memoirs of undoubted value.

The scientific results alone have been considered above, but the moral advantages should not be overlooked. If such a scheme could bring astronomers together, so that all should aim at results independent of personal considerations, a vast gain would be made.

EDWARD C. PICKERING.

[*Extracted from Appalachia, Vol. IV., No. III.*]

Accurate Mountain Heights.

BY EDWARD C. PICKERING.

Read December 11, 1885.

OF the various methods of determining the height of a mountain, the best is undoubtedly that by running a line of levels to its summit. This method is accepted as the standard, and as that by which the errors of the other methods are to be judged. A surprising degree of accuracy can be attained in levelling an ordinary country. Many of the errors compensate, and the final results should generally be accurate within a small fraction of a foot. In ascending a mountain, much greater deviations must be expected. The back sights are usually longer than the fore sights, and therefore errors in the adjustment of the level or in the correction for atmospheric refraction are cumulative. The effect of the mass of the mountain on the level would produce an error which would not be compensated, and might be large enough to be appreciable. Finally, an error in the length of the levelling-rod would enter to its full proportionate amount. For these reasons much reliance should not be placed upon the fractions of a foot, unless the above sources of error have been considered and proper corrections applied. The precise heights as determined have, however, been given below.

The labor and cost of levelling prevent its general application to the determination of mountain heights. A few lines of level have been run up the hills and mountains in this portion of the country, generally by the enterprise and enthusiasm of volunteers. A description of several of these lines has been collected from various sources, generally from the local newspapers. The principal results are published below for permanent reference. Doubtless many similar measurements have been made, and it is hoped that they may be communicated to the writer as material for a second paper. As an example of the danger that such material may be totally lost, it may be mentioned that scarcely any of the results given below are contained in the excellent "Dictionary of Altitudes

of the United States," recently published by the United States Geological Survey.

The following table contains a number for reference, the name of the mountain or other object measured, and its height above the mean tide-level of the ocean. Additional information regarding many of these points is contained in the notes following the table. Nos. 1 to 10 are taken from "The Geology of New Hampshire," Vol. I.; Nos. 11 to 17 from an article by Mr. J. J. Holbrook, "New Hampshire Sentinel," Nov. 22, 1877, where the altitudes of several other points in Cheshire County, N. H., are also given. All of these stations are in New Hampshire; Nos. 18 to 43 are in Vermont, and Nos. 44 to 63 in New York.

		FEET
1.	Mt. Washington	6,293
2.	Upper water-tank, Mt. Washington Railroad	5,800
3.	Second tank (Jacob's Ladder)	5,468
4.	Waumbek Junction	3,910
5.	Ammonoosuc Station	2,668
6.	Half-way House	3,840
7.	Glen House	1,632
8.	Kearsarge (S.)	2,942.79
9.	" Garden	2,622.50
10.	" Plumbago Point	1,705
11.	Monadnock	3,169.3
12.	" Mountain House	2,071.984
13.	John Mann's, near divide	1,487.602
14.	Jaffrey Schoolhouse No. 12 (threshold)	1,231.227
15.	Troy Schoolhouse No. 3 (lowest step)	1,166.112
16.	Beech Hill	1,060.566
17.	" " Reservoir	594.589
18.	Mt. Mansfield (Chin)	4,389.08
19.	Mt. Mansfield (Nose)	4,056.39
20.	Summit House	3,841.64
21.	Ridge southeast of Summit House	3,612.38
22.	Half-way House	2,306.38
23.	Junction of Notch Road	1,291.85
24.	Bench near J. Houston's	955.05
25.	Mansfield House, Stowe	720.27
26.	Methodist Church, Waterbury Centre	712.58
27.	Killington Peak	4,220.87
28.	Summit of the second ridge	3,546.31
29.	Rock, summit of the first ridge	3,385.48
30.	Bench, rock near Manley's barn	2,097.61

		FEET
31.	Bench, rock near R. Maxham's	1,812.72
32.	Junction of the mountain road, Sherburne	1,504.77
33.	Hotel, Sherburne	1,211.21
34.	Congregational Church, Bridgewater	892.39
35.	Mt. Tom (north peak), Woodstock	1,351.22
36.	" (south peak), "	1,244.12
37.	Little Killington	3,951
38.	Base of the Town Hall, Woodstock	697.69
39.	Pico	3,935
40.	Shrewsbury Mountain	3,707
41.	Shrewsbury Peak	3,838
42.	Camel's Hump	4,077
43.	Ascutney	3,163
44.	Whiteface Mt.	4,871.655
45.	" " (spring)	2,817.958
46.	" " (brook, second crossing on trail)	2,023.965
47.	" " (brook, first crossing on trail)	1,959.996
48.	Lake Placid	1,863.715
49.	Mt. Marcy	5,344.243
50.	" " (Hump)	4,998.278
51.	Lake Tear of the Clouds	4,321.958
52.	" " " " (summit of notch)	4,355.313
53.	Panther Gorge	3,353.687
54.	Mt. MacIntyre	5,112.730
55.	Mackenzie Pond Mountain	3,789.322
56.	Mt. Skylight	4,889.626
57.	Gray Peak	4,902
58.	Haystack	4,918.626
59.	Bartlett (west shoulder)	2,785.512
60.	St. Regis Mountain	2,888.298
61.	Lyon Mountain	3,809
62.	St. Regis Lake (Lower)	1,623.162
63.	Raquette Lake	1,774.249

1. The height of Mt. Washington was determined in 1853 by Captain Cram of the United States Coast Survey (C. S. Reports, 1854, p. 39, App. 34; 1870, p. 90). Another determination in 1852, by Mr. William A. Goodwin, gave the elevation as 6,285 ("The Geology of New Hampshire," vol. i. p. 88), and apparently resulted from a line of levels.

8 to 10. Carriage-road survey by Mr. R. S. Howe.

11 to 17. Levelled by Mr. J. J. Holbrook.

18 to 26. Levelled by Mr. Hosea Doton, aided by Messrs. W. W. Ware and J. K. P. Chamberlain. They started from the railway station at Waterbury, and assumed the height of the top of the sleepers at that

point to be 425 feet. Professor Guyot, in 1866, found the height of No. 18 to be 4,386 feet, from a series of barometric measurements made every fifteen minutes for twenty-four hours. Another measurement recently made by him gives the height 4,387.25 (reprinted from Walton's Journal in "Vermont Standard," Oct. 15, 1885, where the elevations of twelve other points on this line are also given). Mr. Doton states that he used an eighteen-inch Y-level made by J. Sawyer, of Yonkers, N. Y., with a "New York" rod. The back and fore sights are usually made as nearly equal as possible without actual measurement. When it was necessary to make the sights unequal, the correction for curvature and refraction was applied. The lines were run up the mountain, and not down. An independent determination of the heights of Nos. 18 and 19 has been made by the Hon. J. P. Bradley. A line of levels gave the height of the Butler House in Stowe to be 851.70. A triangulation from a base measured near this point gave the height of No. 18 to be 4,387.25 and of No. 19, 4,061.46. This is a close agreement, considering the means employed, and a satisfactory proof that no serious error occurs in the line of levels. A line of levels has been run to No. 18 by the Coast Survey, but the result has not yet been published (C. S. Report, 1883, p. 29).

27 to 37. In 1863 Mr. Hosea Doton, assisted by Messrs. W. S. Dewey, J. K. P. Chamberlain, and R. A. Perkins, ran this line of levels, starting from White River Junction. Professor Guyot, by a careful barometric measurement, found the height of No. 27 to be 4,221.39 ("Vermont Standard," Oct. 11, 1866, where the heights of forty-six other points on this line are also given). The height of White River Junction was assumed to be 351 feet. In "The Geology of New Hampshire" (vol. i. p. 251 *et seq.*) it is given as 369.237. If this value is correct, the heights of Nos. 27 to 37 should be increased by 18 feet.

38 to 41. Determined trigonometrically from No. 27, No. 38 is 2.55 miles north; No. 39 is .92 miles southerly; No. 40 is 2.50 miles southeasterly; No. 41, also called Mendon Peak, is 1.44 miles south of west ("Vermont Standard," Oct. 11, 1866). The height of No. 41 is there given erroneously as 3,898.

42. Levelled by Mr. Charles Collins at the time of the building of the Vermont Central Railroad, of which he was one of the engineers ("Vermont Standard," June 8, 1871).

43. Levelled by Messrs. H. F. Dunham and D. C. Bell from a bench in Harland ("Vermont Standard," June 8, 1881). The bench appears to have been the summit of Garvin Hill, which was levelled from the base of the town hall in Woodstock by Mr. Hosea Doton, assisted by Mr. Charles Marsh. For further details, see the "Woodstock Post," Jan. 16, 1874.

44 to 63. All of these heights have been taken from the "Seventh Report of the Adirondack Survey," by Mr. Verplanck Colvin. Unfortunately, this work is entirely out of print. The heights of a large

number of other points are given in the same work. No. 61 was not determined by levelling, but from the mean of two months' observation with the barometer.

The following additional heights of the rail at various railway stations may prove useful; they are taken from "The Geology of New Hampshire," Vol. I.: Winchendon, 992; Peterborough, 744; Keene, 478.58; Bellows Falls, 304.58; White River Junction, 369.237; Woodsville, 448; Manchester, 180.832; Concord, 252.397; Plymouth, 473; Warren, 736; Littleton, 817; Wing Road Junction, 1,019; Bethlehem, 1,187; Lancaster, 870; Groveton Junction, 901; Shelburne, 723; Gorham, 812; Conway Corner, 466; North Conway, 516; Fabyan House, 1,571. The height of the lower Connecticut Lake is 1,618.863. The height of the Twin Mountain Station, as given in "The Geology of New Hampshire," p. 266, is 1,446; as given on pages 274 and 288, it is 1,375. The height of Bacon's Mills is given on page 271 as 922; but is apparently copied erroneously, as 992, in the "New Hampshire Sentinel" for Nov. 22, 1877. In the latter paper attention is called to the fact that the height of Marlborough Station is given (and probably correctly) as 789 feet on page 260, while on page 284 the heights of two bridges in Swanzey are given as 1,072 and 1,022. The water flowing under these bridges (which are only of moderate height) comes from below the Marlborough Station.

4 6

[FROM THE AMERICAN JOURNAL OF SCIENCE, VOL. XXXII, SEPTEMBER, 1886.]

ART. XXVI.—*Comparison of Maps of the Ultra Violet Spectrum;* by EDWARD C. PICKERING.

ONE of the first accurate maps of the ultra-violet portion of the solar spectrum was made in 1873 by the late Dr. Henry Draper. A photograph of a normal diffraction spectrum was prepared and copies printed by the Albertype process. A wide distribution was given to this work, which was published also in this Journal, cvi, p. 401. This map extended from 345 to 394, adopting as a unit the millionth of a millimeter. The scale was such that one unit equaled 0·31^{cm}. Soon after, M. Cornu published in the Annales Scientifiques de l'Ecole Normale Supérieure, III, 421, a steel engraving representing the region extending from 343·5 to 412·5. The map was intended as a continuation of the work of Ångström and was on the same scale, one unit equaling 1^{cm}.

The recent publication of a photograph of the solar spectrum by Professor Rowland furnishes a convenient standard with

which these maps may be compared. The improved apparatus and enlarged scale of Professor Rowland's work rendered it probable that its accidental errors would be unimportant in the present comparison, especially as the measures are only to be carried to hundredths of a unit. As a check upon its systematic errors a comparison has been made with the investigations of Dr. Müller and Dr. Kempf in the Publicationen des Astrophysikalischen Observatoriums zu Potsdam, V. Three hundred lines were measured between the limits of 686 and 389. About two hundred of these are contained in the region covered by Professor Rowland's map. A comparison of a number of them indicate that the accidental errors are very small. The width of the bands or groups of lines into which many of the lines are resolved renders it largely a matter of judgment where the center should be taken. The systematic differences are about 0·01, or more strictly the wave-lengths according to Rowland exceed those of Müller and Kempf by about one eighty thousandth part. The region 588 to 590 occurs on four of the strips of Rowland's map. The positions of forty-five of the lines contained in this region were estimated on all four strips and indicated corrections of +·0007, +·0007, −·0009 and −·0004 respectively. These quantities are probably due to accident. They are inappreciable by the method of measurement employed, as they would not exceed one five hundredth of an inch on the map.

The specimen of Dr. Draper's map with which the comparison was made was not a proof impression, but was taken from a copy of this Journal. The positions of seventy-six lines were read, estimating the divisions to tenths by the aid of a reading glass having a focal length of about 10^{cm}. Preference was given to well defined lines, a few bands with hazy edges being omitted. The corresponding positions of these lines were then read from Professor Rowland's map, estimating the tenths by the unaided eye. The differences were arranged in groups as shown in table I, according to their wave-length. Each group extends over one unit, the middle points having the values given in the first column of the table. The number of lines contained in the group is given in the second column, and the mean of the residuals, Draper *minus* Rowland, in the third column. The fourth column gives the results of a similar comparison with the map of Cornu. The same lines were used except that having a wave-length of 393·76, which is not given on Cornu's map. Some uncertainty exists regarding three or four of the other lines, where the engraving does not agree with the photograph. No large residuals are thus introduced, since lines in the right place are selected even if the intensity is incorrectly represented.

TABLE I.

λ	No.	D—R.	C—R.
372·0	3	+·037	—·087
373·0	5	+·032	—·070
374·0	3	+·037	—·077
375·0	3	+·023	—·070
376·0	3	+·003	—·060
377·0	2	+·005	—·075
378·0	3	+·007	—·093
379·0	3	—·003	—·097
380·0	3	—·007	—·023
381·0	1	+·020	—·070
382·0	3	—·017	—·073
383·0	5	—·030	—·084
384·0	2	—·030	—·090
385·0	6	—·027	—·102
386·0	5	—·026	—·086
387·0	3	—·027	—·133
388·0	2	—·045	—·150
389·0	3	—·067	—·153
390·0	6	—·062	—·128
391·0	5	—·042	—·084
392·0	3	—·057	—·107
393·0	2	—·080	—·100
394·0	2	—·075	—·100

The results of table I are combined in table II, where each group extends over five units. An inspection of these tables

TABLE II.

λ	No.	D—R.	C—R.
375·0	16	+·023	—·070
380·0	13	—·003	—·079
385·0	21	—·028	—·097
390·0	19	—·055	—·119

shows that the maps differ systematically, the wave-lengths according to the map of Dr. Draper being too great for the lines of short wave-lengths. The relation is very closely represented by the formula,

$$D - R = \cdot 0052\ (379\cdot 6 - D),$$

in which D and R represent the wave-lengths according to Dr. Draper and Professor Rowland respectively.

Representing by C the wave-lengths according to Cornu, we have in like manner the correction,

$$C - R = \cdot 003\ (351\cdot 7 - C).$$

Where the wave-lengths are only carried to hundredths of a unit the corrections may be more readily applied by means of table III, which gives the limits within which each correction of one hundredth of a unit is to be applied. The first part of the table gives the corrections of Dr. Draper's map, the second the corrections of M. Cornu's map.

TABLE III.

Limits.	Corr. Dr.	Limits.	Corr. Dr.	Limits.	Corr. C.
370·04 to 371·95	+·05	382·48 to 384·39	—·02	370·00 to 373·33	—·06
371·96 to 372·87	+·04	384·40 to 386·31	—·03	373·34 to 376·66	—·07
372·88 to 374·79	+·03	386·32 to 388·23	—·04	376·67 to 379·99	—·08
374·80 to 376·71	+·02	388·24 to 390·15	—·05	380·00 to 383·33	—·09
376·72 to 378·63	+·01	390·16 to 392·07	—·06	383·34 to 386·66	—·10
378·64 to 380·55	·00	392·08 to 393·99	—·07	386·67 to 389·99	—·11
380·56 to 382·47	—·01	394·00 to 395·91	—·08	390·00 to 393·33	—·12

As an example of the use of this table, if a line on Dr. Draper's map has a wave-length of 371·95, its true wave-length may be assumed to be 372·00. In like manner 390·00 becomes 389·95. The same readings on Cornu's map would become 371·89 and 389·88.

Applying these corrections to the wave-lengths derived from the map of Dr. Draper, we obtain results which agree very closely with those given by Professor Rowland. The mean difference for the seventy-six lines compared was 0·012, corresponding to about one eight-hundredth of an inch upon the Draper map. Probably a remeasurement of the larger differences would still further diminish the average value. No differences were rejected, the two largest having the values 0·05 and 0·06. The mean difference of 0·012 gives a probable error of 0·010, which includes the errors of the two readings and the accidental errors of both maps. We may therefore assume that that the probable error of a wave-length derived from the map of Dr. Draper will not exceed one one-hundredth of a unit if the correction given above is first applied. The minuteness of this quantity is a good illustration of the accuracy attainable from a record obtained automatically by photography. The wave-lengths given on the map of Cornu when corrected in the same way give an average deviation of 0·025, equal to about one one-hundredth of an inch on the map. This is in accordance with the belief of Cornu himself that the probable error of the drawing would not exceed 0·03.

[*Extracted from Appalachia, Vol. IV., No. IV.*]

Heights of the White Mountains.

By Edward C. Pickering.

During the summer of 1876 several thousand measurements were made of the altitudes of various points in the White Mountains. A brief description of this work has already been given in Appalachia, Vol. I. p. 138. The method of zenith distances was employed, the measurements being made by means of a micrometer level. This instrument consists of a telescope to which a delicate level is firmly attached. A micrometer screw serves to raise or lower one end of the telescope, the other end being attached to the base by a hinge. A further discussion of this instrument may be found in the "Proceedings of the American Academy," Vol. XI. p. 256; Vol. XXI. p. 268. Thirty-four stations, most of them mountain summits, were occupied with this instrument in 1876. Measures were made from each station, by directing the telescope to each mountain summit or other object of interest visible, and reading the micrometer screw. At short intervals during this work the instrument was levelled, and the screw again read. Horizontal angles were also measured in order to identify or locate the points observed. To determine the altitude of a given point above the instrument, we must know its angular altitude and its horizontal distance. For want of this last quantity the reduction of the observations described above has been delayed for several years. The preparation of the new Appalachian map of the White Mountains has furnished the means of determining these distances, and has accordingly been employed for this purpose. The horizontal angles have been verified at the same time, and in fact have served to locate many of the observed points. The danger of an error in identification has thus been greatly diminished.

The original readings of the micrometer screw were made to the nearest whole division of the divided head, — that is, to one hundredth of a turn. Later observations have shown that each division, which equals about 14″, should be divided into tenths, to attain the greatest accuracy of which the instrument is capable. The effect on the final result is not,

however, important on account of the presence of other sources of error. The most important of these are: First, variations in the atmospheric refraction; secondly, changes in the instrument, caused by variations in temperature, especially when it is exposed to the sun; thirdly, deviation of the point observed, from the true summit. In many cases the summits measured were wooded, the tops of the trees being observed. The mountains were seldom so flat or so near that a point below the summit would be sighted upon by mistake. This source of error would affect the horizontal angles much more than the vertical. It rarely happened that a signal had been erected on which all the pointings could be made. For ordinary purposes an error of a few feet in the height of a mountain is unimportant. To reduce the readings of the micrometer screw, three quantities must be determined: first, the combined collimation and level error of the instrument, which may be defined as the angle between the level and the telescope, or the inclination of the telescope when the level is horizontal; secondly, the correction for the curvature of the earth and the atmospheric refraction; thirdly, the angular value of one division of the micrometer screw. Each of these quantities may either be measured directly, or derived from the observations themselves. In the present case the latter method has been employed. A large number of reciprocal readings were obtained, since, in general, from each station pointings were made upon all the other stations visible. The difference between two reciprocal readings equals twice the sum of the first and second of the errors, — that is, the sum of the collimation, level, curvature, and refraction errors, — provided that these quantities are the same at the two stations. But the combined curvature and refraction expressed as an angle is nearly proportional to the distance between the points of observation, while the collimation and level errors are, of course, independent of this distance. A comparison of fifty-three pairs of reciprocal readings showed that the collimation and level errors were nearly equal to two divisions of the screw, and that the curvature and refraction equalled about one division for each kilometre of distance between the points of observation. The variation

for different stations did not appear to exceed the other sources of error enumerated above.

The angular value of one division of the micrometer screw may be determined from the observation of any point whose distance and height above the instrument are known. Observations were made between but few pairs of stations, both of which had hitherto been determined with sufficient precision. It was decided, therefore, to employ all pairs of observations which served to determine the height of any station from two others, one high and one low, provided that the height of these last had been determined by levelling.

TABLE I.—STATIONS OCCUPIED.

No.	Desig.	Name.	Vert.	Horiz.	Cl.	Height.
1	—	Israel River Bridge	80	27	F	—
2	—	Plaisted House (piazza) . . .	65	56	D	1396
3	—	" " (window) . .	241	67	D	1406
4	—	R. R. Bridge, Lancaster . .	43	43	F	—
5	—	Hill near Jefferson	32	31	F	—
6	E. 9.1	Bray's Hill	105	107	D	1633
7	—	Jefferson Hill	67	62	F	—
8	D. 16.1	Ball (Boy) Mountain . . .	75	74	D	2233
9	D. 12.1	Starr King	118	115	D	3925
10	E. 4.2	Owl's Head	146	135	C	3270
11	F. 3.1	Mt. Adams	223	215	B	5819
12	—	Mt. Pleasant House	40	34	E	—
13	—	Mrs. Pendexter's, N. Conway	71	57	F	—
14	—	Intervale Station	20	15	A	549
15	P. 1.1	Kearsarge (N.)	194	178	B	3270
16	F. 9.1	Mt. Pleasant.	123	109	C	4781
17	F. 6.1	Mt. Washington	70	66	A	6293
18	F. 4.1	Mt. Jefferson	260	208	C	5736
19	—	Twin Mountain House . . .	64	53	F	—
20	—	Flume House	18	20	F	—
21	I. ?	Mt. Pemigewasset	76	70	D	2561
22	J. 6.1	Mt. Lafayette	185	164	C	5269
23	J. 7.1	Mt. Liberty	172	151	C	4472
24	M. 4.2	S. Doublehead	139	111	E	—
25	O. 1.1	N. Moat	200	184	B	3217
26	O. 1.4	S. Moat	87	85	B	2788
27	Q. 1.1	Chocorua	119	113	B	3508
28	—	Conway Corner Station . . .	27	24	A	466
29	M. 5.1	Thorn Mountain	155	138	E	—
30	K. 4.1	Mt. Willey	197	190	C	4313
31	—	Glen Station	53	48	E	529
32	—	Bemis Station	7	11	E	995
33	—	Upper Bartlett Station . . .	28	26	A	659
34	L. 14.1	Iron Mountain	88	80	C	2736
35	—	Thorn Mountain House . . .	41	37	E	—

The various stations occupied are enumerated in Table I., which gives in successive columns a number for reference, the Appalachian designation, if any, for the station, its current name, the number of vertical and of horizontal angles measured from it, and a letter which designates the class to which it was assigned in the determination of its height. A indicates that the height was determined by levelling. B, C, and D refer to the classes described below in the determination of the heights of the occupied stations. E is applied to other stations whose positions have been found, but whose heights were not used in determining the heights of the other stations. F is applied to those stations whose positions have not yet been determined. The last column gives the height finally adopted for each station. A further description of the stations is given below.

1. The instrument was mounted on the rail of the bridge where the road from Jefferson Hill to Cherry Mountain crosses Israel's River.

2. Near the southern corner of the piazza on the floor of the parlors.

3. Window facing southeast of the room in the eastern corner of the second story.

4. Fifty yards northeast of the bridge of the Boston, Concord, and Montreal Railroad over Israel's River in Lancaster.

5. Ledge on the hill about two miles east of Jefferson Hill.

7. Rock on the hill-side behind Waumbek House, about five hundred feet above it.

12. Window over the front door in the second story.

13. Window-sill in the second story of the house two hundred yards west of Intervale Station.

14. The height of the rails at this station, as also of Nos. 31, 32, and 33, was furnished by the courtesy of John F. Anderson, Esq., Chief Engineer of the Portland and Ogdensburg Railroad Company. They have been diminished by five feet to reduce them to the sea-level at mean tide. The following elevations above mean tide are derived from the same source, and may be valuable for future reference: Line of Bartlett and Hart's Location, 744; Crossing of Sawyer's River, 860; Crossing of Nancy's Brook, 969; Water station at Cow's Brook,

1,455; Station for the Willey House, 1,607; Summit of grade at Crawford's Station, 1,902; Fabyan House, 1,571.

15. Two stations were occupied, as the house on the top prevented an unobstructed view.

17. Elevation determined by the United States Coast Survey (C. S. Report, 1870, p. 90).

19. A knoll between the hotel and railroad.

20. Window-sill in the Flume House facing east in the second story.

24. As trees partially obstructed the view from the top of this mountain, the Coast Survey signal had been erected a short distance from the summit. The station occupied was near this. Unfortunately the position of the signal had not been determined, and its location cannot now be found. As it could not be seen from a distance, reciprocal readings could not be obtained.

28. Elevation determined by levelling ("Geology of New Hampshire," Vol. I. p. 265).

29. As the summit of this mountain was wooded, three stations were occupied on it at some distance apart. For this reason it has not been used in determining the heights of the other stations.

31. See No. 14. Three stations were occupied, as an unobstructed view could not be obtained from either one. For this reason it has not been used in determining the heights of the other stations, although its own height was determined by levelling. The results derived from it by a preliminary computation were somewhat discordant, probably from the shortness of the horizontal distances of the stations observed.

32. See No. 14. Not used in determining the height of the other stations, although its own height was determined by levelling. Its position had not been determined when the computation was made, and No. 16 is moreover the only station visible.

33. See No. 14.

The height given for Mt. Washington, No. 17, is that of the Coast Survey bolt, which was covered by a cairn at the time these observations were made, and now has a tower

erected over it. It is assumed to be the same as the apparent height of the ground around the hotel. Sightings were also made upon the ridge of the hotel from ten stations, and on the top of the chimney connected with the steam-boilers from six stations. Giving weights proportional to the distances of the points of observation, these points were found to be, respectively, 36 and 41 feet above the points observed on the ground. The measures from Mt. Adams and Mt. Jefferson have the greatest weights. The intervals to be measured there equalled nearly half a turn of the screw.

Nine measures of the cupola of the house on Kearsarge (N.), No. 15, in like manner gave its height as 35 feet above the ground. This house has since been destroyed by fire.

Table II. contains the material from which the angular value of one division of the screw was determined. In two cases only was a direct measure made between two stations both of which had been determined by levelling. It seemed best, therefore, to include all those cases where the difference between two such stations could be determined by observations made to or from any intermediate station. Calling the lower, intermediate, and upper stations L, M, and N, respectively, all cases must be considered where L M and M N have been measured by the micrometer level, if L and N have been determined by levelling. The first and second columns of Table II. give the names of the lower and intermediate stations. The upper station was Mt. Washington in each case. In the third column u denotes that the observation was made looking up from the lower to the intermediate station, d that the observation was made in the opposite direction. When observations were taken from each of a pair of stations, two lines are given in the table, since the observations were made on different days and are wholly independent. The next two columns give the number of measures of the line L M, and the mean resulting differences in height, assuming from a preliminary discussion the mean value 0.227 for one division of the screw. The unit is such that this represents the fraction of a foot corresponding to one division of the screw at a distance of one kilometre. The next two columns represent the number of measures along the line M N, and the resulting difference in height. All

of these measures were made from M to N, and none in the opposite direction. The next column gives the difference in height of L and N as determined by levelling, and the last column but one gives the residual found by subtracting this difference in height from the sum of L M and M N given in the preceding columns. When several measures are made of the same line they are seldom successive settings upon the same object. They are generally either settings upon different points, as the ridges and bases of the hotel on Mts. Washington and Kearsarge, or they are settings taken at considerable intervals, other measures being made meanwhile. The final column gives the value of one division of the screw derived from each measure.

TABLE II. — FUNDAMENTAL ALTITUDES.

L.	M.	D.	No	L M.	No.	M N.	Elev.	Res.	Div.
Conway Corner	—	u	4	5836	-	—	5827	+9	0.2274
" "	Adams	u	1	5351	3	470	"	−6	0.2268
" "	Chocorua	u	1	3040	2	2777	"	−10	0.2266
" "	Kearsarge (N.)	u	3	2814.	3	3032	"	+19	0.2278
" "	N. Moat	u	1	2746	3	3076	"	− 5	0.2269
" "	S. Moat	u	1	2319	1	3503	"	−5	0.2269
Intervale	Kearsarge (N.)	u	2	2705	3	3032	5744	−7	0.2268
"	" "	d	1	2713	3	"	"	+1	0.2270
"	N. Moat	u	1	2685	3	3076	"	+17	0.2274
"	" "	d	1	2667	3	"	"	−1	0.2270
"	S. Moat	u	1	2266	1	3503	"	+25	0.2280
"	" "	d	1	2250	1	"	"	+9	0.2272
Upper Bartlett	Kearsarge (N.)	u	1	2599	3	3032	5634	−3	0.2269

The first measure in the table is a direct observation of Mt. Washington from Conway Corner. As no intermediate station was used the columns relating to M are left blank. A measure of Mt. Washington from Glen Station gave a residual of — 8, and the resulting value of one division, 0.2268. The mean of the thirteen values given in the last column of the table is 0.2269, and this value would be somewhat increased if we give greater weight to the observations depending upon the greatest number of settings. The value 0.227, which corresponds to the angular value of 14.″27 for one division of the micrometer, has therefore been adopted.

The heights of the intermediate stations contained in

Table II. are determined with considerable accuracy, since each of them has been compared directly with a station above, and at least one below, whose heights have been determined by levelling. These stations are designated as B in Table I. From Classes A and B the heights of several other stations may be determined; these form Class C. Class D in like manner is formed from A, B, and C. All the more important stations are included in this list, except Nos. 24 and 29, regarding which some uncertainty exists. Several of these heights, however, depend upon a small number of measures, and the accidental errors are likely to be increased by the fact that they are connected with the levelled station only through one or two intermediate stations. A second approximation has therefore been made by determining the height of each station in B, C, and D from each of the observations connecting it with the stations A, B, C, and D, adopting the values just found for the stations B, C, and D, and using the levelled heights of A. The mean of these gave a third approximation to the heights of these stations. A fourth approximation was made in the same way, but did not alter the previous values by a sensible amount.

The results of the successive determinations of the heights of the stations in Classes B, C, and D are contained in Table III. The successive columns give the number of the station, its name, the number of readings employed in the first approximation, the deduced altitude, the average deviation in feet of the separate results, and the probable error of the mean in feet. The correction required by the second approximation is given in the next column, a negative sign denoting that the first result appeared to be too large. The corrections required by the third and fourth approximations are given in the next two columns. The last four columns relate to the value of the height finally adopted. They give the number of settings, their mean result, the average deviations of the separate readings, and the probable error of the mean as indicated by the accordance of the individual values. The three classes B, C, and D are placed in successive portions of the table, and each is followed by a line giving the means of certain of the columns.

TABLE III. — Heights of Stations.

No.	Name.	No.	I.	A. D.	P. E.	II.	III.	IV.	No.	Adopt.	A. D.	P. E.
	Class B.											
11	Adams	5	5821	1.2	0.5	−2	0	0	27	5819	7.6	1.2
15	Kearsarge (N.)	17	3262	7.0	1.5	+9	−1	0	45	3270	11.0	1.4
25	N. Moat	6	3220	5.7	2.1	−2	0	−1	31	3217	10.1	1.6
26	S. Moat	4	2797	9.8	4.6	−9	0	0	20	2788	11.0	2.1
27	Chocorua	3	3513	5.0	3.0	−5	0	0	30	3508	8.3	1.3
	Mean	7.0	—	5.7	2.4	5.4	0.2	0.2	31	—	9.6	1.5
	Class C.											
10	Owl's Head	8	3273	2.0	0.6	−2	0	−1	22	3270	5.2	1.0
16	Pleasant	11	4781	3.9	2.4	+1	0	−1	27	4781	6.9	1.2
18	Jefferson	6	5738	0.8	0.3	−2	0	0	24	5736	6.3	1.1
22	Lafayette	17	5271	13.1	2.8	−3	+1	0	31	5269	13.3	2.1
23	Liberty	8	4464	12.9	4.1	+9	−1	0	20	4472	11.7	2.8
30	Willey	10	4311	14.2	4.0	+2	−1	0	21	4313	10.2	2.0
34	Iron	9	2741	12.4	3.7	−3	−2	0	10	2736	10.6	3.0
	Mean	9.9	—	9.2	2.6	3.1	0.7	0.3	22	—	9.2	1.8
	Class D.											
3	Plaisted	7	1407	6.9	2.2	0	−1	0	7	1406	6.9	3.4
6	Bray	9	1634	6.8	1.9	0	−1	0	10	1633	5.3	1.5
8	Ball	10	2234	11.4	3.2	0	−1	0	10	2233	11.5	3.2
9	Starr King	15	3925	7.2	1.6	0	0	0	16	3925	5.9	1.2
21	Pemigewasset	6	2560	14.5	5.5	0	+1	0	6	2561	10.8	4.1
	Mean	9.4	—	9.8	2.9	0.0	0.8	0.0	10	—	8.1	2.7

An examination of this table shows that the successive approximations give a rapid approach to the final value. The mean values in feet of all the numbers contained in Columns II., III., and IV. are 2.9, 0.6, and 0.2. Another approximation would not change any of the adopted values by one foot. The total number of settings in the table is 357 and of stations 17, or an average of 21 to each station. The average deviation of the separate results has a mean value of very nearly 9 feet, and the probable error of a mean elevation is 2 feet.

To show the degree of accordance indicated by Table III., the observations of one station, North Moat, are given more fully in Table IV. A number for reference is followed by the

number and name of each station serving to determine the height of North Moat, the resulting altitude, and the residual found by subtracting the mean value 3,217.

TABLE IV. — North Moat.

No.	Des.	Name.	Elev.	Res.	No.	Des.	Name.	Elev	Res
1	11	Adams	3253	+36	17	26	S. Moat	3220	+3
2	"	"	3224	+7	18	"	" "	3214	−3
3	14	Intervale	3215	−2	19	27	Chocorua	3204	−13
4	"	"	3234	+17	20	"	"	3206	−11
5	15	Kearsarge (N.)	3194	−23	21	"	"	3204	−13
6	"	"	3208	−9	22	"	"	3216	−1
7	"	"	3205	−12	23	"	"	3221	+4
8	"	"	3205	−12	24	"	"	3229	+12
9	16	Pleasant	3231	+14	25	28	Conway Corner	3212	−5
10	"	"	3231	+14	26	30	Willey	3208	−9
11	"	"	3209	8	27	"	"	3208	−9
12	17	Washington	3216	−1	28	"	"	[illegible]	+16
13	"	"	3222	+5	29	34	Iron	3226	+9
14	"	"	3222	+5	30	"	"	3210	−7
15	22	Lafayette	3232	+15	31	"	"	3210	−7
16	"	"	3207	−10					

Readings 2, 4, 7, 8, 11, 16, 18, 24, 25, 28, 30, and 31 were made upon North Moat from the station named in the third column. The other readings were made from North Moat. No. 5 was made upon the cupola of the house on Kearsarge (N.), and No. 12 on the ridge of the house on Mt. Washington. The first approximation of 3,220 feet, as given in the fourth column of Table III., was derived from Nos. 3, 4, 12, 13, 14, and 25.

Three series of observations were made from the Plaisted House. One, from the piazza, Station No. 2, included the greater portion of the view not cut off by Mt. Starr King. The second from the window, Station No. 3, included a large number of points upon the Mt. Washington Range. The third, also from Station No. 3, is the only one included in Table III. Eight series of observations were made, on seven different days, to determine the effect of the variations in the atmospheric refraction, and other sources of error. In all, 171 settings were made on twenty-seven objects. On taking residuals four were found to be discordant, and to give the values of + 10, − 6, + 11, and − 10 divisions of the micrometer.

Three of these occurred in the second series. All were probably due to errors of reading of the screw-heads, and were doubtless due to carelessness. Of the other residuals but one, which equals 3, exceeds two divisions. The average value of all the residuals is 0.86, or, rejecting the four discordant readings, 0.60. One division varies from two to seven feet, according to the distance of the object. Its average value is about 4.5 feet. A considerable portion of the remaining error is caused by neglecting the tenths of a division. Evidently, for the distances here employed, not exceeding twenty miles, the variations from day to day are inappreciable.

Table V. gives separate residuals for the seven occupied stations observed from the Plaisted House. The number and name of the station are followed by its distance in kilometres, the corresponding value of one division in feet, the mean reading, and the residuals found by subtracting this mean from the individual measures. The residuals are expressed in feet, negative residuals being indicated by Italics. The separate readings for Owl's Head would therefore be 9.31, 9.31, 9.32, 9.31, 9.31, 9.29, and 9.31. The last three columns give the deduced difference in height; the height of the Plaisted window, found by subtracting this difference from the adopted heights of the various stations given in Table III.; and finally, the residuals found by subtracting the mean height, 1,406, from the separate determinations.

TABLE V.—8. Plaisted House (Window).

No.	Name.	Dist.	1 Div.	Rdg.	Resid.	Diff.	Elev.	Res.
10	Owl's Head	8.65	1.96	9.31	00100*2*0	1846	1426	+20
11	Adams	17.85	4.05	10.71	00*1*01*1*110	4421	1400	–6
16	Pleasant	21.7	4.93	6.61	1*12*100*11*	3376	1404	–2
17	Washington	20.97	4.76	10.11	061100	4921	1407	+1
18	Jefferson	17.50	3.97	10.72	0001*1*10*1*	4338	1401	–5
22	Lafayette	31.3	7.11	5.10	010*112*0010*1*1	3860	1408	+2
30	Willey	26.05	5.92	4.65	*1*00011001	2915	1394	–12

It will be seen that the errors of the individual readings are far less than the systematic errors affecting the whole. Thus the separate values of the height as deduced from the settings

upon Owl's Head, are 1,426, 1,426, 1,424, 1,426, 1,426, 1,430, and 1,426, since one division equals 1.96 feet, and a positive residual denotes a large reading, and consequently a large difference in height of the two stations. As these values are all much in excess of those given by the other stations, it is probable that the assumed distance of Owl's Head is a little too great, or that some other similar error affects them all.

Where values had been adopted for the heights of the occupied station, the reduction of the various measures was easily effected.

The original readings of the micrometer screw were corrected for the position of the axis of the instrument and level, by subtracting the readings when the bubble was in the centre of the tube. A correction was at the same time applied for the inclination of the axis, when level readings in different azimuths showed that this was required. All these corrected readings were written in journal form, placing together all the observations taken from a given point. Ledgers were next prepared, which were formed while making the map, bringing together all the readings upon a given object, and entering the name of the station from which the observation was made, and its distance in kilometres and hundredths. The identification of the point was secured from the horizontal angle. The micrometer readings were next increased by two divisions, to correct for the collimation and level errors, and by a number of divisions equal to the distance in kilometres, to correct for the curvature of the earth and refraction. The distance in kilometres was next multiplied by 0.227, which gave the value in feet of one division of the screw, at a distance equal to that of the observed point. The product was then multiplied by the corrected micrometer reading, and gave the height of the observed point above the instrument. Adding to this the height of the station occupied gave the required height of the observed point. The simplicity of the operation is shown by the formula

$$A = 0.227\, k\, (r - l + 2 + k) + B,$$

in which A is the required height, k the distance in kilometres, r the original reading of the micrometer, l its reading

when the bubble is level, and *B* the height of the occupied station.

The actual process is shown in Table VI., which gives the determination of the points measured on J. 3, Mts. Bond and Guyot. The figures are copied directly from the original computation, which was made nearly in this form. The successive columns give the number and name of the station from which the observation was made, the distance in kilometres, and the original reading after correcting it for the level. The next column gives the reading corrected for curvature, refraction, collimation, and level by adding to it two *plus* the number representing the distance in kilometres. The sixth column gives the distance in kilometres multiplied by 0.227, and equals the value in feet of one division of the micrometer head at that distance. The product of the fifth and sixth columns is given in the seventh column, after multiplying by 100. The next column gives the assumed height of the point of observation taken from Table I. The sum of these two, or the corrected height, is given in the ninth column; and the last column gives the residual found by subtracting the mean value from the individual values. This mean value is given in the heading following the name of each object observed.

Two additional measures were made, one from Mt. Pleasant, the other from Mt. Jefferson. The azimuth was nearly that of J. 3.4, but the altitude showed that a nearer point on the main ridge was probably observed.

TABLE VI.—Mts. Bond and Guyot.

No.	Name.	Dist.	Rdg.	Corr. Rdg.	1 Div.	Diff. Elev.	Elev. Base.	Elev.	Res.
				J. 3.1. Bond. 4709.					
18	Jefferson	23.87	−2.16	−1.90	5.42	1030	5736	4706	−3
22	Lafayette	9.08	−2.79	−2.68	2.05	549	5269	4720	+11
"	"	"	−2.79	−2.68	"	549	"	4720	+11
25	N. Moat	28.53	+2.04	+2.34	6.36	1488	3217	4705	−4
26	S. Moat	30.83	+2.40	+2.78	7.00	1911	2788	4699	−10
30	Willey	9.36	+1.78	+1.84	2.12	390	4312	4702	−7
"	"	"	+1.76	+1.87	"	396	"	4708	−1

TABLE VI. *Continued.*—MTS. BOND AND GUYOT.

No.	Name.	Dist.	Rdg.	Corr. Rdg.	1 Div.	Diff. Elev.	Elev. Base.	Elev.	Res.
				J. 3.1 b. N. BOND. 4676.					
15	Kearsarge (N.)	35.42	+1.37	+1.74	8.04	1399	3270	4669	−7
18	Jefferson	23.80	−2.24	−1.98	5.40	1069	5736	4667	−9
22	Lafayette	8.84	−2.94	−2.83	2.01	569	5269	4700	+24
23	Liberty	9.68	+0.80	+0.92	2.20	202	4472	4674	−2
25	N. Moat	28.31	+1.96	+2.26	6.43	1453	3217	4670	−6
30	Willey	9.43	+1.58	+1.69	2.14	362	4312	4674	−2
"	"	"	+1.59	+1.70	"	364	"	4676	0
				J. 3.2. S. GUYOT. 4588.					
15	Kearsarge (N.)	35.90	+1.25	+1.63	8.15	1328	3270	4598	+15
16	Pleasant	16.95	−0.72	−0.53	3.85	204	4781	4577	−6
18	Jefferson	23.19	−2.45	−2.20	5.26	1157	5736	4579	−4
22	Lafayette	8.56	−3.60	−3.49	1.94	677	5269	4592	+9
25	N. Moat	29.09	+1.75	+2.06	6.60	1360	3217	4577	−6
30	Willey	9.37	+1.15	+1.26	2.13	268	4312	4580	−3
"	"	"	+1.15	+1.26	"	268	"	4580	−3
				J. 3.3. N. GUYOT. 4589.					
15	Kearsarge (N.)	35.70	+1.22	+1.60	8.10	1296	3270	4566	−23
16	Pleasant	16.64	−0.68	−0.49	3.77	185	4781	4596	+7
22	Lafayette	8.82	−3.43	−3.32	2.00	664	5269	4605	+16
25	N. Moat	28.99	+1.77	+2.08	6.58	1367	3217	4584	−5
26	S. Moat	31.87	+2.14	+2.48	7.23	1793	2788	4581	−8
30	Willey	9.10	+1.27	+1.38	2.07	286	4312	4598	+9
"	"	"	+1.25	+1.36	"	282	"	4594	+5
				J. 3.4. —— 4528.					
22	Lafayette	7.75	−4.30	−4.20	1.76	739	5269	4530	+2
23	Liberty	8.57	+0.19	+0.30	1.95	58	4472	4530	+2
25	N. Moat	29.20	+1.65	+1.96	6.63	1299	3217	4516	−12
27	Chocorua	31.07	+1.13	+1.46	7.05	1029	3508	4537	+9
				J. 3.5. S. BOND. 4279.					
15	Kearsarge (N.)	35.75	+0.86	+1.24	8.11	1006	3270	4276	−3
22	Lafayette	8.55	−5.20	−5.09	1.94	987	5269	4282	+3
23	Liberty	8.53	−1.18	−1.02	1.94	198	4472	4274	−5
25	N. Moat	28.15	+1.35	+1.65	6.39	1054	3217	4271	−8
27	Chocorua	29.70	+0.84	+1.16	6.74	782	3508	4290	+11
30	Willey	10.58	−0.25	−0.12	2.40	29	4312	4283	+4
34	Iron	24.05	+2.56	+2.82	5.46	1540	2736	4276	−3

TABLE VI. — *Concluded.*

No.	Name.	Dist.	Rdg.	Corr. Rdg.	1 Div.	Diff. Elev.	Elev. Base	Elev.	Res.
J. 3.6. —— 3904.									
18	Jefferson	26.10	−3.40	−3.12	5.92	1847	5736	3889	−15
22	Lafayette	8.50	−7.08	−6.98	1.93	1347	5269	3922	+18
23	Liberty	8.04	−3.23	−3.13	1.83	573	4472	3899	−5
30	Willey	11.20	−1.73	−1.60	2.54	406	4312	3906	+2

The heights of a large number of points were determined by the micrometer level. A portion of the readings has been reduced, some of the objects not having been identified, and the horizontal positions of others not having been determined as yet. Table VII. gives the altitudes of those points which have been determined by measurements from a sufficient number of stations to warrant their publication. The first column gives the Appalachian designation of the point, the second its name, and the third the number of stations from which the height is determined. The fourth column gives the mean value of the height, and the fifth its probable error as indicated from the accordance of the individual readings.

TABLE VII. — Adopted Heights.

Desig.	Name.	No.	Height	P.E.	Desig.	Name.	No.	Height	P.E.
D. 4.2	Pilot	5	3788	1	E. 4.2	Owl's Head	22	3270	1
6.2		8	4186	3	5.2 b		2	1836	0
11.1		3	3916	1	7.2	Dalton	2	2181	2
12.1	Starr King	16	3925	1	9.1	Bray's Hill	10	1633	2
12.3	" "	2	3632	1	F. 2.1	Madison	8	5381	4
12.3 b	" "	5	3325	2	3.1	Adams	27	5819	1
13.1		6	4046	4	3.2	"	4	5615	7
13.2		5	4033	2	3.3	"	3	5431	5
14.1	Pliny	3	3651	6	3.4	"	4	5386	6
16.1	Ball	10	2283	3	4.1	Jefferson	24	5736	1
18.1	Crescent	4	3322	3	5.1	Clay	9	5554	4
18.2	"	3	3246	5	5.2	"	5	5535	2
E. 1.1	Dartmouth	6	3768	3	5.2 b	"	7	5514	2
1.5	Mitten	2	3118	8	5.2 c	"	4	5507	3
3.1	Deception	7	3701	2	6.1	Washington	–	6293	–
3.2	"	4	3722	2	6.2	Boott's Spur	4	5529	4
3.3	"	4	3638	8	7.1	Monroe	6	5397	3
4.1	Cherry	9	3600	2	7.1 b	"	12	5375	6

TABLE VII. — *Continued.*

Desig.	Name.	No.	Height	P.E.	Desig.	Name.	No.	Height	P.E.
F. 7.2	Monroe	13	5216	8	K. 2.1	Tom	6	4078	1
8.1	Franklin	6	4923	2	3.1	Field	11	4355	1
8.2	"	3	5013	3	3.2	"	2	3760	2
9.1	Pleasant	27	4781	1	4.1	Willey	21	4313	2
9.1 b	"	8	4499	8	4.1 b	"	5	4242	4
10.1	Clinton	7	4331	1	6.1	Nancy	9	3944	2
11.1	Jackson	7	4076	4	6.2	"	8	3742	2
12.1	Webster	7	3928	2	7.1	Anderson	8	3748	2
G. 2.1	Bald	5	3752	4	8.1	Lowell	7	3765	3
3.1	Moriah	4	4065	4	9.1	Carrigain	15	4701	1
5.1	Carter	4	4573	2	9.2	"	7	4276	3
5.2	"	5	4614	2	9.3	"	5	3886	4
5.3	"	3	4650	4	9.4	"	9	4435	4
5.3 b	"	4	4640	8	L. 4.1	Giant Stairs	4	3512	2
5.4	"	3	4468	1	5.1 b	Resolution	3	3436	1
6.1	Carter Dome	9	4856	4	6.1	Crawford	5	3130	5
6.2	" "	8	4711	3	8.2	Parker	5	3015	5
H. 1.1	S. Baldface	8	3590	2	9.2	Langdon	3	2439	3
1.2	N. Baldface	8	3608	2	14.1	Iron	10	2736	3
1.5		4	2954	2	M. 1.1	Wildcat	7	4428	3
1.5 b		5	2896	3	1.1 b	"	3	4384	2
2.1	Eastman	9	3559	3	1.2	"	5	4284	2
3.1	Sable	7	3377	4	1.3	"	3	4097	5
4.1	Royce	5	3117	4	4.1	N. Doublehead	3	3072	3
4.2	"	4	3219	5	4.2	S Doublehead	5	2946	3
I. 1.1	Cannon	5	4107	3	4.2 b	" "	5	2950	3
1.2	"	2	3898	7	N. 1.1	Hancock	6	3906	9
2.2	Kinsman	3	4377	7	1.2	"	6	4284	4
2.5	Pemigewasset	6	2561	4	1.3	"	4	4259	4
4.1	Moosilauke	13	4810	3	1.4	"	3	4434	3
J. 1.1	Hale	3	4102	7	1.5	"	5	4056	3
2.1	N. Twin	11	4783	2	2.1 b	Huntington	4	3731	3
2.1 c	" "	4	4626	2	2.2	"	4	3728	2
2.1 d	" "	5	4316	2	2.3	"	3	3820	8
2.1 e	" "	2	3808	0	4.1	Scar Ridge	6	3815	4
2.2	S. Twin	10	4922	2	4.1 b	" "	4	3536	5
2.2 b	" "	3	4801	6	4.2	" "	5	3816	4
2.4	" "	7	4741	3	5.1	Osceola	10	4204	7
2.5	" "	4	4632	3	5.2	"	12	4352	2
3.1	Bond	7	4709	2	5.4	"	7	4259	2
3.1 b	"	7	4676	2	5.5	"	8	4162	2
3.2	S. Guyot	7	4583	2	5.6	"	5	3651	2
3.3	N. Guyot	7	4589	4	5.7	"	5	3658	3
3.4	W. Bond	4	4528	3	8.1	Tecumseh	7	4008	1
3.5	S. Bond	7	4279	2	O. 1.1	N. Moat	31	3217	2
3.6		4	3904	5	1.2	Red Ridge	5	2787	5
4.1	Haystack, or				1.3	Bear Peak	5	2807	4
	Garfield	8	4520	3	1.4	S. Moat	20	2788	2
6.1	Lafayette	31	5269	2	2.1	Attitash	2	2516	0
6.1 b	"	9	5021	4	3.1		5	2985	3
6.2	N. Lafayette	8	5075	4	3.2	Table	4	2726	7
6.3	Lincoln	18	5098	2	3.4		3	2949	3
7.1	Liberty	20	4472	2	3.5		3	2958	2
8.1	Flume	6	4840	3	4.1	Bear	5	3267	4
11.1	Agassiz	6	2401	5	4.1 b	"	4	3271	2
11.2	Round Hill	4	2442	9	4.2	"	3	3225	2

TABLE VII. — *Concluded.*

Desig.	Name.	No.	Height	P.E.	Desig.	Name.	No.	Height	P.E.
O. 4.8	Bear	3	3004	4	3.1	Passaconaway	9	4116	2
5.1	Silver Spring	8	3001	3	3.2	"	4	3402	6
6.1	Tremont	7	3399	4	4.1	Whiteface	8	4057	3
6.2	"	5	3307	8	4.2	"	7	4008	5
6.3	"	5	3010	5	6.1	Tripyramid	8	4189	1
7.1	Green's Cliff	4	2972	4	6.2	"	8	3684	5
P. 1.1	Kearsarge (N.)	45	3270	1	6.3	"	10	4155	3
1.2	Bartlett	6	2650	5	6.4	"	8	4139	2
1.5		3	2628	7	6.5	"	5	3935	3
Q. 1.1	Chocorua	30	3508	1	6.6	"	5	3908	3
1.3	"	8	3375	5	6.7	Kancamagus	4	3774	2
1.4	"	8	3354	3	8.1	Sandwich			
1.5	"	5	3287	5		Dome, or	5	3999	3
2.1	Paugus	7	3248	6	8.2	Black	5	3990	1

In the preparation of this table it appeared that numerous additional heights could be given if the height of Station No. 29 on Thorn Mountain was known. It was accordingly deduced from readings on fifty-four other points, most of them not occupied stations. The mean of these values was 2,283 feet, with an average deviation of 10.7 feet, and a probable error of 1.3 feet. This was adopted as the height of the station on the western ledge, and not of the top of the mountain.

To Mr. Edmands I am indebted for much assistance in relation to the horizontal distances and identifications involved in this work.

From Table VII. it appears that the principal group of the White Mountains is that culminating in Mt. Washington, 6,293 feet. It also contains Mt. Adams, 5,819; Mt. Jefferson, 5,736; Mt. Clay, 5,554; Mt. Monroe, 5,396; Mt. Madison, 5,381; Mt. Franklin, 4,923; Mt. Pleasant, 4,781; Mt. Clinton, 4,331; Mt. Jackson, 4,076; and Mt. Webster, 3,930 feet. The Franconia group contains Mt. Lafayette, 5,269; Mt. Lincoln, 5,098; Mt. Liberty, 4,472; Mt. Flume, 4,340; and Haystack, or Mt. Garfield, 4,520 feet. The Twin Range contains S. Twin, 4,922; N. Twin, 4,783; Mt. Bond, 4,709; Mt. Guyot, 4,589 feet. The Carter Range contains Carter Dome, 4,856; Mt. Carter, 4,650; and Mt. Moriah, 4,065 feet. The Whiteface Range contains Tripyramid, 4,189; Mt. Passaconaway, 4,116; Whiteface, 4,057; and Sandwich Dome (or Black Mountain), 3,999 feet.

Of the other mountains, may be mentioned Mt. Moosilauke, 4,810; Mt. Carrigain, 4,701; Mt. Hancock, 4,434; Mt. Wildcat, 4,428; Mt. Kinsman, 4,377; Mt. Field, 4,355; Mt. Osceola, 4,352; Mt. Willey, 4,313; Mt. Starr King, 3,925; Cherry Mountain, 3,600; Mt. Chocorua, 3,508; Mt. Kearsarge (N.), 3,270; and N. Moat, 3,217 feet.

FORTY-FIRST

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE DECEMBER 7, 1886, AND LAID BEFORE THE BOARD OF OVERSEERS, JANUARY 5, 1887.

CAMBRIDGE, MASS.
PUBLISHED BY THE UNIVERSITY.
1887.

REPORT.

To the President of the University:

Sir, — The munificence of the late Robert Treat Paine has now begun to provide the encouragement which he desired to give to the science of astronomy. The sum of $164,198, comprising about half of his bequest, has been received by the Treasurer of the University, and the income of this fund is now available for the support of work at this Observatory. The previous funds available for that purpose amounted on September 1, 1875, to $164,067, and on September 1, 1885, to $226,988. On September 1, 1886, with the addition of the sum received from Mr. Paine's estate, they had risen to $398,046. This increase in the means of the Observatory will ultimately permit a corresponding extension of its work, but for the moment will largely be required for the publication of observations already made, and for effecting permanent improvements in the condition of the institution which have long been urgently needed. Some of these improvements have already been undertaken, but others will soon be necessary. Among these one of the most important is the remounting of the large telescope in a manner conformable to that provided for similar instruments at observatories more recently founded. The telescope also requires eye-pieces and other appliances of the kind now generally in use. The question of a new building must also soon be considered. The present building is one of the oldest belonging to the University beyond the limits of the College Yard. In many respects it is not adapted to the demands of modern astronomy, and will in any case need extensive repairs. The principal of the invested funds of the Observatory is not available for these improvements, and if they are to be paid for from the income, the work cannot be completed for a series of years and will materially interfere with the prosecution of any new researches. It is therefore to be hoped that special means may be obtained for meeting the want just mentioned.

Another method in which any new funds available for astronomical purposes might be advantageously employed at this Observatory has been pointed out in a recent pamphlet on the extension of astronomical research. This plan contemplates the support of researches

conducted at other places by funds administered here. If the project should commend itself to those desirous of giving further encouragement to astronomy, it is confidently expected that the results would be very large as compared with the expenditure required to obtain them; for instruments and observers now idle might then efficiently contribute to the progress of science in various parts of this country or of the world. The breadth of this plan, since it contemplates aiding astronomy in general rather than this Observatory in particular, may, it is hoped, commend it to those who have not hitherto felt a special interest in Harvard University, but who may desire to aid astronomical science in the most effective way.

The fund of $10,000 established in 1871, the income of which has hitherto been employed in the payment of annuities, has recently become available. This fund will form an important addition to the means of the Observatory, especially as the use of its income is unrestricted. It is now called the Augustus Story Fund, and its amount on September 1, 1886, was $13,380.

The resignation of Professor William A. Rogers has deprived the Observatory of a valued assistant who has devoted many years to laborious astronomical work. Almost all of the observations in the extensive investigations conducted with the Meridian Circle, since its mounting in 1870, have been made by him and reduced under his direction. Fortunately an arrangement has been made by which he will still retain the superintendence of this work, and its publication will be completed as soon as possible. Three volumes of the Annals and a part of a fourth containing these observations have already been published; and three or four more will be required to complete their publication.

The most important extension of the work of the Observatory which has recently been made, was effected last spr'ng by the liberality of Mrs. Henry Draper. As a memorial to the late Professor Henry Draper, the study of the photography of stellar spectra undertaken by him is now being carried on at this Observatory on a scale appropriate to the advance in this department of science. This work will be considered more in detail in the following description of the various investigations now in progress here.

East Equatorial.

Eclipses of Jupiter's Satellites. — The photometric observation of these eclipses has been continued upon the system adopted in 1878. The total number of eclipses thus observed is now three hundred and fifty-eight, thirty-nine of which have occurred since the end of October, 1885.

Comparison Stars for Variables. — The observation of these stars with the wedge photometer has been continued through the year upon the plan described in the last report, and has formed the principal work of the instrument whenever its use for some special purpose was not required. All observers of variable stars are again requested to communicate to this Observatory the comparison stars which they have employed, so far as these are not already contained in the lists published by Argelander and others.

New Stars in Orion and Andromeda. — A cable message announcing the discovery of a new star near χ^1 *Orionis* was received in the afternoon of December 16, 1885. After transmitting this intelligence to other American astronomers, observations were at once commenced upon the new object and continued throughout the greater portion of the night. The position of the new star was determined with the equatorial and meridian circle; its brightness was measured with the wedge photometer and the meridian photometer, and was also estimated. Its spectrum was examined with the large equatorial. Two photographs of the star, and one of its spectrum, were also taken. The results of these observations made up to $1^h\ 30^m$ A.M. on December 17th were at once communicated to the press, and appeared in the newspapers of the same morning. On subsequent nights these observations were repeated, measures being made with the meridian circle on seven nights, with the meridian photometer on twenty nights, and with the wedge photometer on eight nights. Photographs were taken of the region containing this object on thirteen nights. The absence of the star from one of these showed that it must have been much fainter on November 9, 1885, than when it was discovered six weeks later. The spectrum was also photographed on five nights.

Observations of the region containing the temporary star in the nebula of Andromeda were continued until January 7, 1886. The total number of dates on which the region was observed was twenty-three.

Photographic Experiments. — The East Equatorial was frequently employed during the year in experiments upon photographing the spectra of stars. Usually a prism was placed before the object-glass of the instrument, but the effect was also tried of attaching a large spectroscope to the tail-piece.

Comets. — The occasional observation of comets by Mr. Wendell has continued, as in former years, to form part of the work done by the East Equatorial. Comet 1885 V. was observed on five nights; comet 1886 I. on seven nights; Barnard's comet, discovered December 3, 1885, on six nights; the three comets discovered by Brooks on April 27, April 30, and May 22, 1886, were respectively observed on

four, two, and two nights; and those discovered by Finlay and by Barnard on September 26 and October 4, 1886, were each observed on two nights.

Meridian Circle.

Owing to circumstances explained at the beginning of this report, the reduction and publication of work already done with this instrument is at present more desirable than the prosecution of new series of observations. The work of reduction has been actively continued through the year and has resulted in the publication of Volume XV., Part I., and Volume XVI. of the Annals of the Observatory. The first of these publications contains the annual results for the fundamental stars observed during the years 1870 to 1879, inclusive, and the individual results for the years 1883 to 1886. It also includes the results from the separate observations of stars belonging to various special classes, and the catalogue of 1213 stars, separately published last year.

Volume XVI. contains a tabular statement of the instrumental constants and a journal of the observations, beginning with the mean results obtained in right ascension and declination from the separate wires and microscopes, and resulting in the place of the star for the beginning of the year of observation.

The second part of Volume XV. will contain the catalogue of the zone stars, and will be printed as part of the general catalogue of stars in the northern hemisphere undertaken by the Astronomische Gesellschaft. The completion of the manuscript has been delayed by the comparison undertaken between the results obtained here and those found in previous catalogues.

A volume corresponding to Volume XVI., but relating to the zone stars instead of to the fundamental stars, will be required to complete the publication of the zone observations. The observations for absolute right ascension and declination, made by Professor Rogers during the period 1879 to 1883, inclusive, will furnish material for a separate volume, which will complete the work of the meridian circle still requiring publication.

Meridian Photometer.

During the year ending November 1, 1886, 209 series of measures have been made with this instrument by Mr. Wendell and myself. The total number of separate photometric comparisons is 59,800. The corresponding numbers in the two previous years are 27,500 and 50,000. The instrument continues to give entire satisfaction and leaves little to be desired as a means of measuring the bright-

ness of stars of the ninth magnitude or brighter. It possesses various advantages over the small instrument of the same kind employed in the Harvard Photometry. This is shown in the average deviation of the one hundred circumpolar stars employed as standards. The value of this quantity, which was .16 of a magnitude with the older instrument, has been reduced to .12 with the present instrument. Fainter stars are measured with still greater accuracy, the average deviation of stars from the fifth to the ninth magnitude but little exceeding a tenth of a magnitude. The rapidity of the observations is such that 179 stars have been measured by a single observer in one evening, four settings being made on each star. The rate of forty stars an hour is frequently maintained during the entire evening, and when there is no difficulty in identifying the stars, the rate of a star a minute is often exceeded. This rapidity is attained by saving time at every point except in making the actual settings. It is understood that the observer is always to take as much time as he desires for this part of the work.

Various tests have been applied to detect the presence of systematic errors in this instrument, so far with negative results. A comparison of the ten brightest with the ten faintest of the standard stars served to show whether the value of one magnitude was the same for this instrument as for that previously used. All of the observations made during the past four years with the present instrument give the same results within less than an hundredth part as that derived from the similar observations with the original meridian photometer. Another possible source of error was investigated by using a red and a blue star as alternate standards of comparison for other objects. No appreciable difference was detected over a range of more than seven magnitudes in the observations of either of the observers.

A comparison of the seven hundred stars common to the observations of Wolff, of Pritchard, and the Harvard Photometry showed that our results differed on the average from Wolff, after allowance for systematic differences, by .140 of a magnitude; from Pritchard by .145; while Wolff and Pritchard differed from each other by .192. A comparison of the fifty-five stars proposed by Professor Pritchard as standards, and measured by him on several nights, showed that the average deviation from the Harvard Photometry was only .104.

Dr. E. Lindemann, of the Pulkowa Observatory, who is conducting an extensive series of observations in stellar photometry by means of the Zöllner photometer, has sent lists of stars observed by him, for the purpose of comparison. These stars have been observed with the meridian photometer. A comparison between the results obtained at Pulkowa and Cambridge shows that the average deviation of a meas-

urement of the difference in brightness between two stars observed at both places does not exceed one tenth of a magnitude.

The principal work of the meridian photometer, the revision of the Durchmusterung magnitudes, is now approaching completion, nine tenths of the observations having already been made.

During the coming year similar observations will be made upon stars situated in the first twenty degrees of south declination. All stars which have been used as comparison stars for variables, if brighter than the ninth magnitude, are also being measured. Various pieces of miscellaneous work have been undertaken. Among these are measures of the brighter known and suspected variables, the stars observed by Rosén, and various stars employed by different astronomers as standards of reference.

STELLAR PHOTOGRAPHY.

Bache Investigation. — The investigation in stellar photography undertaken with the aid of the Bache Fund and described in the last report is now nearly completed. The principal results obtained include photographs of the entire sky north of — 30° on which all stars bright enough to leave trails without the aid of clock-work are depicted. One series of plates exhibits the effect of atmospheric absorption on nearly every night of observation for a year. A large number of photographs of stellar spectra were also taken. Provision has been made for reducing most of the latter photographs, but the reduction of the others has not yet been undertaken. Among the miscellaneous observations may be mentioned some experiments in the application of photography to transit instruments, which showed that the accidental errors did not reach one half of those affecting eye-observations. Various photographs were taken of the nebula of Orion to show the relative brightness of different portions of this object. The nebulae in Andromeda, in Lyra, and in the Pleiades were also photographed. An attempt was made to photograph a satellite of Jupiter while undergoing eclipse, and thus determine the time of this phenomenon.

Henry Draper Memorial. — An investigation has been undertaken which promises to lead to an important extension of our knowledge of the stellar universe. As has been already stated, by the aid of Mrs. Henry Draper the study of the photographic spectra of the stars is being carried on with appliances which are probably unequaled elsewhere. Three researches are now in progress. The first includes a general survey of stellar spectra. Each spectrum is photographed with an exposure of not less than five minutes, and these photographs generally exhibit the spectra of all stars brighter than the sixth magnitude with sufficient distinctness for measurement. The greater

portion of the sky north of —30° has been surveyed in this work, which will be repeated during the coming year. 151 plates have been measured and 5431 spectra examined and classified. Of these 4148 have been identified and the name and position of the corresponding star entered opposite each. The completed work will form a catalogue probably containing three or four thousand stars, each photographed on several plates.

The second research relates to a determination of the spectra of the fainter stars. Each photograph taken in the course of this research receives an exposure of one hour, so that the spectra of all the stars not fainter than the eighth or ninth magnitude, and included in a region ten degrees square, are represented upon the plate. On 58 plates 2416 spectra have been measured, and of these 2359 have been identified.

In both of these investigations the eight-inch Bache telescope has been employed. The third research relates to a more careful study of the spectra of the brightest stars. For this work Mrs. Draper has lent the 11-inch photographic lens employed by her husband. She has also furnished an admirable mounting for the instrument, and a small observatory to contain it. Two prisms have been constructed to place in front of the object-glass, the large one having a clear aperture of eleven inches square, and an angle of nearly 15°, the other being somewhat smaller. The preliminary results attained with this apparatus are highly promising. The account of them, however, will more properly belong to the report of the following year.

Miscellaneous.

Variable Stars. — Messrs. Parkhurst, Eadie, Hagen, and Zaiser, have continued their observations in coöperation with this Observatory, as heretofore, and have still further increased the large amount of material collected by their industry for the study of the changes occurring in variable stars. Mr. Parkhurst has also made various interesting researches upon photometric questions, the results of which he has frequently appended to his records of observation. Mr. E. F. Sawyer, of Cambridgeport, has also reported the results of observations of variable stars made by him. Communications, which have been recorded in a report upon the subject published early in the year, were received from the following foreign observers of variable stars: Mr. T. W. Backhouse, of Sunderland, England; Dr. N. C. Dunér, of Lund, Sweden; Rev. T. E. Espin, of Wolsingham, England; Mr. J. E. Gore, of Ballysodare, Ireland; Dr. E. Hartwig, of Bamberg, Germany; Mr. George Knott, of Cuckfield, England; Professor Safarik, of Prague, Austria.

Estimated Magnitudes. — A valuable series of comparisons by estimation between stars of various magnitudes, made by M. P. Stroobant, of Brussels, was kindly furnished by the observer for reduction and publication at this Observatory. The copy of these comparisons sent by M. Stroobant was part of the mail carried by the steamship Oregon, and was supposed to have been lost by the sinking of that vessel. But after a submersion of about fifteen weeks it was recovered in an entirely legible condition by the skill of the American Wrecking Company, and forwarded to the Observatory.

Time Signals. — The increasing difficulties in transmitting these signals, and the efforts made to improve the service, were mentioned in the last report. The methods adopted have proved efficient, and the distribution of the signals throughout the year has gone on in a satisfactory manner. The Boston Time Ball, now mounted on the building of the United States Post Office, was dropped at noon on week days 295 times. On five other occasions, having failed to drop at noon, it was dropped five minutes later, according to the custom in such cases.

Telegraphic Announcements. — The telegraphic distribution of important astronomical discoveries or data has been continued during the year under the management of Mr. Ritchie, as before. Announcements have been made of the discovery of ten asteroids, nine comets, one meteoric shower, and one new star. There have been sent to the European union of astronomers, or received therefrom, twenty cable messages, and the number of telegrams sent in this country was 450, an increase of 190 as compared with the previous year.

The Associated Press has been regularly notified of the facts thus distributed and the information has been widely spread by means of the newspapers. Among other items of information may be noted the receipt of various positions of comets observed at the United States Naval Observatory, Washington. These positions were published as soon as possible after their receipt and have proved useful in the computation of orbits.

Buildings and Grounds. — Various important repairs in the main building of the Observatory will soon become necessary, unless means can be secured for replacing it by a new structure. The entire exterior, and most of the interior of the building, requires painting; new floors are needed, and additional space will become imperative. Besides the usual care of the grounds, a number of rare trees and shrubs have been planted which in a few years will greatly improve the appearance of the place. The most important improvement is the construction of a substantial fence on the north side of the grounds, which was needed to protect the numerous valuable instruments now

located in small buildings near the Observatory. A drain has also been laid on the southern and eastern sides of the main building.

PUBLICATIONS.

The principal publications of the year were Volume XV., Part I., and Volume XVI. of the Observatory Annals, which have been described above. Two treatises in quarto form, on the apparent position of the zodiacal light, and on stellar photography, have also appeared in the Memoirs of the American Academy of Arts and Sciences, as is shown in the list which follows.

These publications have appeared either as official communications from the Observatory or as papers prepared by its officers individually.

Fortieth Annual Report of the Astronomical Observatory of Harvard College.

A Plan for the Extension of Astronomical Research. By Edward C. Pickering. Cambridge, 1886.

Faint Stars for Standards of Stellar Magnitudes. By Edward C. Pickering. Sidereal Messenger, iv. 24; Astronomical Register, xxiii. 39.

Photographic Study of Stellar Spectra. Henry Draper Memorial. By Edward C. Pickering. Circular, reprinted in Nature, xxxiii. 535, and Science, vii. 278.

Accurate Mountain Heights. By Edward C. Pickering. Appalachia, iv. 215; Science, vii. 423.

Comparison of Maps of the Ultra Violet Spectrum. By Edward C. Pickering. American Journal of Science, cxxxii. 223.

Draper Memorial Photographs of Stellar Spectra exhibiting Bright Lines. By Edward C. Pickering. Nature, xxxiv. 439.

Comet 1883 I. (Brooks). By O. C. Wendell. Sidereal Messenger, v. 92.

On the New Nebula discovered in the Pleiades by MM. Henry. By Edward C. Pickering. Astronomische Nachrichten, cxiii. 399.

Photometric Observations of the New Star near χ^1 *Orionis*. By Edward C. Pickering. Id. cxiv. 283.

Orbits of Meteors. By O. C. Wendell. Id. cxiv. 285; Sidereal Messenger, v. 111.

Comet-Meteor Radiants. By O. C. Wendell. Astronomische Nachrichten, cxiv. 329; Sidereal Messenger, v. 152.

Magnitudes of Comparison Stars employed in Dr. Müller's determination of the Phases of Asteroids. By Edward C. Pickering. Astronomische Nachrichten, cxiv. 413.

Orbit of Comet 1886 (Brooks 2). By O. C. Wendell. Id. cxiv. 415; Sidereal Messenger, v. 220.

Positions of the New Star near χ^1 *Orionis*. By Edward C. Pickering. Astronomische Nachrichten, cxv. 31.

Early Experiments in Telegraphing Sound. By Edward C. Pickering. Proc. Am. Acad. of Arts and Sciences, xxi. 262.

Atmospheric Refraction. By Edward C. Pickering. Id. xxi. 268.

A New Form of Polarimeter. By Edward C. Pickering. Id. xxi. 294.

Observations of Variable Stars in 1885. By Edward C. Pickering. Id. xxi. 319.

The Apparent Position of the Zodiacal Light. By Arthur Searle. Memoirs of the Am. Acad. of Arts and Sciences, xi. 135.

An Investigation in Stellar Photography. By Edward C. Pickering. Id. xi. 179.

The Longitude of the McGill College Observatory. By Professor W. A. Rogers, Harvard College Observatory, and Professor C. H. McLeod, McGill College Observatory. Trans. Roy. Soc. Canada, Section iii., 1885, page 111.

EDWARD C. PICKERING, *Director.*

XX.

OBSERVATIONS OF VARIABLE STARS IN 1886.

BY EDWARD C. PICKERING.

Communicated March 9, 1887.

THE present publication is the fourth in a series of annual statements relating to variable stars, which was begun in 1884. In the fifth statement, to be published in 1888, it is proposed to review the entire period since the discovery of each variable star, giving the number of observations made by each observer during each year, so far as this information can be obtained. All persons who have any facts of this kind at command are urgently requested to communicate them to the Observatory of Harvard College, so that the proposed publication may be as complete as possible.

Some difficulty has been experienced in preparing the present report, from the circumstance that variable stars are occasionally designated only by letters and constellations, without their numbers in any published catalogue or their places for a given date. The recommendation made in previous reports is accordingly here renewed, that the place of each star should always be given when there is no other means of identification than the name. The number in a designated catalogue, such as that printed with these reports, will of course be a sufficient substitute for the place.

In view of the extended publication proposed for next year, the present report may be made comparatively brief. The names of the observers, their methods of observation, and the abbreviations by which they are designated in Tables I. and II. are mainly the same as in the report for last year; Messrs. Backhouse, Dunér, Eadie, Espin, Gore, Hagen, Knott, Parkhurst, Safarik, and Zaiser, being designated as before by B., D., Ee., En., G., Hn., K., P., Sk., and Zr., while M. denotes the work of the meridian photometer. An eyepiece magnifying 90 times has often been employed in Mr. Parkhurst's observations, in addition to the powers of 56 and 150 mentioned last

year. Father Hagen has begun a systematic search for variables of the fourth class in a great circle whose pole is $12^h\ 40^m$, $+28°$, and has examined the DM. chart No. 32 in accordance with suggestions given in "Variable Stars of Short Period." See these Proceedings, XVI. 277, 281. He suspects the following stars to be variable: DM. $+55°$ 2587, $+44°$ 3368, $+44°$ 3402. Dr. Hartwig has not made any statement with regard to his observations in 1886. Professor Safarik has been prevented by illness from making a complete statement of his work, but has sent a list containing a large number of observations, which are entered as usual in Tables I. and II. Statements have also been received from some observers who have not contributed to previous reports. The additional abbreviations thus required in Tables I. and II. are explained in the following paragraphs.

B_1. These observations were made by Mr. Joseph Baxendell, at Birkdale, Southport, England. The telescope used in the observations is an achromatic refractor of 6 inches' aperture, made by Cooke and Sons, of York; and the magnitudes of the variables are determined by comparisons with neighboring stars whose magnitudes have been determined by the method of limiting apertures.

B_2. These observations were made by Mr. Joseph Baxendell, Jr. The place, instrument, and method of observation were the same as described under the heading B_1.

Eq. These observations were made with the equatorial telescope of the Observatory of Harvard College. The aperture and focal length of the instrument are respectively 15 and 279 inches. The magnifying power employed was ordinarily 103. The observers were Messrs. Arthur Searle and O. C. Wendell. The observations consist in measurements, made with the wedge photometer, of the comparison stars known to have been employed by previous observers; but when the variable stars themselves were visible, they were incidentally compared with others by estimate, according to the method of Argelander, and were also observed with the wedge. The work will be continued during the coming year, and it is desired to make it include as many as possible of the comparison stars which have been employed by any observer. The list now in use is chiefly derived from the published work of Argelander, Schönfeld, and Oudemans. Observers are requested to send lists of the comparison stars not included in these publications which they have themselves employed, or which have been employed to their knowledge by others. It is very desirable that not only the places of these stars, but also their designations in the

Durchmusterung, when they occur in that catalogue, should be entered in the lists thus sent.

Sn. These observations were made by Mr. T. S. H. Shearmen, of Brantford, Canada. The instruments employed were an opera-glass and a two-inch refractor. Each variable was compared with stars differing little from it in brightness. Many suspected variables have been photographed.

Table I. indicates the progress of observation for stars included in Table I. of previous reports. Other stars, whether known or suspected to be variable, are included in Table II. All the columns of Table I. except the last are repeated from the statement of the previous year. The first column of the left-hand pages gives a provisional number for designating the star. This number is taken from Schönfeld's Catalogue when the star occurs there; in other cases, a letter is added to the number. The second column contains numbers from the Photometric Catalogue called Harvard Photometry, and published in Volume XIV. of the Annals of the Harvard College Observatory. The following columns contain the usual designation of the star, its right ascension and declination for 1875, magnitude at maximum and minimum, and period in days.

The first column of the right-hand page repeats the number to be used for the provisional designation of the star. The second gives the class to which the star belongs, upon the system of classification employed in the Proceedings of the American Academy of Arts and Sciences, XVI. 257. Upon this system, Class I. includes temporary stars; Class II., stars undergoing large variations in periods of several months; Class III., irregularly variable stars, undergoing but slight changes in brightness; Class IV., variable stars of short period, like *β Lyræ* or *δ Cephei;* Class V., Algol stars, or those which at regular intervals undergo sudden diminutions of light, lasting for a few hours only. The third column gives the name of the discoverer, and the fourth column the date.

The last column contains the number of nights on which each star was observed by the astronomer whose designation is attached to the number. The abbreviations employed have been explained above.

Table I. is followed by a series of remarks containing observed dates of maximum and minimum, and other information derived from the observers with regard to particular stars.

Table II. indicates the progress of observation of stars suspected or known to be variable, but not included in Table I. for reasons explained in previous reports. The provisional numbers given in the

first column for many of the stars refer to Mr. Chandler's unpublished catalogue, as in previous years. The second and third columns give the right ascensions and declinations of the stars for 1875. The fourth column gives the number of observations made by each observer, as in the last column of Table I. The abbreviations are likewise the same. The letters in the last column refer to the remarks on page 395.

TABLE I. — Variable Stars.

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
0a	—	Ceti	0 15 26	−20 45.1	5.2	7.0	—
1	51	T Cassiopeiæ	16 20	+55 5.9	6.5 — 7.0	11 — 11.2	436
2	54	R Andromedæ	17 28	+37 53.0	5.6 — 8.6	<12.8	404.7
3	—	S Ceti	17 42	−10 1.3	7.0 — 8.0	<10.7	323.6
4	—	B Cassiopeiæ	17 52	+63 27.2	>1	[illegible]	—
5	—	T Piscium	25 31	+13 54.6	9.5 — 10.2	10.5 — 11.0	Irr.
6	94	α Cassiopeiæ	33 25	+55 51.1	2.2	2.8	Irr.
6a	—	U Cephei	51 18	+81 12.1	7.0	9.5	2.5
7	—	S Cassiopeiæ	1 10 30	+71 57.2	6.7 — 8.5	<13	615
8	—	S Piscium	11 2	+ 8 16.3	8.8 — 9.8	<13	406.6
8a	—	Piscium	16 22	+12 12.7	10	14	—
8b	—	Ceti	19 31	− 4 36.6	6.5	7.8	—
8c	—	R Sculptoris	21 18	−33 11.5	5¾	7¾	207
9	—	R Piscium	24 12	+ 2 14.1	7.4 — 8.8	<12.5	345
10	—	S Arietis	57 55	+11 55.5	9.1 — 9.8	<13	288.8
11	—	R Arietis	2 9 1	+24 28.4	7.6 — 8.5	11.9 — 12.7	186.2
12	370	o Ceti	13 1	− 3 32.7	1.7 — 5.0	8 — 9	331.3
13	—	S Persei	13 54	+58 0.8	8.5?	<9.7	—
14	—	R Ceti	19 39	− 0 44.6	7.9 — 8.7	<12.8	167.1
15	—	T Arietis	41 22	+16 59.8	7.9 — 8.2	9.4 — 9.7	324
16	469	ρ Persei	57 10	+38 21.3	3.4	4.2	Irr.
17	406	β Persei	3 0 2	+40 28.4	2.2	3.7	2.9
18	—	R Persei	22 6	+35 14.3	8.1 — 9.2	12.5	208.8
19	657	λ Tauri	53 45	+12 8.2	3.4	4.2	4.0
20	—	T Tauri	4 14 43	+19 14.3	9.2 — 11.5	12.8 — <	Irr.
21	—	R Tauri	21 27	+ 9 52.9	7.4 — 9.0	<13	325.6
22	—	S Tauri	22 22	+ 9 40.1	9.9	<13	378
22a	—	Doradus	35 19	−62 19.4	5¼	6¼	—
23	—	V Tauri	44 48	+17 19.6	8.3 — 9.0	<12.8	168.6
24	—	R Orionis	52 13	+ 7 56.3	8.7 — 8.9	<13	378.8
25	877	ε Aurigæ	53 0	+43 38.2	3.0	4.5	Irr.
26	880	R Leporis	53 55	−14 59.7	6 — 7	8.5?	437.8
27	—	R Aurigæ	5 7 12	+53 26.6	6.5 — 7.4	12.5 — 12.7	465
27a	—	S Aurigæ	18 52	+34 2.3	9.4	<13	—
28	—	S Orionis	22 50	− 4 47.5	8.3?	<12.8	—
29	1005	δ Orionis	25 37	− 0 23.6	2.2?	2.7	Irr.
29a	—	Orionis	29 42	− 5 33.5	10	13	—
30	1091	α Orionis	48 24	+ 7 23.3	1	1.4	Irr.
31	1160	η Geminorum	6 7 20	+22 22.4	3.2	3.7 — 4.2	229.1
31a	—	Monocerotis	16 26	− 2 8.1	7	<10	—
32	1205	T Monocerotis	18 29	+ 7 9.1	6.2	7.6	26.8
33	—	R Monocerotis	32 21	+ 8 50.7	9.5	11.5	Irr.
34	1256	S Monocerotis	34 6	+10 0.5	4.9	5.4	3.4
35	—	R Lyncis	50 59	+55 30.2	9?	<12.3	—
36	1384	ζ Geminorum	56 41	+20 45.1	3.7	4.5	10.2
37	—	R Geminorum	59 49	+22 53.8	6.6 — 7.8	<12.3	371.0
38	—	R Canis min.	7 1 50	+10 13.1	7.2 — 7.9	9.5 — 10.0	335.0
38a	—	Puppis	9 43	−44 26.2	3¼	<6	135
38b	—	V Geminorum	16 10	+13 21.8	8.5	12 — 13¼	276
38c	1417	U Monocerotis	24 50	− 9 31.0	6.0	7.2	46.0
39	—	S Canis min.	25 56	+ 8 35.0	7.2 — 8.0	<11	332.2
40	—	T Canis min.	27 3	+12 0.6	9.1 — 9.7	<13	335.2
40a	—	Canis min.	34 34	+ 6 40.2	8¼	13.5	405
41	—	S Geminorum	35 32	+23 44.6	8.2 — 8.7	<13	294.2

TABLE I. — VARIABLE STARS.

No.	Class.	Discoverer.	Date.	Observations, 1886.
0*a*	—	Chandler	1881	8 M.
1	II.	Krüger	1870	8 B_1. 8 Ee. 2 Eq. 9 M. 80 Sk.
2	II.	Argelander	1858	8 Ee. 2 Eq. 3 M. 9 P. 10 Sk.
3	II.	Borelly	1872	9 Ee. 4 Eq. 3 M. 12 P.
4	I.	Tycho Brahe	1572	8 M.
5	II.	Luther	1855	12 Ee. 3 Eq. 4 M. 4 P.
6	III.	Birt	1831	10 M. 1 Sn.
6*a*	V.	Ceraski	1880	1 B_1. 2 B_2. 12 Hn. 1 M.
7	II.	Argelander	1861	2 B_1. 27 B_2. 17 Ee. 2 Eq. 2 G. 6 M. 11 P.
8	II.	Hind	1851	10 Ee. 2 Eq. 3 M. 6 P. [82 Sk.
8*a*	—	Peters	1880	13 P.
8*b*	—	Gould	1874?	2 M.
8*c*	II.	Gould	1872?	—
9	II.	Hind	1850	2 Eq. 3 M. 8 P.
10	II.	Peters	1865	1 Eq. 4 M. 11 P.
11	II.	Argelander	1857	14 B_1. 3 Eq. 9 Hn. 14 P. 2 Sk.
12	II.	Fabricius	1596	50 B. 3 B_1. 22 G. 12 K. 21 Sk. 5 Sn.
13	II.	Krüger	1873	2 Eq. 5 Hn. 1 P. 30 Sk.
14	II.	Argelander	1866	8 Eq. 13 P. 10 Sk.
15	II.	Auwers	1870	3 Eq. 17 Hn. 26 Sk.
16	II.?	Schmidt	1854	1 G.
17	V.	Montanari	1669	—
18	II.	Schönfeld	1861	10 B_1. 8 Eq. 6 Hn. 7 P. 5 Sk.
19	V.	Baxendell	1848	1 B. 9 Zr.
20	—	Hind	1861	8 B_1. 8 Eq. 3 K. 12 P. 2 Sk.
21	II.	Hind	1849	8 Eq. 10 P. 3 Sk.
22	II.	Oudemans	1855	6 Ee. 3 Eq. 11 P. 3 Sk.
22*a*	—	Gould	1874?	—
23	II.	Auwers	1871	3 Eq. 18 P. 12 Sk.
24	II.	Hind	1848	3 Eq. 1 K. 3 Sk.
25	III.	Fritsch	1821	—
26	II.	Schmidt	1855	8 B_1. 2 Eq. 1 K. 17 Sk.
27	II.	At Bonn	1862	2 En. 2 Eq. 4 G. 4 Hn. 1 P. 3 Sk.
27*a*	II.	Dunér	1881	19 D. 4 Eq. 11 P.
28	II.	Webb	1870	9 B_1. 19 Ee. 3 En. 2 Eq. 8 K. 1 P. 24 Sk.
29	III.	J. Herschel	1834	—
29*a*	—	Bond	1863	17 Ee. 1 P.
30	III.	J. Herschel	1836	11 Zr.
31	II.?	Schmidt	1866	2 B.
31*a*	—	Schönfeld	1883	6 P.
32	IV.	Gould	1871	—
33	II.	Schmidt	1861	2 Eq. 1 Sk.
34	IV.	Winnecke	1867	—
35	II.	Krüger	1874	3 Eq. 19 P. 21 Sk.
36	IV.	Schmidt	1844	5 M. 18 Zr.
37	II.	Hind	1848	3 Eq. 2 K. 16 P. 8 Sk.
38	II.	At Bonn	1854	3 Eq. 15 Hn. 1 K. 25 Sk.
38*a*	II.	Gould	1872	—
38*b*	II.	Baxendell	1880	24 B_1. 3 Eq. 6 K.
38*c*	II.?	Gould	1878	11 En. 3 G.
39	II.	Hind	1856	1 B_1. 1 Ee. 8 Eq. 3 K. 19 Sk.
40	II.	Schönfeld	1865	2 Eq.
40*a*	II.	Baxendell	1879	21 B_1. 1 Ee. 1 Eq. 5 K.
41	II.	Hind	1848	2 B_1. 1 Eq. 10 P. 20 Sk.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
42	—	T Geminorum	7 41 48	+24 2.7	8.1 — 8.7	<13	288.1
42a	—	S Puppis	43 6	—47 8.8	7½	9	—
43	—	U Geminorum	47 41	+22 19.7	8.9 — 9.7	13.1	Irr.
43a	—	Puppis	55 0	—12 32	8½	<14	310
44	—	R Cancri	8 9 40	+12 6.5	6.2 — 8.8	<11.7	354.4
45	—	V Cancri	14 36	+17 40.9	6.8 — 7.2	<12	272
46	—	U Cancri	28 37	+19 19.5	8.2 — 10.4	<13	305.7
47	—	S Cancri	36 48	+19 29.0	8.2	9.8	9.5
48	—	S Hydræ	47 8	+ 8 32.4	7.5 — 8.5	<12.2	256.4
49	—	T Cancri	49 32	+20 19.7	8.2 — 8.5	9.3 — 10.5	484.2
50	—	T Hydræ	49 35	— 8 39.8	7.0 — 8.1	<12.5	289.4
50a	—	R Carinæ	9 29 6	—62 14.2	4.4	9.8	313
51	—	R Leonis min.	38 4	+35 5.2	6.1 — 7.5	<11.0	374.7
52	1752	R Leonis	40 50	+12 0.5	5.2 — 6.4	9.4 — 10.0	312.6
52a	—	l Carinæ	41 49	—61 55.9	3.7	5.2	31.2
52b	—	Leonis	53 8	+21 51.6	8½	8.6<13	280?
52c	—	Antliæ	10 4 22	—37 7.1	6½	<8	—
52d	—	Carinæ	5 23	—60 56.3	6½	9	—
52e	—	U Leonis	17 21	+14 38.1	9½	Inv.	—
52f	1869	Hydræ	31 22	—12 44.1	4½	6	—
53	1880	R Ursæ maj.	35 47	+69 25.9	6.0 — 8.1	12	303.4
54	—	η Argus	40 13	—59 1.6	>1	6.3	Irr.
54a	—	T Carinæ	50 18	—59 51.2	6.2	6.9	—
55	—	R Crateris	54 25	—17 39.2	>8	<9	—
56	—	S Leonis	11 4 28	+ 6 8.5	9.0 — 9.7	<13	187.6
57	—	T Leonis	32 2	+ 4 8.9	10 ?	<13	—
58	—	X Virginis	55 27	+ 9 46.1	7.8?	<10	—
59	—	R Comæ	57 51	+19 28.8	7.4 — 8.0	<13	363
60	—	T Virginis	12 8 12	— 5 20.4	8.0 — 8.8	<13	337
61	—	R Corvi	13 10	—18 33.5	6.8 — 7.8	<11.5	318.6
61a	—	Virginis	27 26	— 8 43.8	8	14	210±
62	—	T Ursæ maj.	30 42	+60 10.6	7.0 — 8.3	12.2	255.6
63	2147	R Virginis	32 10	+ 7 40.6	6.5 — 7.5	10.0 — 10.9	145.7
63a	—	R Muscæ	34 28	—68 43.3	6.6	7.3	0.9
64	—	S Ursæ maj.	38 28	+61 46.7	7.7 — 8.2	10.2 — 11.1	224.8
65	—	U Virginis	44 46	+ 6 14.0	7.7 — 8.1	12.2 — 12.8	207.4
66	—	W Virginis	13 19 35	— 2 43.4	8.7 — 9.2	9.8 — 10.4	17.3
67	—	V Virginis	21 21	— 2 31.4	8.0 — 9.0	<13	251
68	2275	R Hydræ	22 53	—22 38.0	4.0 — 5.5	10?	469.3
69	2289	S Virginis	26 29	— 6 33.0	5.7 — 7.8	12.5	374.0
69a	—	Virginis	14 3 37	—12 42.7	9	14	—
69b	—	R Centauri	7 35	—59 19.8	6	10	—
70	—	T Bootis	8 14	+19 39.1	9.7?	<13	—
71	—	S Bootis	18 41	+54 22.7	8.1 — 8.5	13.2	272.4
72	—	R Camelopardi	27 8	+84 23.8	7.9 — 8.6	12?	266.2
73	2445	R Bootis	31 41	+27 16.9	5.9 — 7.5	11.3 — 12.2	223.0
73a	2459	Bootis	37 56	+27 8.6	5.2	6.1	370?
73b	—	Bootis	48 33	+18 12.1	9.1	12.0 — 13.6	173.8
74	2506	δ Libræ	54 18	— 8 1.2	4.9	6.1	2.3
74a	—	Libræ	15 8 37	—19 38.9	10	<13.5	700±
74b	—	R Triang. Austr.	8 37	—66 2.1	6.6	8.0	3.4
75	—	U Coronæ	13 6	+32 6.4	7.6	8.8	3.5
76	—	S Libræ	14 13	—19 56.1	8.0	12.5?	—
77	—	S Serpentis	15 48	+14 45.9	7.6 — 8.6	12.5?	361.0

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1886.
42	II.	Hind	1848	11 P. 32 Sk.
42*a*	—	Gould	1874?	—
43	II.?	Hind	1855	18 B_1. 26 B_2. 3 Eq. 21 K. 9 P. 45 Sk.
43*a*	II.	Pickering	1881	—
44	II.	Schmidt	1829	1 Ee.
45	II.	Auwers	1870	2 Eq. 12 P. 20 Sk.
46	II.	Chacornac	1853	12 Hn. 5 Sk.
47	V.	Hind	1848	8 Ee. 64 Hn.
48	II.	Hind	1848	6 Sk.
49	II.	Hind	1850	2 En. 2 Eq. 11 P. 19 Sk.
50	II.	Hind	1851	—
50*a*	II.	Gould	1871	—
51	II.	Schönfeld	1863	1 B_1. 2 M. 22 P. 16 Sk.
52	II.	Koch	1782	11 B_1. 2 M. 1 En. 22 Sk. 2 Sn.
52*a*	—	Gould	1871	—
52*b*	II.	Becker	1882	—
52*c*	—	Gould	1872	—
52*d*	—	Gould	1871	—
52*e*	—	Peters	1876	1 Eq. 1 M.
52*f*	—	Gould	1871	8 D. 9 Zr.
53	II.	Pogson	1853	2 B_1. 20 B_2. 5 Eq. 12 K. 4 M. 21 Sk.
54	II.?	Burchell	1827	—
54*a*	—	Thome	1872	—
55	II.	Winnecke	1861	1 B_1. 1 Hn. 2 M. 17 Sk.
56	II.	Chacornac	1856	—
57	II.	Peters	1865	2 M. 5 Sk.
58	II.	Peters	1871	1 Ee. 1 Eq. 2 M.
59	II.	Schönfeld	1856	1 Eq. 1 K. 1 M. 13 P. 1 Sk.
60	II.	Boguslawski	1849	12 Sk.
61	II.	Karlinski	1867	3 Eq. 2 Sk.
61*a*	II.	Henry	—	—
62	II.	Hencke	1856	6 B_1. 27 B_2. 22 Sk.
63	II.	Harding	1809	1 Eq. 10 G.
63*a*	IV.	Gould	1871	—
64	II.	Pogson	1853	22 B_2. 12 K. 8 Sk.
65	II.	Harding	1831	1 Ee. 16 Sk.
66	II.?	Schönfeld	1866	—
67	II.	Goldschmidt	1857	8 P. 3 Sk.
68	II.	Maraldi	1704	9 Sk.
69	II.	Hind	1852	1 Eq. 1 K. 1 Sk.
69*a*	II.	Palisa	1880	1 Eq. 8 P.
69*b*	—	Gould	1871	—
70	I.?	Baxendell	1860	3 M. 22 Sk.
71	II.	At Bonn	1860	3 B_1. 24 B_2. 5 Ee. 20 Hn. 9 P. 10 Sk.
72	II.	Hencke	1858	5 B_1. 29 Ee. 11 P. 86 Sk.
73	II.	At Bonn	1858	7 B_1. 1 Eq. 35 Sk.
73*a*	—	Schmidt	1867	1 G. 10 Zr.
73*b*	II.	Baxendell	1880	11 B_1. 2 Eq.
74	V.	Schmidt	1859	14 Zr.
74*a*	II.	Palisa	1878	1 Eq.
74*b*	IV.?	Gould	1871	—
75	V.	Winnecke	1869	15 Hn.
76	II.	Borelly	1872	1 Eq. 5 P. 5 Sk.
77	II.	Harding	1828	10 Ee. 11 Hn. 2 P. 4 Sk.

TABLE I. — *Continued.*

No.	H.P.	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
78	2553	S Coronæ	15 16 18	+31 49.1	6.1 — 7.8	11.9 — 12.5	361.0
78a	—	Libræ	34 46	—20 46.5	9	<14	—
79	2639	R Coronæ	43 25	+28 32.5	5.8	13.0	Irr.
80	2647	R Serpentis	44 56	+15 30.8	5.6 — 7.6	<11	357.6
80a	—	V Coronæ	45 4	+39 57.0	7.7	12	300.0
81	—	R Libræ	46 32	—15 51.7	9.2 — 10.0	<13	723
82	2678	T Coronæ	54 16	+26 16.5	2.0	9.5	—
83	—	R Herculis	16 0 37	+18 42.5	8.0 — 9.0	<13	319.0
83a	—	W Scorpii	4 28	—19 48.6	10	<13	224 8
84	—	T Scorpii	9 36	—22 39.9	7	<10	—
85	—	R Scorpii	10 12	—22 38.2	9 ? — 10.5	<12.5	223
86	—	S Scorpii	10 13	—22 35.2	9.1 — 10.5	<12 5	176.9
86a	—	Ophiuchi	14 40	— 7 24.0	9.0	<13.5	328
87	—	U Scorpii	15 16	—17 35.3	9 ?	<12	—
87a	—	Ophiuchi	19 46	—12 8.5	7.5	10.5	365
88	—	U Herculis	20 16	+19 10.8	6.6 — 7.7	11.4 — 11.6	406.3
89	2772	g Herculis	24 32	+42 9.6	5	6 2	Irr.
90	—	T Ophiuchi	26 35	—15 51.8	10	<12.5	—
91	—	S Ophiuchi	27 4	—16 53.7	8.3 — 9.0	<12.5	233.8
91a	—	W Herculis	30 48	+37 35.6	8.0	<14.5	289
91b	—	Ursæ min.	31 40	+72 31.9	8.6	10.5	180?
91c	—	R Draconis	32 22	+67 0.7	7.2	13<	245.9
92	2828	S Herculis	46 13	+15 9.2	5.9 — 6.8	11.5 — 12.2	303
93	2839	Ophiuchi	52 30	—12 42.0	5.5	12.5	—
93a	—	V Herculis	53 41	+35 15.5	9.0	11.7	—
94	—	R Ophiuchi	17 0 36	—15 55.5	7.6 — 8.1	<12	302.4
95	2879	α Herculis	8 57	+14 32.1	3.1	3.9	Irr.
95a	2883	U Ophiuchi	10 12	+ 1 21.0	6.1	6.8	0.9
96	2890	u Herculis	12 42	+33 14.1	4.6	5.4	38.5
97	—	Serpentarii	23 9	—21 22.4	>1	?	—
98	2972	X Sagittarii	39 41	—27 46.8	4	6	7.0
99	3035	W Sagittarii	57 2	—29 35.1	5	6.5	7.6
100	—	T Herculis	18 4 22	+31 0.1	7.2 — 8.3	11.4 — 12.1	165.1
101	—	T Serpentis	22 43	+ 6 13.1	9.1 — 10.0	<12.8	342.3
102	—	V Sagittarii	24 4	—18 20.9	7.5?	9.5 ?	—
103	—	U Sagittarii	24 32	—19 12.7	7.0	8.3	6.7
104	—	T Aquilæ	39 45	+ 8 36.9	8.8	9.5	Irr.
105	3176	R Scuti	40 49	— 5 50 2	4.7 — 5.7	6.0 — 8.5	71.1
105a	—	κ Pavonis	44 3	—67 23.2	4.0	5.5	9.1
106	3193	β Lyræ	45 28	+33 13.0	3.4	4.5	12.9
107	3224	R Lyræ	51 32	+43 47.1	4 3	4.6	46.0
108	—	S Coron. Aust.	52 43	—37 7 2	9.8	11 5?	6.1
109	—	R Coron. Aust.	53 29	—37 7.2	10.5 — 11.5	<12.5	31
110	—	R Aquilæ	19 0 21	+ 8 2.6	6.4 — 7.4	10.9 — 11.2	345.1
111	—	T Sagittarii	9 1	—17 11.2	7 6 — 8.1	<11	381
112	—	R Sagittarii	9 21	—19 31 5	7.0 — 7.2	<12	270.0
113	—	S Sagittarii	12 7	—19 15.1	9.7 — 10.4	<12.7	230
114	3395	R Cygni	33 28	+49 55.1	5.9 — 8.0	13	425.3
115	—	11 Vulpeculæ	42 26	+27 0.5	3	?	—
116	—	S Vulpeculæ	43 16	+26 58.7	8.4 — 8.2	9.0 — 9.5	67.5
117	3434	χ Cygni	45 46	+32 36.0	4.0 — 6.0	12.8	406.5
118	3436	η Aquilæ	46 6	+ 0 41.2	3.5	4.7	7.2
119	—	S Cygni	20 2 53	+57 37.6	8.8 — 9.5	<13	322.8
120	—	R Capricorni	4 17	—14 38.2	8.8 — 9.7	<13	347

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1886.
78	II.	Hencke	1860	12 B_1. 1 Eq. 1 G. 12 Hn. 6 K. 11 P. 27 Sk.
78a	—	Peters	1878	8 P.
79	II.?	Pigott	1795	1 B_1. 1 En. 2 Eq. 3 G. 13 Ee. 15 P. 23 Sk.
80	II.	Harding	1826	1 Eq. 6 Ee. 9 P.
80a	II.	Dunér	1878	20 D. 2 Eq. 7 Ee. 13 P. 26 Sk.
81	II.	Pogson	1858	2 K.
82	I.	Birmingham	1866	17 B. 1 Eq. 8 Hn. 1 K. 3 M. 11 Sk.
83	II.	At Bonn	1855	9 Ee. 1 Eq. 7 Hn. 9 P. 11 Sk.
83a	II.	J. Palisa	1877	8 P.
84	I.	Auwers	1860	3 M. 8 P.
85	II.	Chacornac	1858	8 K. 8 P.
86	II.	Chacornac	1854	1 K. 8 P.
86a	II.	Schönfeld	1881	2 Eq.
87	I.?	Pogson	1868	—
87a	—	Dunér	1881	7 D. 1 Eq.
88	II.	Hencke	1860	5 B_1. 1 Eq. 34 Sk.
89	III.	Baxendell	1857	10 Sk.
90	II.	Pogson	1860	3 Eq.
91	II.	Pogson	1854	3 Eq.
91a	—	Dunér	1880	14 D. 13 Ee. 1 Eq. 6 Hn. 11 P.
91b	II.	Pickering	1881	5 B_1. 22 Ee. 1 K. 2 P. 69 Sk.
91c	II.	Geelmuyden	1876	7 B_1. 26 Sk.
92	II.	At Bonn	1856	7 B_1. 1 Eq. 6 Hn. 11 P.
93	I.	Hind	1848	6 M.
93a	II.	Baxendell	1880	8 Ee. 2 Eq. 10 P.
94	II.	Pogson	1853	1 Eq.
95	III.	W. Herschel	1795	14 Zr.
95a	V.	Sawyer	1881	—
96	III.	Schmidt	1869?	10 Zr.
97	I.	Fabricius	1604	4 M.
98	IV.	Schmidt	1866	—
99	IV.	Schmidt	1866	—
100	II.	At Bonn	1857	11 B_1. 6 Hn. 3 P. 20 Sk.
101	II.	Baxendell	1860	3 B_1. 5 Ee. 2 Eq. 12 Sk.
102	II.	Quirling	1865	2 Eq.
103	IV.	Schmidt	1866	2 Eq.
104	II.	Winnecke	1860	1 Eq. 1 M.
105	II.	Pigott	1795	5 G. 2 M.
105a	IV.	Thome	1872	—
106	IV.	Goodricke	1784	16 G. 19 Zr.
107	II.?	Baxendell	1856	—
108	IV ?	Schmidt	1866	2 Eq.
109	II.?	Schmidt	1866	2 Eq.
110	II.	At Bonn	1856	8 Ee. 2 Eq. 6 G. 39 Sk.
111	II.	Pogson	1863	2 Eq. 4 P. 17 Sk.
112	II.	Pogson	1858	2 Eq. 8 P. 10 Sk.
113	II.	Pogson	1860	2 Eq. 8 P. 9 Sk.
114	II.	Pogson	1852	17 B_1. 4 Hn. 1 K. 15 P. 13 Sk.
115	I.	Anthelm	1670	—
116	II.	Hind	1861	10 B_1. 4 Hn.
117	II.	Kirch	1686	13 B_1. 3 G. 15 P. 6 Sk.
118	IV.	Pigott	1784	3 G. 9 Zr.
119	II.	At Bonn	1860	4 B_1. 2 Eq. 8 K. 19 P. 20 Sk.
120	II.	Hind	1848	2 Eq. 4 P.

TABLE I. — *Continued.*

No.	H P	Name.	R. A. 1875.	Dec. 1875.	Max.	Min.	Per.
			h. m. s.	° ′	m.	m.	d.
121	—	S Aquilæ	20 5 52	+15 14.9	8.9 — 9.0	10.7 — 11.8	147.2
122	—	R Sagittæ	6 22	+16 21.0	8.5 — 8.7	9.8 — 10.4	70.4
123	—	R Delphini	8 53	+ 8 42.7	7.6 — 8.5	12.8	284.0
124	3547	P Cygni	13 11	+37 38.7	3 — 5	<6	—
125	—	U Cygni	15 44	+47 30.1	7.8 ?	9.8 ?	—
126	3557	R Cephei	19 52	+88 45.0	5 ?	10 ?	—
126a	—	— Cygni	37 17	+47 41.8	8	12	423
127	—	S Delphini	37 19	+16 38.4	8.4 — 8.6	10.4 — 11.1	275.6
128	—	T Delphini	39 34	+15 56.7	8.2 — 8.9	<13	331.4
129	—	U Capricorni	41 11	—15 14.4	10.2 — 10.8	<13	203.5
130	3654	T Cygni	42 12	+33 55.0	5.5 ?	6 ?	—
131	—	T Aquarii	43 20	— 5 36.5	6.7 — 7.0	12.4 — 12.7	203.3
132	—	R Vulpeculæ	58 40	+23 19.5	7.5 — 8.5	12.5 — 13.0	137.5
132a	—	Capricorni	21 0 19	—24 25.5	9½	14	—
132b	—	T Cephei	7 52	+67 58.9	5.6	9.6	382
133	—	T Capricorni	15 6	—15 41.4	8.9 — 9.7	<13	269.4
134	—	S Cephei	36 46	+78 3.6	7.4 — 8.5	11.5	485
134a	—	Nova Cygni	37 2	+42 18.2			—
135	3845	μ Cephei	39 41	+58 12.4	4 ?	5 ?	Irr.
136	—	T Pegasi	22 2 48	+11 55.7	8.8 — 9.8	<12.5	367.5
137	3981	δ Cephei	24 32	+57 46.6	3.7	4.9	5.4
137a	—	Lacertæ	37 48	+41 43.0	8.6	<13.5	315
138	—	S Aquarii	50 25	—21 0.6	7.7 — 9.1	<11.5	279.4
139	4078	β Pegasi	57 45	+27 24.2	2.2	2.7	Irr.
140	—	R Pegasi	23 0 22	+ 9 52.1	6.9 — 7.7	12 ?	382.0
141	—	S Pegasi	14 14	+ 8 14.2	7.6	<12.2	—
142	4193	R Aquarii	37 21	—15 58.7	5.8 — 8.5	11 ?	388.0
143	4234	R Cassiopeiæ	52 4	+50 41.5	4.8 — 6.8	<12	425.9

REMARKS.

27. Spectrum of type III. En.

27a. Max. 1886, March 7; magn. 9.5. In middle of May, magn. 10.5; October 27, magn. 8.7. Variation irregular, or law very complicated. D.

43. Max. 1886, December 1. K.

63. Max. 1886, April 9. G

64. Max. 1886, August 10; magn. 7.7. Min. 1886, Dec. 2; magn. 13.1. K.

80a. Period 357^d.03. Magn. at max. 7.2; at min. 10.3. D.

85. Max. 1886, July 28; magn. 9.8. K.

87a. Period 300 days (uncertain). Magn. at max. 7.1; at min. 9.6. D.

TABLE I. — *Continued.*

No.	Class.	Discoverer.	Date.	Observations, 1886.
121	II.	Baxendell	1863	29 B_1. 18 Ee. 19 K.
122	II.?	Baxendell	1859	29 B_1. 24 Ee. 3 Hn.
123	II.	Hencke	1859	2 Eq. 2 Hn. 1 P. 34 Sk.
124	I.	Janson	1600	11 Sk. 11 Zr.
125	II.	Knott	1871	16 B_1. 17 Ee. 14 En. 1 Eq. 17 G. 11 K [39 Sk
126	II.?	Pogson	1856	6 B_2. 16 Ee. 13 Sk.
126a	II.	Birmingham	1881	18 B_1. 5 En. 9 K. 12 P. 37 Sk.
127	II.	Baxendell	1860	15 B_1. 22 Ee. 2 Eq. 10 P. 6 Sk.
128	II.	Baxendell	1863	13 B_1. 10 Ee. 1 Eq. 10 K. 7 P. 12 Sk.
129	II.	Pogson	1858	2 Eq. 5 P.
130	—	Schmidt	1864	—
131	II.	Goldschmidt	1861	2 Eq. 19 P.
132	II.	At Bonn	1858	12 Ee. 1 Eq. 15 K. 1 M. 4 P.
132a	—	Peters	1867	5 P.
132b	II.?	Ceraski	1878	17 Ee. 2 Eq. 21 K. 3 M. 52 Sk.
133	II.	Hind	1854	2 Eq. 4 P.
134	II.	Hencke	1858	20 Ee. 3 M. 56 Sk.
134a	I.	Schmidt	1876	4 M.
135	III.?	Hind	1848	70 G. 8 M. 31 Sk. 28 Zr.
136	II.	Hind	1863	3 M. 15 P.
137	IV.	Goodricke	1784	11 G. 28 Zr.
137a	—	Deichmüller	1883	2 Eq. 5 M. 9 P.
138	II.	Argelander	1853	2 Eq. 4 M. 7 P. 6 Sk.
139	III.	Schmidt	1847	14 M.
140	II.	Hind	1848	1 Ee. 1 Eq. 12 M.
141	II.	Marth	1864?	1 Eq. 11 M.
142	II.	Harding	1811	2 Eq. 3 M. 6 P. 15 Sk.
143	II.	Pogson	1853	6 B_1. 8 D. 7 Ee. 7 G. 10 M. 1 P. 31 Sk

91a. Period 281^d.2. Magn. at max. 8.0; at min. 11.5. D.

91b. 1886, December 1, magn. 9.1; ruddy. K.

121. Max. 1886, October 4?, magn. 9.2. Min. 1886, July 7?, magn. 11.[illegible] also December 2; magn. 10.75.

125. Max. 1886, September 19. G. — Max. 1886, August 17±; magn. 7.9. [illegible]

132. Min. 1886, July 5; magn. 13.6. K.

132b. Max. 1886, March 26±; magn. 6.2. Min. 1886, October 16; mag[illegible] 9.6. K.

135. No well marked max. in 1886. Near min. 1886, August 22 to 28, Se[illegible]tember 16 to 30, October 1 and 16, and November 1. G.

TABLE II. — ADDITIONAL STARS.

No.	R. A. 1875.	Dec. 1875.	Observations, 1886.	Rem.
	h. m.	° ′		
—	0 35.9	+40 37	4 En. 1 P.	
9	37.8	+ 6 37	14 P.	
—	1 4.4	+ 8 58	1 G.	
21	6.2	+80 53	4 M.	
23	7.7	+34 57	3 M.	
25	15.0	+ 9 2	6 P.	
—	16.4	+ 6 45	3 M.	
31	19.5	— 4 37	5 M.	
37	25.7	— 7 22	4 M.	
—	28.2	+11 55	11 G. 2 M.	A.
43	33.5	—29 40	4 M.	
45	39.1	+ 7 56	5 M.	
47	47.8	+ 8 10	2 B. 1 G. 5 M.	
—	48.9	+22 58	6 G.	
57	2 0.8	— 9 11	4 M.	
59	10.4	+58 22	4 M.	
61	15.2	+54 47	1 G. 4 M.	
63	19.0	+ 9 56	4 M.	
—	19.7	+81 5	6 G.	B.
—	31.9	+49 1	1 G.	
—	52.5	+80 59	7 G.	C.
73	3 37.6	+ 9 0	3 P.	
77	41.8	— 0 17	2 M.	
83	46.9	— 0 52	1 M.	
87	57.8	+23 38	3 P.	
93	4 14.6	+19 31	25 Sk.	
117	5 1.4	— 8 49	4 M.	
123	5.3	+ 0 22	2 M.	
—	5.6	—12 0	1 G.	
—	5.9	—12 2	5 En. 5 Sn.	
129	6.6	— 0 15	3 M.	
135	23.0	— 0 9	4 M.	
—	23.4	— 1 12	5 G.	D.
139	23.8	— 1 8	8 P.	
143	27.4	+21 52	3 M. 4 P.	
145	28.3	+10 10	9 G.	E.
151	29.3	— 3 20	2 M.	
—	35.8	+ 2 18	2 G.	
—	42.5	+37 16	1 G.	
—	48.4	+20 9	14 B_1. 16 B_2. 19 D. 26 Ee. 36 G. 5 Hn. 21 M. 39 P. 22 Sn.	F.
—	56.1	+42 59	8 G.	G.
159	6 10.6	+ 5 8	4 M.	
161	11.2	— 1 32	4 M.	
165	27.9	—27 51	1 G. 1 M.	H.
—	45.1	—27 11	1 G.	I.
—	7 22.0	—11 18	8 G.	J.
177	23.0	— 1 39	3 M.	
179	24.8	— 9 31	8 M.	
—	35.0	+ 3 55	2 G.	
199	55.0	—12 32	3 M.	
205	8 2.4	+19 48	5 P.	
—	48.8	+17 42	1 G.	
—	9 1.2	+15 18	8 M.	

TABLE II. — *Continued.*

No.	R. A. 1875.	Dec. 1875.	Observations, 1886.	Rem.
	h. m.	° ′		
—	9 14.6	+14 52	3 M.	
229	20.1	+14 50	3 M.	
—	21.4	— 8 7	1 G.	
248	30.4	+15 48	8 M.	
—	40.8	+ 7 13	6 G. 8 M.	K.
—	53.2	+ 3 59	2 G.	
—	10 5.3	+18 13	4 M.	
—	11.6	+18 21	3 M.	
—	31.4	—12 30	6 En.	
293	45.5	—20 35	4 M.	
294	47.2	+14 23	11 P.	
303	11 10.0	— 3 22	3 M.	
—	54.5	—18 58	19 Sn.	
311	12 7.5	+ 0 17	4 M.	
315	10.7	+80 49	5 M.	
—	14.1	—21 31	18 Sn.	
—	18.8	+ 1 28	1 G. 2 Sn.	
—	18.8	—10 55	1 G.	
327	24.0	+ 5 6	1 M.	
331	26.9	—19 56	1 M.	
—	32.0	+ 2 38	1 G.	
337	32.7	+17 12	4 M.	
345	37.0	—13 10	1 G.	
—	40.7	+ 6 38	3 G.	
348	44.6	+82 23	1 M.	
361	13 24.0	— 8 55	1 M.	
375	47.8	+11 41	3 M.	
381	56.4	— 1 47	3 M.	
383	58.2	— 8 36	11 P.	
—	14 8.1	+13 38	17 Sn.	
—	8.7	+10 41	18 Sn.	
—	13.3	+ 0 58	2 G.	
—	15.5	— 1 25	8 B.	
—	18.3	+16 53	8 B_1.	
—	24.7	+39 25	22 D. 4 G.	L.
—	29.5	+37 11	1 G.	
407	42.7	+ 6 29	14 Hn.	
—	43.6	+ 8 30	2 G.	
413	45.4	—11 49	3 M.	
429	15 10.7	— 3 43	3 M.	
—	13.0	+27 18	9 G.	M.
437	29.0	—20 45	9 P.	
—	30.7	+15 31	4 G.	
441	30.8	—15 46	5 P.	
—	33.5	+36 39	3 G.	
447	36.5	—10 31	3 M.	
451	39.2	—20 44	8 P.	
—	49.4	—17 57	3 M.	
459	16 1.2	—21 11	9 P.	
—	3.8	+ 1 9	1 G.	
465	9.1	+11 50	3 M.	
471	22.4	—19 14	3 P.	
479	31.7	+72 32	4 M.	
483	44.7	— 5 58	3 M.	

TABLE II. — *Continued.*

No.	R. A. 1875.	Dec. 1875.	Observations, 1886.	R
	h. m.	° ′		
491	16 53.2	— 4 2	8 M.	
503	17 37.6	—18 36	3 M.	
509	18 2.7	+28 44	11 B. 27 Hn.	
517	28.0	+36 54	3 M.	
—	32.4	+ 8 43	17 En. 3 M.	
521	43.1	— 8 3	3 M.	
—	50.0	+ 4 3	9 Zr.	
529	52.8	+14 12	2 M.	
—	55.3	+31 58	1 Sn.	
535	57.7	— 5 52	8 M.	
—	19 3.4	+23 59	8 G.	
—	7.6	+ 5 18	8 G.	
—	9.3	+56 39	7 G.	
—	16.1	+17 25	12 En.	
545	23.9	+ 2 39	2 G.	
549	27.1	+17 28	11 Ee. 3 M.	
—	28.4	—25 0	2 Sn.	
—	32.3	—23 42	8 Sn.	
—	32.6	—23 43	8 Sn.	
555	35.3	+12 53	11 Ee. 3 G. 3 M.	
—	50.3	+16 17	55 En. 71 G. 3 M. 39 Sn.	
—	55.0	—28 3	3 Sn.	
—	20 5.7	+47 29	19 En. 7 M.	
—	6.1	+44 39	- Hn.	
567	7.8	—22 21	9 P.	
—	10.7	+44 39	- Hn.	
—	12 4	+39 59	15 G.	
—	21.7	—18 13	3 M.	
—	24.3	+39 34	15 En. 3 M.	
—	39.7	+17 38	35 G.	
—	41.6	+ 5 32	2 G.	
—	46.1	+27 47	49 G. 55 Sn.	
—	49.2	+27 35	5 Sn.	
—	21 0.2	—16 15	3 M.	
601	1.4	—21 51	8 P.	
—	9.6	+59 36	8 G.	
—	22.4	+55 58	- Hn	
—	31.3	+44 49	77 G.	
—	36.8	+34 56	6 G.	
—	40.0	— 2 47	3 En. 1 G.	
613	45.2	+ 6 4	2 M.	
615	56.5	—17 14	7 P.	
—	22 31.7	+57 47	18 M.	
—	47.5	+ 1 11	1 G.	
631	23 1.7	— 7 1	3 M.	
635	14.8	+55 26	4 M.	
645	33.3	— 1 26	3 M.	
—	39.9	+ 2 47	6 En. 4 G.	
651	51.5	— 9 39	12 P.	
653	54.9	+59 40	9 M.	
—	58.1	—11 12	2 G.	

REMARKS.

A. Only slight variation observed. G.
B. Below magn. 8 in all observations. G.
C. Variation of about half a magnitude observed. G.
D. Only slight variation. G.
E. T *Orionis*. Only small variation observed. G.
F. U *Orionis*. Discovered by Gore in 1885.
G. Variation of three or four steps observed. G.
H. Invisible with binocular, March 5, 1886. G.
I. Estimated magn. 7.4, March 5, 1886. G.
J. No variation detected. G.
K. Below magn. 8 in March and April, 1886. It lies south following DM. +7° 2181. G.
L. V *Bootis*. Period $266^d.5$. Magn. at max. 6.7; at min. 9.5. D.
M. Certainly variable to some extent. G.
N. Only slight variation observed. G.
O. Slight variation. G.
P. Magn. 7.5 or 7.6 September 3, 6, and 22. G.
Q. S (10) *Sagittæ*. Maxima, 1886, July 19; August 30; September 7, 15; October 1, 10, 27; November 4, 30; December 16, 25. Minima, June 12; August 8; October 7, 24; November 1, 10; December 4. G.
R. No variation observed. G.
S. Practically invariable. Spectrum of type IV. En.
T. Min. August 29±. G.
U. T *Vulpeculæ*. Near max. 1886, September 14, 27; October 10; November 10, 15; December 16, 24. Min. 1886, August 29. G.
V. Estimated magn. 7.3, 1886, September 2 and 8. G.
W. W *Cygni*. Max. 1886, May 19, August 28±. Min. 1886, Feb. 14±. G.
X. Only slight variation observed. G.
Y. Nearly equal to κ on September 15. En. Near max. September 15. G

Professor Safarik has made 12 observations of the minor planet Vesta, in continuation of his interesting researches upon the light of asteroids, mentioned at the close of the report made last year.

FORTY-SECOND

ANNUAL REPORT

OF THE

DIRECTOR

OF

THE ASTRONOMICAL OBSERVATORY

OF

HARVARD COLLEGE.

BY

EDWARD C. PICKERING.

PRESENTED TO THE VISITING COMMITTEE DECEMBER 2, 1887.

CAMBRIDGE, MASS.
PUBLISHED BY THE UNIVERSITY.
1887.

REPORT.

To the President of the University:

Sir, — The work of the Observatory has been largely increased during the past year by means of three important accessions to its resources. The plans for the study of stellar spectra conducted as a memorial to the late Dr. Henry Draper have been greatly enlarged by Mrs. Draper's continued liberality. The fund left by the late Uriah A. Boyden for the establishment of a mountain observatory has been transferred by its Trustees to the President and Fellows of Harvard College, and the researches undertaken by its means are now directed at this Observatory. Finally, the income of the large bequest of the late Robert Treat Paine has been available for the support of Observatory work during the entire year. Each of the sources of income above enumerated is greater than that often provided for the entire maintenance of an independent observatory, and, in comparison with what is ordinarily attainable, this institution can no longer be regarded as inadequately endowed. It would be a serious mistake, however, to infer from this circumstance that the want of additional funds with which important and, from the scientific point of view, urgently required researches may be supported has ceased to be felt. All endowments require increase as time passes, since the rate of income from invested property continually tends to diminish in countries where wealth is increasing. But apart from this consideration, the new resources of the Observatory are already so much absorbed in the work which has been undertaken with their aid that it is extremely difficult to spare any part of them for the improvement of the building or for the publication of the results obtained by observation. Yet, as has been remarked on former occasions, attention to these wants of the Observatory can hardly be delayed. It is probable that there has never been a time in the history of the institution when so large a return could be obtained for a given additional expenditure of money as would now result from the command of means sufficient for the desired improvements. The principal building, as was stated in the last report, is inadequate to the requirements of modern astronomy and to the present needs of the Observatory. A temporary remedy for the want of space which is now seriously felt would be to occupy the east wing of the building, now used as the residence of the Director, for scientific purposes. In

this case it would be necessary to build a new dwelling-hous the Observatory. This addition is also desirable on the groun the east wing of the Observatory is ill adapted for use as a dw

In order to prevent an undue increase in the length of these r it will be necessary to describe the progress of the work Observatory more concisely than heretofore. It will be given three principal headings, the Observatory Instruments, the Memorial, and the Boyden Fund.

The Observatory is indebted to Professor William A. Rog the contribution of a portion of the expense of constructing parator made under his supervision, and also for a careful st the errors of a thermometer belonging to the Observatory.

Observatory Instruments.

East Equatorial. — This instrument remains in good conditi still in great want of facilities for its more convenient use, s are now commonly provided for telescopes of its class. Its pr work has been the continuation of the observations on ecli Jupiter's satellites and on the comparison stars for variables. weather proved unusually unfavorable to the first of these rese Out of 68 eclipses which might have been observed if the weath permitted, only 16 could be observed fairly well; 7 more we served imperfectly through clouds, so that the total number is the year, and 381 in all. The observations of comparison st variables were nearly complete, so far as regards the list o objects originally compiled, when the series was terminated accidental breaking of the wedge. The wedge photometer no ployed in the continuation of the work and in other investigati that mentioned in the Fortieth Report as made in England f Observatory under the superintendence of Professor Pritchar the details of the apparatus have been somewhat modified sin instrument was received. It was investigated by Professors L and Young, as will be mentioned in describing the publications year, and after its return to this place the wedge was remove the remainder of the apparatus and mounted in the focal plane telescope in a manner proposed by Professor Searle. The sta observed is viewed between two bars placed at right angles wedge and rigidly connected to the index of the scale, but m in the field for convenience of adjustment. In this form the ment gains in facility of use, and the inaccuracy to be appre from a larger field of view and the occasional presence of other or general brightness of the sky, does not seem to be importa is probably counterbalanced by the additional security that the

beam of light from the eyepiece is properly received by the eye when the field is visible. From observations made on 25 nights the average deviation of one observation from the mean of a set of six, expressed in magnitudes, is 0.19; if one night is excluded, on which the observations were thought to be affected by variable haze, the result is 0.15. The average deviation of the result for a single star upon one night from the mean of three nights is 0.09. It is proposed to employ the instrument in the observation of zones of DM. stars, and in the investigation of the phases of asteroids.

Occasional observations of comets have been made by Mr. Wendell as in former years. Comet 1886 VII. was observed on one night; Comet 1886 VIII. on one night; Comet 1886 IX. on one night; Comet 1887 II. on two nights; Comet 1887 III. on four nights; Comet 1887 IV. on thirteen nights. The comet recently discovered by Brooks (a return of Olbers' Comet) has been observed on four nights. At the request of Mr. R. Bryant the asteroid (80) Sappho was observed by Professor Searle on seven nights.

Meridian Circle. — The only observations made with this instrument at present are those needed for the determination of clock error, since the accumulation of unpublished results obtained with it still demands the use of the resources applicable to this department of work. But at the request of Dr. Auwers this Observatory has agreed to undertake the observation of one of the zones required in the proposed revision of the Southern Durchmusterung. These observations will be begun with the meridian circle as soon as possible after the definite assignment of the zones to be revised, which has not yet been made by the Astronomische Gesellschaft.

The reduction of the observations still unpublished has been actively continued throughout the year, under the superintendence of Professor William A. Rogers, according to the arrangement mentioned in the last report. The preparation of the catalogue of zone stars between the declinations $+50°$ and $+55°$ is approaching completion. The results of the observations of these stars, including those observed in the revision made in 1883–85, have been reduced to the epoch 1875.0. The preliminary computation of the precession and secular variation has been completed, with the aid of tables previously prepared for the purpose. From twenty-eight published catalogues, 13,469 observations of stars in the zone have been brought forward to 1875.0, and the work of comparing these places with the zone observations is far advanced.

Meridian Photometer. — The work of observing the zones at intervals of five degrees undertaken with this instrument in the region

covered by the Southern Durchmusterung is about half con
and will probably be finished within a year. The similar obser
in the northern hemisphere are now substantially completed
ditional observations of groups of stars, also observed by Dr.
mann of the Pulkowa Observatory, have likewise been mad
many stars have been observed at the request of Mr. H. M.
hurst, to enable him to connect his photometric observations w
system of magnitudes here adopted. Similar observations wo
gladly undertaken to assist the work of other astronomers wh
desire them. The observations of the year have been made, as
by Mr. Wendell and myself. They consist of 156 series, con
in all 47,476 settings. The average deviation, expressed in t
stellar magnitude, for stars observed at the upper culminat
0.117.

Eclipse Observations. — The policy of the Observatory is ge
to undertake large pieces of routine work in which some v
result is certain to be attained rather than to make consi
expenditure for eclipses or other accidental phenomena in
failure could result from clouds.

The total eclipse of the sun on August 19, 1887, gave an u
opportunity to study the solar corona at two points widely sep
and thus indicate its fixity or motion during an interval of two
A plan was accordingly prepared by Mr. W. H. Pickering to t
question. By the courtesy of Professor C. A. Young, photog
apparatus was sent to Russia under his charge, and Professo
comb kindly authorized Professor Todd to have similar obser
made in Japan. The plates were prepared here, tested
exposure of a small part of each to a standard light, and were t
been developed and discussed by Mr. W. H. Pickering. Unfortu
clouds prevented a successful result. The total cost to th
servatory did not exceed three hundred dollars. Had the w
permitted, photographs would have been obtained at each
with apertures of ten inches, giving images of the sun nearly
in diameter. In Japan photographs would also have been ob
on plates seventeen inches square, in which the sun would
diameter of five inches without enlargement.

Henry Draper Memorial.

The progress of this work was detailed last spring in a
widely circulated and copied in various scientific journals.
various researches undertaken, the photographs of the spectra
brighter stars north of —25° are now completed. Each por
the heavens should appear on at least four plates, of which 63

have been made. Each plate covers a region 10° square, from two to five exposures of five minutes each being made on every plate. The total number of spectra contained in this work is 27,803. The greater portion of these have been measured, the character of the spectra indicated, and the corresponding stars identified. The places of these stars for 1900 have also been computed and the preparation of a catalogue will shortly be undertaken.

A beginning has also been made in the study of the photographic brightness of these spectra. As the effect of color is thus eliminated, this method, if successful, may supersede all photometric determinations and eye estimates of stellar magnitude. A second research will give to each plate an exposure of an hour, and thus include fainter stars. The entire sky north of — 20° would be covered twice by 655 plates, if no defective plates were taken. 192 plates have so far been measured, containing 9880 spectra, of which 9065 have already been identified. These photographs will be completed in one or certainly in two years from the present time. It is then proposed to send the instrument to the southern hemisphere and thus complete the work to the south pole. A third research is carried on with the 11-inch refractor formerly owned by Dr. Henry Draper. This instrument has been mounted in a wooden building surmounted by a dome of twenty feet in diameter. Four prisms, each nearly a foot square, cover the object-glass and give spectra about five inches long. All of the stars brighter than the third magnitude have been photographed with this apparatus. These photographs are then enlarged by the aid of a cylindrical lens so as to form images several inches in length, and of any desired width. Several hundred lines are visible in many of these spectra. For a more detailed study of the spectra of the fainter stars Mrs. Draper has sent to Cambridge the 15 and 28 inch reflectors constructed by Dr. Draper. They are to be mounted in a building recently erected to contain them. They will be employed in studying the spectra of the variable stars, of the banded stars, and of those stars in which the first investigation mentioned above indicates any peculiarity of spectrum. Besides an observer and corps of computers for this work, Mrs. Draper has furnished a second observer, who maintains the work during the latter part of the night until interrupted by daylight. The total number of photographs so far taken with the 8-inch telescope is 1792, with the 11-inch 732. The photographs are developed in a cottage recently rented by the Observatory and especially fitted up for photographic work. Various novelties are contained in it; for instance, an arrangement for exposing the plates automatically to a standard of light for exactly one second, the enlarging apparatus, and other appliances constructed especially for this work.

BOYDEN FUND.

A fund, now amounting to about $238,000, was left by Uriah A. Boyden for conducting astronomical observations a height as to be free, so far as possible, from the injurious eff to the atmosphere. Last spring the trustees of the fund conc transfer it to the President and Fellows of Harvard Colleg W. H. Pickering was appointed as assistant in charge of t and since then has devoted his time to devising, construct testing the new class of instruments required. An object twelve inches aperture was purchased and mounted on a support, so that it could be directed to the pole-star. Two in this object were formed by a double-image prism and their d measured under various atmospheric conditions. Photograph trail of the spectrum of the brighter stars also served to mea steadiness of the air in various localities. The clearness o was determined by similar observations near the horizon, and photographing the trail of stars near the north pole of the and near the horizon upon the same plate. Another test of t ness of the sky is found by photographing upon the same p sky near the sun, and at a distance from it, and also expo plate to a light of constant intensity.

A study of the transparency of the air to the violet rays w with a spectroscope whose lenses and prisms were made of The meteorological conditions of the atmosphere are exam self-recording instruments. Besides the usual forms of ther and barograph a form of sunshine recorder has been devise indicates on a single sheet all the morning hours during a n which the sun is shining. An instrument has also been con for recording the hours during which the sky is clear at ni photograph of the trail of the pole-star is taken every eveni small telescope, which is closed automatically every mornin twilight begins.

All of these instruments, except the pole-star recorde tried by Mr. Pickering, two assistants, and myself in the cle Colorado. They were mounted and carefully tested at an alt 14,000 ft. on Pike's Peak, at 11,000 ft. at Seven Lakes, and ft. at Colorado Springs. A comparison with the results a bridge near the sea level exhibits the effect of varying the of the place of observation to the extent of nearly three miles

Visits were also made to Mt. Lincoln, 14,300 ft., Mt. Bross, and various points reached by railroad at heights exceeding ft., to determine the availability of these points for a pe station.

Important aid was rendered in the study of the climate of Colorado by Professor F. H. Loud, who was enabled to aid officially in this work by the courtesy of the trustees of Colorado College. With his assistance stations have been established upon Mt. Lincoln, Mt. Bross, and at various lower points. By the coöperation of the U. S. Signal Service self-registering instruments are now in operation at Pike's Peak, Colorado Springs, and Denver. A large number of vertical and horizontal angles were obtained by means of a micrometer level, showing the positions of the mountains seen from various points in Colorado. It is expected that important additions will thus be made to our knowledge of their heights and locations. Photographs of the solar spectrum in Colorado permit prints to be made extending to the wave-length 292 and showing many lines not contained in any work hitherto published.

A telescope of thirteen inches aperture and fifteen feet focus, so constructed that it can be used for either visual or photographic purposes, is now nearly completed by the firm of Alvan Clark & Sons. A second telescope of eight inches aperture, eleven feet focus, will be mounted upon the same stand. This last lens is a doublet, and is intended to make charts of the stars 5° square, on the same scale as the charts of Peters and Chacornac.

A dome is now nearly completed on the Observatory grounds in which these instruments will be mounted and thoroughly tested. A second building, forty feet long, is also being erected for spectroscopic and photographic purposes. A large collection of books, maps, and photographs relating to mountains in all parts of the world has been made, and an extensive correspondence on the same subject has also been opened. Next summer a second expedition will be made to Colorado, and it is hoped that observations may be maintained there permanently in connection with this Observatory. Later it is purposed to test some insular climate like that of the Sandwich Islands, and plans are already proposed for observations in Peru, where two railroads reach the heights of 15,800 and 14,600 ft. respectively.

MISCELLANEOUS.

Variable Stars. — Messrs. Parkhurst, Eadie, Hagen, and Zaiser have continued their coöperation with this Observatory in collecting fresh material for the study of the variable stars. An interesting research undertaken during the year by Mr. Parkhurst relates to the variations of asteroids due to distance and phase, and possibly also to rotation or other causes as yet undetermined. It is expected that a preliminary series of these observations will soon appear as one of the sections of the eighteenth volume of Annals. Communications,

which have been recorded in a report upon the subject p
during the summer, were received from the following for
servers of variable stars: Mr. T. W. Backhouse, of Sun
England; Messrs. Joseph Baxendell and Joseph Baxendell
Southport, England; Dr. N. C. Dunér, of Lund, Sweden; Re
Espin, of Wolsingham, England; Mr. J. E. Gore, of Ball
Ireland; Mr. George Knott, of Cuckfield, England; P
Safarik, of Prague, Austria; Mr. T. S. H. Shearmen, of Br
Canada. It is proposed that the corresponding report, to be p
during the coming year, shall contain an index to all observa
variable stars, so far as their existence may be known,
published or not. All who can furnish information tending
this report more complete, especially with regard to the unp
observations, are requested to communicate it to this Observa

Time Service. — Two important improvements have bee
during the year in this department of the work of the Obse
A new mean time clock, Ballou No. 103, has been test
delivered to the Observatory according to contract. It h
used as the origin of the signals since June, 1886. The
amount of its error at the times of observing star transits h
$0^s.30$, the average interval between the times of these determ
being 2.1 days. The average error of the clock at 10 A.M. h
$0^s.26$, while the average change in the daily rate from each da
next has been $0^s.19$. The clock can obviously be so manag
diminish either of the amounts last named, if the other is all
increase. A more rapid compensation of the error, howeve
be inconvenient to the watchmakers who use the signals.
Observatory now possesses two mean time clocks, it will b
ticable to make use of one of them as a standard time-piece, w
other is employed as the origin of the signals. Moreover, whe
of the clocks requires repairs, the signals can be continued
other. The second improvement above mentioned consists
considerable additions made to the line of wire conveying the
The work has been planned so that it will tend to the
establishment of two Observatory lines of wire to Boston, w
tions interchangeable between switch boards accessible by tel

Telegraphic Announcements. — The telegraphic distribu
astronomical intelligence has been continued during the yea
the management of Mr. Ritchie. Announcements have been
the discovery of seven asteroids and six comets. There ha
sent to the European union of astronomers or received th
fifteen cable messages, and the number of telegrams sent
country has been 325.

Buildings and Grounds.—The need of a new building has been pointed out in the beginning of this report, and as additional space for computation and the preservation of records will continue to be required, it is expected that the inadequacy of the present building will constantly become more apparent. A temporary remedy has been sought in securing the use of two small houses on Madison Street. One of these belongs to the College and has been hired by the Observatory. It serves as a photographic laboratory and for other purposes, generally connected with the work of the Draper Memorial. The other house has been purchased, and will chiefly be serviceable in connection with the work of the Boyden Fund. In supplying the want of additional room for new instruments, it has appeared best to follow the modern practice of providing each with its own building. Two buildings surmounted by domes about twenty feet in diameter have been erected to accommodate the 11-inch refractor and the 28-inch reflector of Dr. Draper, and a similar building is in course of construction for the instrument to be employed in experimental work under the Boyden Fund.

Publications.—Volume XVII. of the Annals of the Observatory has been published during the year. It contains Mr. Chandler's treatise on the Almucantar, which explains the theory of this instrument and exhibits the results of the observations made with it by the author. Volume XVIII. will consist of comparatively short separate articles, which will be successively distributed as soon as published, in order to place them earlier in the hands of astronomers. Two numbers of the series have already appeared. The first, entitled "Magnitudes of Stars employed in various Nautical Almanacs," contains a list of the stars for which ephemerides are given in the almanacs published by government authority in the United States, England, France, Germany, and Spain, with the magnitude of each star as determined by the Harvard Photometry, the Uranometria Oxoniensis, Wolff's photometric observations, and the Uranometria Argentina. The almanacs of the United States, France, and Spain intend to employ these magnitudes hereafter, and the American Ephemeris for 1890, in which this change has been made, has already appeared. The second number of Volume XVIII. contains a discussion of the Uranometria Oxoniensis.

A treatise in quarto form, by Professors S. P. Langley, C. A. Young, and E. C. Pickering, was published in the Memoirs of the American Academy of Arts and Sciences. It describes observations undertaken to test the applicability to photometric uses of the wedge of tinted glass made for this Observatory in England, and already mentioned in describing the work of the East Equatorial.

The following list contains the titles of other publication
have appeared during the year, either as official communicatio
the Observatory or as papers prepared by its officers individu

Forty-first Annual Report of the Astronomical Observatory of
College.

Photographs of Stellar Spectra. By Edward C. Pickering.
xxxiv. 570.

New Form of Construction of Object-glasses intended for
Photography. By Edward C. Pickering. Ibid. xxxvi. 562.

Recommendations with respect to the proposed Photographic
the Sky. By Edward C. Pickering. Letter, translated into Fre
published at page 93 of the work entitled "Congrès Astrophotogr
International tenu à l'Observatoire de Paris pour le Levé de la
Ciel," Paris, 1887.

Observations of Variable Stars in 1886. By Edward C. Pi
Proc. Am. Acad. of Arts and Sciences, xxii. 380.

Boyden Fund. Circular, describing the objects of this fund. By
C. Pickering.

Boyden Fund. Circular No. 2. Meteorological Observations. By
C. Pickering.

Henry Draper Memorial. First Annual Report of the Photo
Study of Stellar Spectra conducted at the Harvard College Obse
Edward C. Pickering, Director. With Plate. Cambridge, 1887. R
in Nature, xxxvi. 31, and Memorie della Società degli Spettr
Italiani, xvi. 93.

Heights of the White Mountains. By Edward C. Pickering. App
iv. 305.

Observations of Comets (1886 VIII. and 1887 II.). By O. C. W
Astronomische Nachrichten, cxvi. 191.

Observations of Comet 1887 III. By O. C. Wendell. Ibid. c
Also, Astronomical Journal, vii. 79.

Orbit of Comet 1887 III. By O. C. Wendell. Astronomisch
richten, cxvi. 317. Also, Sidereal Messenger, vi. 161.

The Luminous Bands Visible in and near the Zodiac. By Arthur
Astronomische Nachrichten, cxvi. 323.

Observations of (80) Sappho. Communicated by Edward C. Pi
Ibid. cxvi. 373.

Orbit of Comet 1886 IX. By O. C. Wendell. Ibid. cxvii. 59.

Orbit of Comet 1887 IV. By O. C. Wendell. Ibid. cxvii. 119.

Magnitudes of Circumpolar Stars determined at the Observat
Moscow and of Harvard College. By Edward C. Pickering. Ibi
139.

Proposed Index to Observations of Variable Stars. By Edw
Pickering. Ibid. cxvii. 167. Also, Monthly Notices of the R. Astr.
xlvii. 443, and Observatory, x. 301.

Observations of Comet 1887 IV. By O. C. Wendell. Astron
Nachrichten, cxvii. 243. Also, Astronomical Journal, vii. 111 and

EDWARD C. PICKERING, *Dir*